10. Post Main-Sequence Evolution

10.1 Isothermal Cores

On the lower main sequence, stars develop a small inert He core which becomes isothermal. Can this core support the star?

Apply the Virial theorem to the core:

$$2U + W = 3P_cV_c \quad (214)$$

$$P_c = \frac{1}{3V_c}\left(2U + W\right) \quad (215)$$

where U is the internal energy, W the gravitational energy, and $3P_cV_c$ originates from the pressure at the surface of the isothermal core, from the surrounding envelope.

$$W = -\frac{3}{5}\frac{GM_c^{\ 2}}{R_c} \quad (216)$$

$$U = \frac{3}{2}\frac{M_c}{\mu}kT_c \quad (217)$$

$$V_c = \frac{4}{3}\pi R_c^{\ 3} \quad (218)$$

so,

$$P_c = \frac{3}{4\pi R_c^{\ 3}}\left(\frac{3M_c}{\mu}kT - \frac{3}{5}\frac{GM_c^{\ 2}}{R_c}\right) \quad (219)$$

The maximum pressure occurs for

$$\frac{\partial P_c}{\partial R_c} = 0$$

for

$$R_1 = const \times \frac{M_c\mu_c}{T_c} \quad (220)$$

Substituting into equation 219:

$$P_c\left(\max\right) = c_1\frac{T_c^{\ 4}}{M_c^{\ 2}} \quad (221)$$

The maximum surface pressure of an isothermal core in equilibrium $\propto T_c^{\ 4} / M_c^{\ 2}$.
P_c must balance the pressure from the envelope outside the core, P_e.
At the bottom of the envelope:

$$P_e \propto \frac{M_*^{\ 2}}{R^4}$$

$$T_e \propto \frac{M_*}{R}$$

therefore,

$$P_e \propto \frac{T_e^{\ 4}}{M_*^{\ 2}} \quad (222)$$

where M_* is the total mass of the star.

At the surface of the core,

$R_c = R_e$

$T_c = T_e$

$P_e = c_2 \frac{T_c^{\ 4}}{M_*^{\ 2}}$ (223).

For the core to be stable, we require $P_e \le P_c(\max)$.

Combining equations (221) and (223), we get:

$\frac{M_c}{M_*} \le const = q_{sc}$

where q_{sc} is the Schonberg-Chandrasekhar limit.

$q_{sc} = 0.37\left(\frac{\mu_{env}}{\mu_{core}}\right) \approx 0.13$ (225)

If $M_c / M_* < q_{sc}$, the core is stable.

If $M_c / M_* > q_{sc}$, the core will collapse. P, T rise.

The Schonberg-Chandrasekhar limit assumes ideal gas pressure and isothermal core.

Collapse is halted by:

1. Electron degeneracy pressure
2. And/or helium ignition

During the thick-shell burning phase,

- Initially $M_c / M_* << q_{sc}$
- Core grows as H exhausted in inner shell
- Thick shell moves out
- T_c increases

The shell burning releases more energy than the original core.

- The star expands
- The effective temperature drops

For the sun, when point 4 is reached (end of main sequence), ~ 12% of the H is converted to He.

10.2 Post Main Sequence Evolution of a Low Mass Star

Thick shell burning:

- Luminosity increases
- Core temperature increases
- Surface temperature decreases

Thin shell burning:

- Star moves right and up in the HR diagram.
- Meets Hyashi track and moves steeply upwards.
- Outer layers become convective
- The star is now on the Red Giant Branch.

Evolution proceeds as follows:

1. Thick H-shell burning

2. Thin H-shell burning: luminosity increases, shell temperature increases. Star expands and becomes convective.
3. Core mass reaches Schonberg-Chandrasekhar limit → core collapses and electrons become degenerate $\left(M_* \leq 3M_\odot\right)$
4. Shell temperature has been rising, and so has core temperature.
5. Core temperature reaches ignition temperature for He, $\sim 10^8 k$ → He starts to burn via the triple α process:
   $^4He + {}^4He \leftrightarrow {}^8Be$
   $^8Be + {}^4He \rightarrow C + \gamma$
6. But the matter is degenerate → $P \neq P(T)$. T_c rises due to helium ignition, but P_c does not rise. Core does not expand.
7. Reaction rates $R \propto \rho^a T^n$. T increases → R increases → T increases → R increases, etc.
8. The result is an explosion: the Helium Flash. Finally T_c rises so far that the degeneracy is removed: core now expands and cools.
9. The energy released during the Helium flash is $\sim 10^{11} L_\odot$. This is the luminosity of a whole galaxy! It only lasts a few seconds, and is not seen at the surface.
10. The core settles into core He burning.

