Neutrinoless double-beta decay

If we have fermions with spin-1/2, then they are either Dirac particles – the fermion and its antiparticle are not identical, $f \neq \bar{f}$, for example the electron and the positron – or Majarana particles – the fermion and its antiparticle are identical, $f = \bar{f}$. In the latter case, the particle should have no 'charges' (referring to all internal quantum numbers, or additive quantum numbers, e.g. charge, flavour.). This is because CPT is conserved in $f \rightarrow \bar{f}$.

In most fermions, because they have charge they must be Dirac particles. The Neutrino is a possible exception to this, however. It could be a Majorana particle (like the photon in the Boson set). This would immediately violate the Lepton flavour – with that, neutrinos have +1 lepton number, antineutrinos -1.

Why 'Majorana'? Named after the Italian Majorana (1912-1938).

We assume that $m_\nu = 0$.

Helicity $H = \pm 1$. This is the projection of the spin vector on the momentum vector (spin can be either parallel to the momentum vector, or antiparallel – right-handed and left-handed respectively), defined by $H = \frac{\underline{\sigma} \cdot \underline{p}}{|\underline{p}|}$. If $H = -1$, ν_- ; if $H = +1$, ν_+ .

Chirality is the same as the helicity if $m_\nu = 0$, however it is not if $m_\nu > 0$. Chirality gives ν_L and ν_R . It is different because it is possible to find a Lorentz frame where you can overtake the neutrino (assuming $m_\nu \neq 0$) and 'look back at it' and see that it has apparently changed spin – this is covered by Chirality, but not Helicity.

Under CPT, $\nu_- \rightarrow \bar{\nu}_+$ (the spin is axially symmetric, so doesn't change sign under parity, while the momentum is a normal vector, so does change sign under parity). Note that the T part does not do anything in this case.

Lorentz transformation (LT):
If $m_\nu > 0$, then a Lorentz transformation can flip the spin, by going to a reference frame where we 'overtake' the neutrino. This would give $\nu_- \rightarrow \nu_+$.

<u>Two cases:</u>
$\nu_+ = \bar{\nu}_+$ - Majorana particle
$\nu_+ \neq \bar{\nu}_+$ - Dirac particle
We can only distinguish the two for $m_\nu > 0$. In other words, in the standard model (for $m_\nu = 0$) we know that there is a left-handed doublet and right-handed singlet:

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}_L ; \; e_R .$$

We only have $\nu_{e,L}$ and $\bar{\nu}_{e,R}$, but no $\nu_{e,R}$ or $\bar{\nu}_{e,L}$ (C and P violation). There are two possibilities:

1. ν_R , $\bar{\nu}_L$ doesn't exist
2. They are 'sterile' – we can't see them.

[We're trying to identify two properties – the spin, and the particle/antiparticle. These features could be linked, so we'll never be able to get the mass difference between them.]

Dirac Particle

This is described by a Dirac spinor,

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

[4 elements, because of particle/antiparticle, spin-up/spin-down]. For $m_\nu = 0$, we can split this up into Weyl spinors:

$$\psi_L = \frac{1}{2}\begin{pmatrix} \psi_1 - \psi_3 \\ \psi_2 - \psi_4 \end{pmatrix}; \psi_R = \frac{1}{2}\begin{pmatrix} \psi_1 + \psi_3 \\ \psi_2 + \psi_4 \end{pmatrix}$$

We get this by applying a projection operator. All of these spinors obey the Dirac equation. (There is no difference for $m_\nu = 0$.)

See-Saw

The Lagrangian for a Dirac particle

$$L_D = -m_D \bar{\psi}\psi$$

where the ψ 's are the Dirac spinors.

In a Majorana model, we assume that we have a light left-handed neutrino ν_L, and a heavy right-handled neutrino N_R (sic).

	Majorana	Dirac
Light ν_L	$m_L \sim 0$	$m_D \sim 1GeV$ [Lepton/quark masses]
Heavy N_R	$m_R >> m_D$	

$$L_{DM} = -\frac{1}{2}\left(\bar{\nu}_L, \bar{N}_L{}^C\right)\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix}\begin{pmatrix} \nu_R{}^C \\ N_R \end{pmatrix}$$

where the superscript C represents charge conjugation. We now have a mass matrix, which we can diagonalise to get the eigenvalues of the masses. This gives an equation for the masses:

$$m_L = \frac{m_D{}^2}{m_R}$$

We say that the heavy neutrino N_R must be very heavy because we haven't seen it yet. We're saying that it's at the GUT scale, so $10^{15}GeV$. The See-Saw mechanism says that because m_R is unbelievably heavy, then m_L is unbelievably light. This is the main possibility for why the neutrinos are so light in the standard model.

[See powerpoint]

Difference between solar and reactor neutrinos: the latter is solely anti-neutrinos (from beta decay).

Beta decay: $u \to p + e^- + \bar{\nu}_e$ - look at the end-point of the decay. There would be a small difference at the end of the decay curve (see PPT), which can provide the mass of the neutrino. Ultimate sensitivity is $\sim 0.2eV$.

Neutrinoless double-beta decay. "Experimentum crucis"