PC4691 – Frontiers of Astrophysics – Solar Corona

Lectures 11 & 12
Lecturer: Veckstein

1. A bit of history
2. What is the corona and what is the problem of coronal heating?
3. Magnetic coronal heating: the role of magnetic reconnection
4. Nanoflares scenario of coronal heating: how it works and how it can be probed observationally

Reading list:
1. E.N. Parker, "Nanoflares and the solar corona", ApJ. v330, 474-479, 1988, July 1
2. Y. Katurakawa and S. …, "Small fluctuations of coronal X-ray intensity and a signature of nanoflares", ApJ, v557, 343-350, 2001, August 10

Easy reading for general interest: … "Nearest Star", Harvard University Press, 2001

Helium – first discovered in the stellar corona 1868 (?)

1869 – another unknown spectral line discovered, called Choronium. From calculations of the gravitational height, the mass of Choronium should be much less than Helium – a puzzle, when compared with the periodic table. Resolved in 1939; spectral line was actually a line of highly ionized iron. This meant that the temperature in the corona should be very high – a million k or so – which was a new puzzle, as the surface of the sun is much less than this and temperature decreases proportional to R. This problem is still not entirely solved.

Corona:
$T_c \sim 10^6 k$

Quiet areas of the corona (i.e. not many flares):

Number density of particles: $n_c \sim 10^8 cm^{-3}$

Energy supply required: $q_c\left(erg / cm^2 s\right) \sim 3\times 10^5$

Active regions of the corona (flare areas):

Number density of particles: $n_c \sim 10^9 cm^{-3}$

Energy supply required: $q_c\left(erg / cm^2 s\right) \sim 10^7$

Solar luminosity: $L_\odot \sim 6.3\times 10^{10} erg / cm^2 s$.

Due to the solar corona's temperature, the radiation is given off as IR and X-ray.

"Old" theory of coronal heating: Acoustic heating
Main drawbacks:
- Observed acoustic energy flux is too small

- Strong correlation between the X-ray activity in the corona and the magnetic field → magnetic nature of solar coronal activity → Magnetohydrodynamics

Magnetic field on the surface of the sun can be measured using the Zeeman splitting of spectral lines.
Sun spot – area of cooler gas – caused by the magnetic field holding the hotter gas back.

$\beta << 1$ - dynamics is governed by magnetic field.
$\beta >> 1$ → magnetic field is determined by convective motions.
$B \sim 10^2 G$, $L \sim 10^3 cm$ → $v_a \sim 10^3 km/s$ → $\tau_A \sim \frac{L}{v_A} \sim 10s$

$$\tau_{ph} \sim \frac{\ell_{ph}}{v_{ph}} \sim \frac{10^3 km}{1km/s} \sim 10^3 s >> \tau_A$$

(DC currents) quasi-static evolution – force free magnetic field: $\underline{j} \parallel \underline{B}$
Magnetically open region (coronal hole) → generation of … waves propagating upwards (AC currents)
A viable energy source – the maximum Poynting flux

$$S_{\max} \sim v_{ph} \frac{B^2}{4\pi} \sim 10^8 erg/cm^2 s >> q_c$$

Magnetohydrodynamics:
$\underline{j}$:
$\underline{\nabla} \times \underline{B} = \mu_0 \underline{j}$
→ $\underline{j} = \frac{1}{\mu_0}(\underline{\nabla} \times \underline{B})$

$\rho, \underline{v}, p$

$$\begin{aligned}
\rho \frac{d\underline{v}}{dt} &= -\underline{\nabla}P + \left(\underline{j} \times \underline{B}\right) \\
&= -\underline{\nabla}P + \frac{1}{\mu_0}(\underline{\nabla} \times \underline{B}) \times \underline{B} - \frac{1}{\mu_0}\underline{B} \times (\underline{\nabla} \times \underline{B}) \\
&= -\underline{\nabla}P - \frac{1}{\mu_0}\left(\nabla \frac{B^2}{2} - (\underline{B} \cdot \underline{\nabla})\underline{B}\right) \\
&= -\underline{\nabla}\left(P + \frac{B^2}{2\mu_0}\right) + \frac{1}{\mu_0}(\underline{B} \cdot \underline{\nabla})\underline{B}
\end{aligned}$$

where $\left(\underline{j} \times \underline{B}\right)$ is the magnetic force, $\nabla \frac{B^2}{2}$ is the magnetic pressure and $\left(\underline{B} \cdot \underline{\nabla}\right)\underline{B}$ the magnetic tension. Call the last version of this equation (1).

$$\beta = \frac{P}{B^2 / 2\mu_0}$$

$\beta << 1$ → magnetic force dominates

$\beta >> 1 \rightarrow$ thermal pressure dominates.

We need another equation which governs how B evolves with time – magnetic induction equation.

Ohm's law: $\underline{E} = \eta \cdot \underline{j} \rightarrow I = \frac{V}{R}$

Electric force = resistivity x current

Add magnetic term: $(\underline{v} \times \underline{B}) + \underline{E} = \eta \cdot \underline{j}$

$\underline{E} = -(\underline{v} \times \underline{B}) + \eta \underline{j}$

Maxwell's equation: $-(\underline{\nabla} \times \underline{E}) = \frac{\partial \underline{B}}{\partial t}$

Hence, we have:

$$\frac{\partial \underline{B}}{\partial t} = \underline{\nabla} \times (\underline{v} \times \underline{B}) + \frac{\eta}{\mu_0} \nabla^2 \underline{B}$$

(NB: we have assumed $\mu_0 = 1$)

RHS: first term: motion of the magnetic field. Second term: due to resistivity. Resistive diffusion?

