Spiral Stability
The dispersion relation shows whether the disk is stable against the self-gravitation of the spiral wave.

Consider $m=0$ (a bar, limiting case)

$$\omega^2 = \kappa^2 - 2\pi G\Sigma|k| + k^2 {v_s}^2$$

The disk is stable for $\omega^2 > 0$ and unstable for $\omega^2 < 0$. The dividing line $\omega = 0$ is

$$0 = \kappa^2 - 2\pi G\Sigma|k| + k^2 {v_s}^2$$

Stability for all wavelengths k requires that this has no solution. This happens if (use quadratic formula)

$$Q = \frac{v_s \kappa}{\pi G \Sigma} > 1$$

For stellar disks, a similar result is obtained

$$Q = \frac{\sigma_R \kappa}{3.36 G\Sigma} > 1$$

For smaller Q, self-gravity of the spiral wave becomes more and more important.

The dispersion relation leads to a wave traveling at its group velocity,

$$v_g(R) = \frac{\partial\omega(k,R)}{\partial k} = sign(k)\frac{|k|{v_s}^2 - \pi G\Sigma}{\omega - m\Omega}$$

In the Galaxy, within co-rotation, a leading wave $(k<0)$ will travel outwards: the spiral arm will unwind.

A trailing wave will travel inward: the spiral arm winds up.

Thus, a leading spiral arm will eventually become a trailing arm, and quickly wind up. The time scale is $\sim 2\times 10^8\,yr$.

The analysis suggests that a spiral arm is *not* a stable phenomenon. This contradicts the observations. However, perhaps grand design spirals are special cases, and the less regular spirals are more typical.

Spiral arm theories

- The Lin-Shu quasi-steady density wave
- Chaotic density waves: fragments of spiral arms form and dissolve continuously.
- Tidal arms due to encounters between galaxies
- A bar can cause a spiral structure *but only in the gas*

There is probably not a unique explanation for spiral arms.