Gravitational Lenses

Slides will be on web…

<u>From blackboard</u>

Lens equation: $\theta - \beta = \alpha \frac{D_{ls}}{D_s}$

We would like to know things between the vectors in the source plane, and vectors in the image plane. θ is basically a vector in the image plane, while β is a vector in the source plane. So we can define a vector $\underline{y}$ in the source plane, and a vector $\underline{x}$ in the image plane.

$\underline{x} - \underline{y} = \underline{\alpha}$

where $\underline{\alpha}$ is some sort of relational vector. "For fun", differentiate that wrt x.

$$\delta_{ij} - \frac{\partial \underline{y}}{\partial \underline{x}} = \frac{\partial \underline{\alpha}}{\partial \underline{x}}$$

(Differentiating a vector wrt itself will give the Kronecker delta.) We are after the second term, but we need to know the last term.

As $\alpha = \nabla \psi$, then $\frac{\partial \alpha}{\partial x}$ will give the second derivatives of ψ. Now define three things

1. Convergence $\kappa = \frac{1}{2}\nabla^2\psi \left(= \frac{1}{2}\frac{\partial^2\psi}{\partial x_1{}^2} + \frac{\partial^2\psi}{\partial x_2{}^2} \right)$
2. Shear (part 1) $\gamma_1 = \frac{1}{2}\left(\frac{\partial^2\psi}{\partial x_1{}^2} - \frac{\partial^2\psi}{\partial x_2{}^2} \right)$
3. Shear (part 2) $\gamma_2 = \frac{\partial^2\psi}{\partial x_1 \partial x_2}$

Hence we get:

$$\frac{\partial \underline{\alpha}}{\partial \underline{x}} = H = \begin{pmatrix} \frac{\partial^2\psi}{\partial x_1 \partial x_1} & \frac{\partial^2\psi}{\partial x_2 \partial x_1} \\ \frac{\partial^2\psi}{\partial x_1 \partial x_2} & \frac{\partial^2\psi}{\partial x_2 \partial x_2} \end{pmatrix} = \begin{pmatrix} \kappa - \gamma_1 & \gamma_2 \\ \gamma_2 & \kappa + \gamma_1 \end{pmatrix}$$

Hence,

$$\frac{\partial \underline{y}}{\partial \underline{x}} = I - H = \begin{pmatrix} 1 - \kappa + \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa - \gamma_1 \end{pmatrix}$$

(I is the identity matrix)

So if we know ψ, then we can work out the relation between vectors in the source and image frame, which subsequently lets you calculate the magnification.