10.3 Life after the He Flash

After the He flash the star has:

- Stable He burning in its core. $M_c \approx 0.45 M_\odot$
- H burning in a shell
- Surrounded by a H + He envelope.
  $M_* = M_c + M_e$
- Luminosity drops to $L \approx 50 M_\odot$
- L independent of M_*
- Temperature depends on envelope mass
  - M_e small → high T_{eff}
  - M_e high → low T_{eff}

Star is now on the Horizontal Branch (HB).

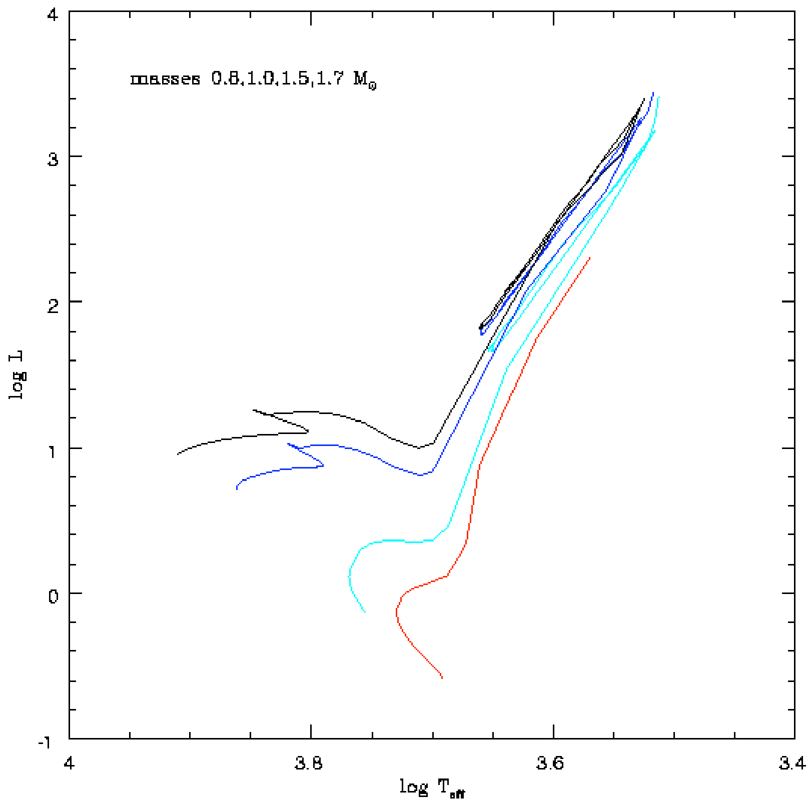
Lifetime of helium core burning is short:

