1. Observations

1.1 Parameters

- Distance d: measured by triangulation (parallax method), or the amount that the star has dimmed (if it's the same type of star as the Sun…)
- Brightness or flux f: energy received on Earth per square meter per second
- Luminosity L: energy emitted by star per second $L = 4\pi d^2 f$
- Temperature T : Plank's or Wien's law
- Radius R : from angular size and distance
- Mass M : from Kepler's 3rd law. But you have to find something orbiting around it… Use a second star nearby to the first.

1.2 Distance
Baseline: distance from the Earth to the Sun $(1AU)$

Position shift θ: $d = \theta^{-1}$. This requires a distance in parsecs, and θ in arcseconds.

1.3 Flux and Luminosity
Flux received f .

Energy emitted $L = flux \times area$

$$L = f \times 4\pi d^2 \tag{1}$$

Flux density – energy per wavelength / frequency per area per second, f_ν or f_λ .

$$f = \int_0^\infty f_\lambda d\lambda W m^{-2}$$

$d =$ distance
$f =$ flux at Earth.

1.4 Effective Temperature
For a black body:

$$F_\nu \propto \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1} \tag{2}$$

If this is needed in the exam, it'll be provided.

$$F = \int_0^\infty F_\lambda d\lambda = \sigma T^4$$

If the spectrum deviates from a true black body, then we define the effective temperature T_{eff} :

$$F = \sigma T_{eff}{}^4$$

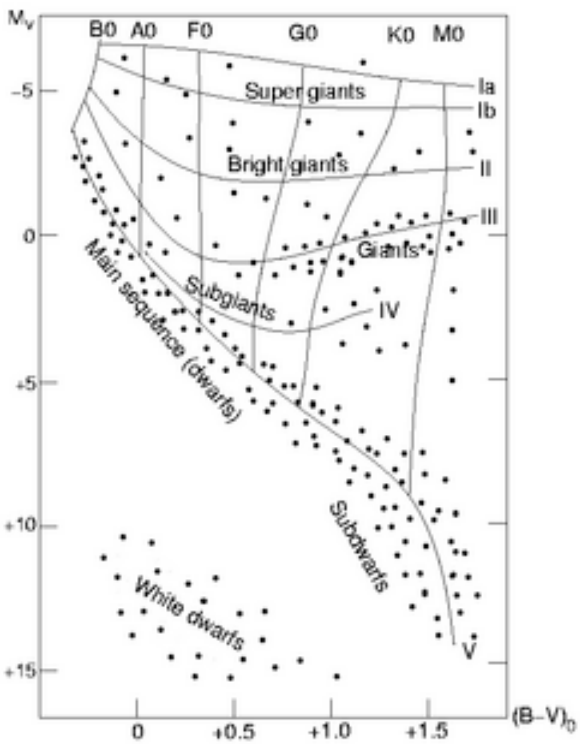
Wien's Law: cooler stars emit at longer wavelength.

$$\lambda_p T = const.$$

where λ_p is the peak wavelength.

1.5 Hertzsprung-Russell Diagram
Plot for each star: Temperature vs. Luminosity
x-axis: Temperature: log scale, reversed
y-axis: Luminosity: log scale
NB: log in astronomy is always to base 10.
Requires that stars are at known distance – or at the same distance.
Majority of stars show a tight relation between temperature and luminosity; the main sequence. Giants and supergiants are at the top right, while white dwarfs are at the bottom left.
Kinks in the line are due to internal structures of the stars, esp. the energy transport.

Open clusters: young, and expanding.
Globular clusters: old. Dense; held together by their own gravity.



- Red dwarfs
- Sub dwarfs
- Main sequence
- Subgiants
- Hertzsprung gap
- Cluster-type variables
- Red giants (Types 1 and 2)
- Supergiants
- Nuclei of planetary nebulae

The more mass the star has, the shorter it lives.
$L \propto M^{\alpha}$, $\alpha > 1$

1.6 Spectroscopy

Spectral type	Characteristics
O (3-9)	Ionized He. Strong UV
B (0-9)	Neutral He in abs.
A (0-9)	Hydrogen at maximum strength for A0 decreasing after
F (0-9)	Noticeable metal lines
G (0-9)	Strong neutral metallic atoms and ions – solar type
K (0-9)	Metallic lines dominate. Red continuum
M (0-10)	Molecular bands of titanium oxide noticeable
L	Cool
T	Cool

Temperature decreases as you go down this table.
OBAFGKM

All spectral types sorted into sub-types, 0-9, with the exception of O (3-9), and M (0-10) [for historical reasons].
Classified so that the temperature can easily be read off… (one-to-one relationship between temperature and spectral class)

