Lecturer: Brian Cox

[There are handouts… hopefully available on the net?]

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}\begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}\begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

- There are 10^9 remnant ν / m^3 from the big bang
- There are $10^{15} m^{-3} s^{-1}$ from the Sun at the Earth's surface from the reaction $p + p \to D + e^+ + \nu_e$, $E_\nu = 0 - 0.42 MeV$,
  and $10^{11} m^{-2} s^{-1}$ from $^8B \to 2He + e^+ + \nu_e$, $E_\nu = 0 \to 14.6 MeV$
- There are e and μ neutrinos from cosmic rays

Detecting Neutrinos

The Sudbury Neutrino Observatory (SNO)

- $\nu_e + D \to e + p + p$
  D = Deuteron = $n + p$.
  "Charge current" – sensitive only to ν_e.
  Flux $\phi_e \approx 30$ / day (predicted)
- $\nu_e + e \to \nu_e + e$
  "Electron scattering", sensitive to $\phi_e + 0.15\phi_{\mu,\tau}$
  Flux ~ 3 per day (predicted)
- $\nu + d \to \nu + n + p$
  "Neutral current", sensitive to $\phi_e + \phi_{\mu,\tau}$
  Flux ~ 30 per day (predicted)

The weak interaction is weak. For a $100 GeV$ ν, the mean free path in iron is $3 \times 10^9 m$. But the cross section increases with energy. For $10^8 GeV$ ν, the earth is opaque.

The sun only produces ν_e. SNO measures $\phi_{\mu,\tau} = \left(3.41^{+0.66}_{-0.64}\right) \times 10^6 cm^{-2} s^{-1}$, coming from the sun, even though we know that there are none produced. So clearly there are ν_μ, ν_τ reaching the Earth from the sun.

The standard solar model (SSM) predicts: $\phi_{SM} = \left(5.05^{+1.01}_{-0.81}\right) \times 10^6 cm^{-2} s^{-1}$.

SNO measures from $\nu + d \to \nu + n + p$, $\phi_{e,\mu,\tau} = \left(5.09^{+0.64}_{-0.61}\right) \times 10^6 cm^{-2} s^{-1}$.

The total ν flux agrees with SSM, but $\dfrac{\phi_e}{\left(\phi_e + \phi_{\mu,\tau}\right)} \approx \dfrac{1}{3}$. i.e. more than half of the ν_e flux created in the solar core changes flavour on their way to Earth.

Super Kamiokande

Atmospheric ν are decay products of π (and K) mesons created in interactions of cosmic rays in the upper atmosphere.

$\pi^\pm \to \mu^\pm + \nu_\mu\left(\overline{\nu_\mu}\right)$; $\mu^\pm \to e^\pm + \overline{\nu_e}\left(\nu_e\right) + \nu_\mu\left(\overline{\nu_\mu}\right)$

So you expect $\frac{\nu_\mu}{\nu_e} \sim 2$. The key result:

$$\frac{\phi_\mu(up)(-1.0 < \cos\theta < -0.2)}{\phi_\mu(down)(+0.2 < \cos\theta < 1.0)} = 0.54 \pm 0.045$$

- Given that cosmic rays are isotropic, ν_μ must be disappearing on their way through the earth.
- Note also that they appear not to be turning into ν_e (ν_τ are very difficult to see).

<u>K2K</u>
Pure ν_μ produced at KEK (12GeV proton accelerator).
$L = 250km$, $E = 1.3GeV$

$$\sin^2\left(1.27\Delta^2_{matm}\left(ev^2\right)\frac{L(km)E(GeV)}{}\right) \sim \frac{1}{3}$$

Observed 108 events in SK (expected 150).
→ $\Delta m^2_{k2k} \sim 3\times10^{-3} eV$, $\left(\sin^2 2\theta\right)_{k2k} = 1.0$

KamLAND
~180km from reactor $\overline{\nu_e}$ sources.

$$\frac{\phi_{\overline{\nu_e}}}{\phi_{\overline{\nu_e}}(expected)} = 0.686 \pm 0.044(stat) \pm 0.045(syst)$$

Reactor $\overline{\nu_e}$ disappear!