Estimate: $\underline{\nabla} \times (\underline{v} \times \underline{B}) \sim \frac{vB}{L}$ and $\frac{\eta}{\mu_0} \nabla^2 \underline{B} \sim \frac{\eta B}{\mu_0 L^2}$

Hence: $\dfrac{\frac{vB}{L} \mu_0 L^2}{vB} \sim \frac{\mu_0 v L}{\eta}$ - Magnetic Reynold's Number R_m.

($R = \frac{vL}{\nu}$ - Reynold's Number)

Typically, $R_m >> 1$.

If we neglect the last term of the equation, we get

$$\frac{\partial \underline{B}}{\partial t} = \underline{\nabla} \times (\underline{v} \times \underline{B}) \quad (2)$$

This can be seen as a frozen-in magnetic field: the magnetic field flows with the fluid. It cannot change its geometrical structure / topology. This is a very strong constraint. Called "Ideal MHD" (Ideal MagnetoHydroDynamics)

Return to (1). Take the LHS and the first part of the RHS.

$$\rho \frac{d\underline{v}}{dt} = -\underline{\nabla} \underline{P}$$

$$\rho \frac{\Delta v}{\Delta t} = -\frac{P}{1\ell}$$

$$v \sim \sqrt{\frac{P}{\rho}}$$

This is the case when $\beta << 1$. When $\beta >> 1$, then we have:

$$\rho \frac{v}{\Delta t} \sim \frac{B^2}{\mu_0 \Delta L}$$

$v = \sqrt{\frac{B^2}{\mu_0 \rho}} \sim \frac{B}{\sqrt{\mu_0 \rho}}$ - Alfven Velocity

$\left(\underline{j}\times\underline{B}\right)=0$ means that the current is flowing along the magnetic field lines, → called force-free magnetic fields.

A viable energy source?

We know that we need to provide $10^7\,erg\,/\,cm^2 s$, or $10^4\,Jm^{-2}s^{-1}$, to the surface for active regions. Does magnetic heating provide this?

Use the Poynting vector.

$$S=\frac{1}{\mu_0}\left(\underline{E}\times\underline{B}\right)$$

Assume that the magnetic field is frozen, and there is no η - so we have $\underline{E}=-\left(\underline{v}\times\underline{B}\right)$. Hence:

$$\begin{aligned}S&=-\frac{1}{\mu_0}\left(\underline{v}\times\underline{B}\right)\times\underline{B}\\&=\frac{1}{\mu_0}\underline{B}\times\left(\underline{v}\times\underline{B}\right)\\&=\frac{1}{\mu_0}\left[\underline{v}B^2-\underline{B}\left(\underline{v}\cdot\underline{B}\right)\right]\\&\approx\frac{vB^2}{\mu_0}\end{aligned}$$

Put in some numbers…

$$S\sim\frac{10^3\left[ms^{-1}\right]\times\left(10^{-2}\left[T\right]\right)^2}{4\pi\times10^{-7}}\sim10^5\,Jm^{-2}s^{-1}$$

So we have enough energy. Only a fraction of this will actually be turned into heat, but there is enough available to do this.

Magnetic Reconnection

Remember that we have

$$\frac{\partial\underline{B}}{\partial t}=\underline{\nabla}\times\left(\underline{v}\times\underline{B}\right)+\frac{\eta}{\mu_0}\nabla^2\underline{B}.$$

How do we increase the dissipation?

Dissipation in a wire is $Q=I^2R$. In this case, we have $Q=\eta j^2$. So if we have a small η, we need to get a large j.

From $\underline{\nabla}\times\underline{B}=\mu_0 j$, we have $j\sim\dfrac{B}{\mu_0\ell}$.

If we use the full time-derivative equation, then the second term allows the breaking of magnetic field lines. This allows the reconnection of magnetic field lines.

Take a volume of magnetic field. The total energy will be $W_M \sim \frac{B^2}{2\mu_0}L^3$. The dissipation power will be $\dot{W}_M \sim \eta j^2 L^3 \sim \eta L^3 \frac{B^2}{\mu_0{}^2 L^2} \sim \eta L \frac{B^2}{\mu_0{}^2}$. So the time to dissipate all the energy will be:

$$\tau_\eta \sim \frac{W_M}{\dot{W}_M} \sim \frac{L^2\mu_0}{\eta}$$

For the solar corona, this time turns out to be circa a million years. Reconnection can make this much quicker.

$$u \cdot B \sim \frac{L}{\mu_0}\frac{B}{\Delta}$$

$$uL \sim v_A\Delta \rightarrow u = \frac{v_A\Delta}{L}$$

$$\rightarrow v_A \frac{\Delta}{L} \sim \frac{h}{\mu_0\Delta}$$

$$\Delta = \sqrt{\frac{\eta L}{\mu_0 v_A}}$$

$$\rightarrow u \sim v_A\sqrt{\frac{\eta}{L\mu_0 v_A}}$$

$$S = L\mu_0 u B^2 \sim L\mu_0 B^2 v_A \sqrt{\frac{\eta}{L v_A \mu_0}}$$

Total magnetic energy is $U_B \sim \frac{B^2}{2\mu_0}L^2$

So the energy dissipation through reconnection is $\tau_B = \frac{U_B}{S} = \frac{L/2\mu_0}{\mu_0 v_A\sqrt{\frac{\eta}{L v_A \mu_0}}} << \tau_\eta$