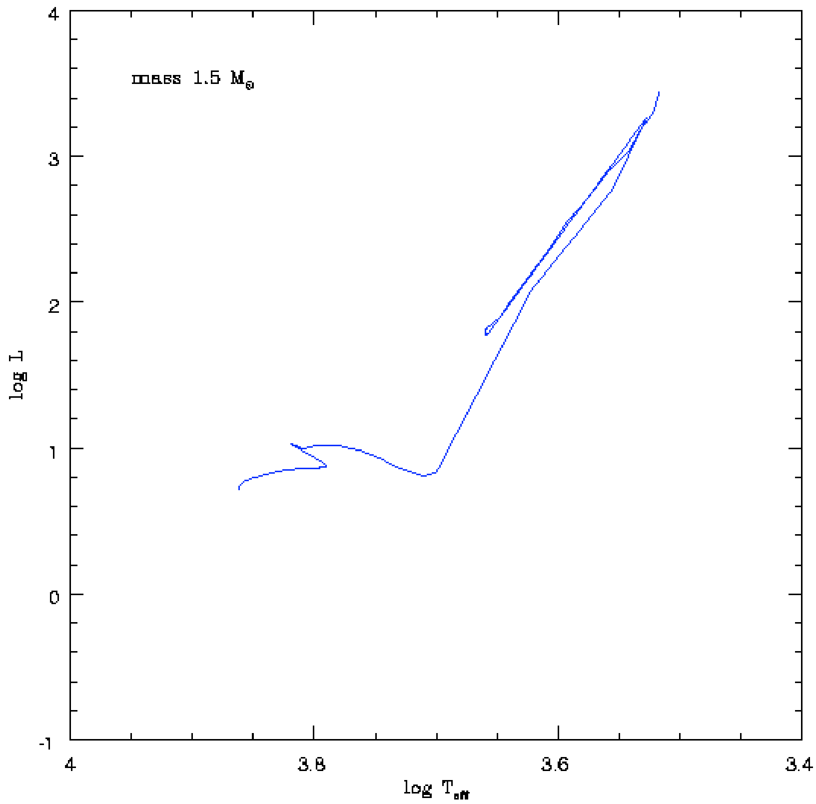
- Helium generates much less energy than hydrogen
- Luminosity is much higher than on the main sequence

He becomes depleted in the centre of the core: an isothermal C, O core forms and grows.

1. He burning continues in shell. H-shell extinguishes.
2. Core collapses and becomes degenerate
3. Thin-shell He burning
4. Star ascends the Giant Branch for the second time
5. Star becomes convective – again.

The star is now on the Asymptotic Giant Branch.





Eventually He-burning shell reaches outer edge of He-enriched area.

- He-burning stops
- H shell reignites
- He shell reforms
- He reignites, H-burning stops

- Sequence repeats every $10^4 - 10^5 yr$.

Helium reignitions are called Thermal Pulses.

Star consists of:
- Degenerate C, O core
- Thin He shell
- H envelope

When $L \sim 7000 L_\odot$, the star develops a very strong stellar wind which depletes the envelope.

$$\frac{dM_*}{dt} = -\frac{dM_w}{dt}$$

$$\frac{dM_e}{dt} = -\frac{dM_w}{dt} - \frac{dM_c}{dt} \quad (226)$$

$$\frac{dM_w}{dt} = f(L)$$

$$L = f(M_c) \quad (227)$$

Wind removes entire envelope within $\sim 10^5 yr$
- Nuclear burning ceases
- Naked C, O core becomes a white dwarf.

10.4 Post Main Sequence Evolution of a Massive Star

For $M > 3M_\odot$, evolution differs in two ways:
1. Convective core; the full core becomes exhausted at once.
   No think-shell phase
2. Helium core ignites non-degeneratively.
   No helium flash

Example: a $5M_\odot$ star:
- $\sim 5.6 \times 10^7 yr$: H is exhausted
  Fast evolution to right in HR diagram: Hertzsprung Gap
- $\sim 5.9 \times 10^7 yr$: He core burning starts
- $\sim 7.0 \times 10^7 yr$: He core exhausted.
  Star joins the AGB.

Notes:
1. After H-exhaustion, core contracts rapidly on the Kelvin-Helmholtz time:
   $\sim 3 \times 10^6 yr$ for a $5M_\odot$ star
   $\sim 5 \times 10^5 yr$ for a $7M_\odot$ star
2. He core burning is centrally condensed (~5% of the mass of the star): most energy is supplied by the H shell.
3. Luminosity already close to the Eddington limit: stars do not increase in luminosity as much.
4. Outer regions become convective and reach H-burning products: dredge up.

5. Core He burning ends with build up of ^{12}C, ^{16}O and ^{20}Ne (depending on abundance ratios, temperature, reaction rates).

$m_* > 9M_\odot$

Ignition after H-burning:

M_*	He	C-O
$< 2.3M_\odot$	Degenerate	No
$2.3 \rightarrow 9M_\odot$	Non-degenerate	No
$> 9M_\odot$	Non-degenerate	Non-degenerate

High mass stars show successive cycles:

1. Core ignites
2. Exhaustion of fuel
3. Core contracts
4. Core heats
5. Core ignites

Etc.