Theory
Flavour eigenstates are not necessarily mass eigenstates. Neutrinos are produced in a state of definite flavour, but they propagate through space as states of finite mass (mass eigenstates).

$$|\nu_\alpha\rangle = \sum_i u_{\alpha i}{}^* |\nu_i\rangle$$

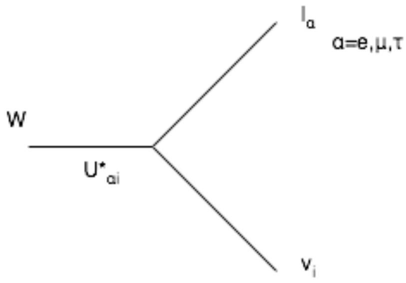
where ν_α is a neutrino of definite flavour, and ν_i is a neutrino of definite mass m_i. There are at least three neutrino states of definite, increasing mass –there could be more.

Inversely,

$$|\nu_i\rangle = \sum_\alpha u_{\alpha i} |\nu_\alpha\rangle$$

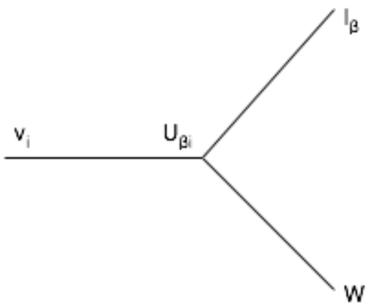
u is the leptonic mixing matrix. It is unitary (i.e. total number of ν is conserved).

Creation



Create a linear combination of ν_i such that they add up to flavour α

Detection

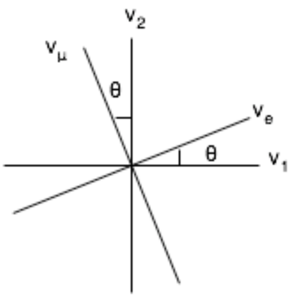


Only the ν_β component contributes:
An e is made by a ν_e
A μ is made by a ν_μ
A τ is made by a ν_τ

Flavour $\propto$ fraction of ν_i is $\left|\langle \nu_\alpha | \nu_i \rangle\right|^2 = |u_{\alpha i}|^2$.
For simplicity, consider 2 neutrino species.

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$



At creation $(t=0)$,

$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

At later time t (using $i\frac{d}{dt}|\nu_1\rangle = H_0|\nu_1\rangle = E_1|\nu_1\rangle$, where $\hbar = c = 1$),

$$\begin{aligned}|\psi\rangle &= -\sin\theta e^{-iE_1t}|\nu_1\rangle + \cos\theta e^{-iE_2t}|\nu_2\rangle \\ &= \left(\cos^2\theta e^{-iE_1t} + \sin^2\theta e^{-iE_2t}\right)|\nu_e\rangle + \sin\theta\cos\theta\left(e^{-iE_2t} - e^{-iE_1t}\right)|\nu_\mu\rangle\end{aligned}$$

through substitution for $|\nu_1\rangle$ and $|\nu_2\rangle$.

The probability to oscillate into $|\nu_e\rangle$ is

$$\begin{aligned}P_{osc} &= \left|\langle\nu_e|\psi_{(t)}\rangle\right|^2 \\ &= \frac{1}{2}\sin^2 2\theta\left[1-\cos(E_2-E_1)t\right]\end{aligned}$$

Use $E_1 = \sqrt{p^2+m_1^2} \approx p + \frac{m_1^2}{2p}$ and $\frac{t}{p} = \frac{tc}{pc} = \frac{L}{E}$

$$\begin{aligned}P &\approx \frac{1}{2}\sin^2 2\theta\left(1-\cos\left(\frac{(m_2^2-m_1^2)L}{E}\right)\right) \\ &= \sin^2 2\theta\sin^2\left(1.27\Delta m^2\frac{L}{E}\right)\end{aligned}$$

where the constant 1.27 comes from recovering from $c = \hbar = 1$.

The Vacuum Oscillation Formula

$$P = \sin^2 2\theta\sin^2\left(1.27\Delta m^2\frac{L}{E}\right)$$

$\frac{L}{E}$ is the time elapsed in the neutrino's rest frame during the journey.

This depends on 2 experimental parameters:
- L – the distance from the source to the detector (km)
- E - the energy of the neutrino (GeV)

and 2 fundamental parameters
- $\Delta m^2 = m_1^2 - m_2^2$ (ev^2)
- $\sin^2 2\theta$

(See K2K figure)

In words;
As the neutrino travels from source to detector the mass eigenstate components propagate with different frequencies because the masses are different. So a ν_e need not necessarily stay as a ν_e because the components that make it up 'shift' during the journey.