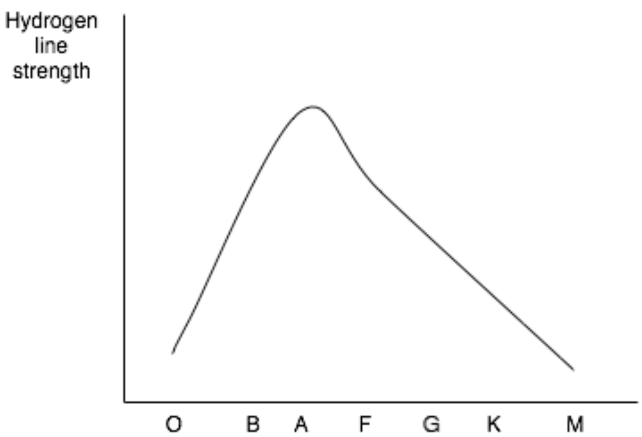


Diagram: H atom energy levels + emission series

There are two competing effects: Boltzmann (the hotter it is, the higher the electrons are [in general – more an equality between the population of lower and higher energy states]) + Saha == combined.

Population of the $n = 2$ level of hydrogen: only above $T = 15{,}000k$ in Boltzmann.
Problem: if the temperature is too high, then atoms will be ionized. This distribution is shown through the Saha equation. So subtract the Saha from Boltzmann, and you get the correct distribution – a peak at around 10,000, falling to 0 at <7500 and >25000k (approx)

The absorption to $n = 2$ in general is very small – orders of one in a million (1-9 in a million). 90% of atoms by number in the universe are hydrogen. 10% is helium. The next is oxygen, with 10^{-5} %. So overall, we have around the same ratio for absorption to $n = 2$ of hydrogen (as there is so much) as you can for other elements. Same for other energy levels…

Some stars don't fit this classification. Most stars have lots of oxides – lots of oxygen. But also lots of carbon. So why not carbides? First to form is CO – which is the most stable molecule known. But there is more O than C, and everything else forms from things which are left over. There are some stars which have more C than O. Once this happens, then you get a completely different spectra, due to the absorption from carbides.

Note that O stars are a lot more luminous than M stars, and they also have a higher temperature. Remember that $L = 4\pi R^2 \sigma T^4$. But it varies more than that – so the O stars are also bigger. For F-class, they are much cooler, emit the same amount as some O-stars. Thus they must be huge.

1.7 Typical Values

Parameter	**Technique**	**Range**
Mass	Orbits of binary stars	$0.08 \rightarrow 100 M_\odot$
Radius	Directly, occultation's	$0.3 \rightarrow 10 R_\odot$
Luminosity	Directly (distance)	$10^{-3} \rightarrow 10^6 L_\odot$
Temperature	Spectra	$3000 \rightarrow 5 \times 10^4 K$
Rotation rates	Spectra	
Magnetic fields	Spectra	
Surface Gravity	Spectra	

1.8 Determining Masses and Radii

Consider a binary system with circular orbits.

First star: mass m_1, distance from CM r_1.

Second star: mass m_2, distance from M r_2

From the definition of the centre of mass $m_2 r_2 = m_1 r_1$ and gravitational force equals the centrifugal force.

$$\frac{G m_1 m_2}{(r_1 + r_2)^2} = m_1 {\omega_1}^2 r_1 = m_2 {\omega_2}^2 r_2$$

$$\omega_1 = \omega_2 = \omega = \frac{2\pi}{P}$$

$$\therefore \frac{G m_1 m_2}{(r_1 + r_2)^2} = m_1 r_1 \frac{4\pi^2}{P^2}$$

P is the period.

$$M = m_1 + m_2 = m_2\left(\frac{r_2}{r_1} + 1\right) = \frac{m_2}{r_1}(r_1 + r_2)$$

$$\frac{m_2}{r_2} = \frac{M}{r_1 + r_2}$$

$$m_1 + m_2 = M = \frac{(r_1 + r_2)^3}{P^2}\frac{4\pi^2}{G}$$

- Kepler's third law

1.9 Doppler Effect

- Distance independent

Rotation v. At an inclination angle θ, v_r.

So the light should go from being red-shifted (coming closer to you), to blue-shifted (going away).

$$v_r = v\cos\theta$$

$$v_1 = \frac{2\pi r_1}{P}$$

$$v_2 = \frac{2\pi r_2}{P}$$

$$\frac{v_1}{v_2} = \frac{r_1}{r_2} = \frac{m_2}{m_1}$$

Combined with Kepler's third law, we can determine $m_1 + m_2$ and m_1/m_2. How do you separate out the two different luminosities? Look at the spectrum. Either two different spectra, or one spectra which has lines splitting and coming back together over time (as one is blue-shifted, one is red-shifted…)

Problem: with Doppler shift, you only see the velocity which is radial. How you get the total velocity, you need to know the inclination of the system. Hence $v_r = v\sin i$. $M_{obs} = M\sin^3 i$.

Is there a way around this problem? Yes – occultation… You can see when one star is in front of the other, for when that happens you won't see the star at the back. So twice during one orbit, you get an eclipse. This can take various forms, depending on the angle of the system, as well as the brightness of the stars (shallow, deep, shallow, deep → one star is much brighter than the other one).