Successive phases are short-lived:

- Fusion of heavier nuclei releases decreasing energy per nucleon
- High energy losses from neutrinos

Duration of burning for stars of:

$15M_\odot \; \left(L \approx 10^4 L_\odot\right)$

$25M_\odot \; \left(L \approx 3\times10^5 L_\odot\right)$

Burning	$15M_\odot$	$25M_\odot$
C	$6.3\times10^3 yr$	$1.7\times10^2 yr$
Ne	7	1.2
O	1.7	0.51
Si	0.017	0.004

Cycles end with ^{28}Si burning to ^{56}Fe.

10.5 Supernovae

Iron core contracts:

- Electrons become degenerate
- Core temperature reaches $\sim 10^{10} k$

Instabilities develop:

1. Iron core photo-disintegrates
   $^{56}Fe \rightarrow 13\,^4He + 4n$
2. Helium disintegrates to photons and neutrons
3. Relativistic electrons $\left(kT = 1.7 m_e c^2\right)$ are captured by photons

Number of particles and temperature decrease, therefore pressure decreases.

Core collapse to a neutron star begins.

The core initially resembles an iron white dwarf, with density $r \approx 10^{13} kgm^{-3}$.

The free fall time is:
$$t_{ff} = \frac{1}{\sqrt{G\rho}} = 40ms \quad (228)$$
The core of the star collapses on this time scale.

Why the free fall timescale and not Kelvin-Helmholtz?
- The energy is coming out in neutrinos
- Neutrinos suffer very low opacity: escape freely.
- Instantaneous energy loss.

Energy released ΔE is about,
$$\begin{aligned}\Delta E &\approx W_{ns} - W_{wd} \\ &= GM_c^2\left(\frac{1}{R_{ns}} - \frac{1}{R_{wd}}\right) \sim \frac{GM_c^2}{R_{ns}} \\ &\sim 3\times10^{46} J \quad (229)\end{aligned}$$

Once $\rho \sim 10^{17} kgm^{-3}$, the core becomes optically thick to neutrinos. The core now bounces.

The rebounding core material hits the outer envelope. The binding energy of the envelope is $\sim 10^{43} J << \Delta E$: the bouncing core blows out the envelope

This blow-out is the supernova. The core ends up as a neutron star, or collapses to a black hole.

10.6 Supernova Observations

10.6.1 Type I
- Light curve declines rapidly for about 30 days after maximum, and then exponentially.
- Strong Si absorption in spectrum. Little hydrogen.
- Similar rates in spiral and elliptical galaxies.
- Rate about 1 in 300 years.

→ progenitors old, low mass stars.

White dwarf accretes matter from a companion star and reaches the Chandrasekhar mass.

10.6.2 Type II
- Light curve more variable between objects – declines for about 25 days followed by 50-100 day long plateau and then rapid decline.

- No Si absorption. Hydrogen lines.
- Maximum about 1 magnitude brighter than type 1 SN.
- Only seen in spiral arms of spiral galaxies.
- Rate of about 1 per 100 years per spiral galaxy.

→ progenitors young massive $\left(M > 10M_{\odot}\right)$ stars.

Core collapse in massive stars.

10.6.3 SN1987A

- First naked-eye supernova in 300 years.
- Located in the Large Magellanic Cloud
- Core collapse of 20 $M_{\odot}$ star
- Detected by neutrino detectors, in 12 second burst
- Several hours before the visual explosion

10.6.4 Nucleosynthesis

- Shock wave around the core reaches $T \approx 5 \times 10^9 k$
- Statistical equilibrium leads to ^{56}Ni
    - Most bound element with equal number of protons and neutron
- Lighter elements produced when shock wave reaches the oxygen layer
- ^{56}Ni is unstable: decays with half-life of 6.1 days.
  $^{56}Ni \rightarrow {}^{56}Co$
- ^{56}Co decays half-life of 77 days.
  $^{56}Co \rightarrow {}^{56}Fe$
- Decay occurs after the envelope is ejected.

Cobalt decay energizes the supernova during its decline. This is the origin of the 100-day plateau phase.

Explosive nucleosynthesis is the origin of most of the heavier elements in the universe.

Exam: 2 out of 3 questions. More similar to previous UMIST course than UoM.
Dimensional analysis, and repeating of derivations, will probably be needed.
Expect things in exams which we've never seen before!