6. Expansion of H*II* Regions

When a new OB star ionizes its molecular cloud, T goes from $\sim 10k \rightarrow 8000k$.
Number density n goes from $n_{H_2} \rightarrow n_p + n_e$ (i.e. 4 times the number of particles per unit volume).

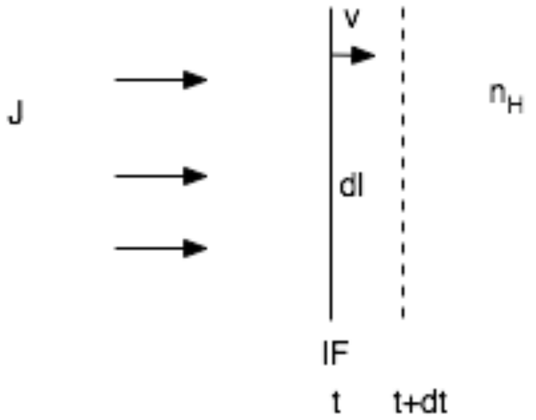
→ The pressure increases by around 3000 times.

Initial nebula formed is far out of pressure balance, so expands at its internal sound speed $c_{iso} >> c_s$ in molecular gas. → shock wave.

But also:

6.1 Ionization Front

Interface between neutral and ionized gas, but now let a significant ionizing flux J reach the neutral gas.



Front advance set by the number of ionizing photons $m^{-2} = Jdt$ = the number of atoms ionized $m^{-2} = n_H dR$

i.e. $\dfrac{dR}{dt} = \dfrac{J}{n_H}$

(in frame where upstream gas is stationary).

6.2 Stages of Expansion

Suppose star starts emitting ionizing photons at time $t = 0$.

6.2.1 First stage: rapid ionization

IF sweeps through molecular gas at $\dfrac{dR}{dt} >> c_s$.

→ upstream gas has no chance to respond.

$J = \dfrac{S_*}{4\pi R^2}$ initially

where S_* is the number of ionizing photons generated by the star.

So $\dfrac{dR}{dt} \propto R^{-2}$.

But as ionized region gets larger, photons are used up canceling out recombinations in the gas that has already been ionized, so J falls faster than R^{-2}.

If $R = R_s$ the Strongrem radius, all the ionizing photons are used up inside the nebula so $\frac{dR}{dt} = 0$.
Detailed solution (DW 7.1.4):
IF slows exponentially as $R \to R_s$, i.e. creeps towards the strongrem radius but never quite gets there.
At some point, $\frac{dR}{dt}$ falls below sound speed in neutral gas.

6.2.2 Intermediate Stage
High pressure region consisting of ionized gas as well as a surrounding shell of neutral gas that has been compressed by a shock wave. The whole region tries to expand at its' sound speed.
$c_{iso} \approx 13 km\, s^{-1} >> c_s$ in cold gas.
Shock wave sphere around the gas, with a slightly smaller sphere within it of the ionized gas H*II* with number density $n_i < 2n_0$. In between is the compressed neutral gas. Outside, number density n_0.
As ionized gas expands, its density n_i falls → recombination rate inside the nebula falls. Hence we have more photons surviving to reach the ionization front.
→ IF continues to advance into the neutral gas.

Example: Idealized Intermediate Case:
Asume:
$n_0 = const.$
$\underline{u_0} = 0$
$V_s >> c_{S0}$ (strong shock)
- Swept up gas forms a thin shell, so $R_{IF} = R_{shock} \equiv R$
In practice, shock is radiative, though T is lower than classical "isothermal" shock ionized gas.
→ hence compression $> \times 4$.
→ $\Delta R << 10\%$
- Uniform pressure P_i throughout ionized region and in neutral compressed shell.
→ not so good at start, as $V_s \approx c_{s_i}$. So gas can't smoothly adjust. Gets better as shock slows.
- Advance speed of IF into the swept-up gas $<< V_s$.
i.e. "Strongrem" condition applies almost exactly.

Then: $P_i = \left(n_e + n_p\right) kT_e = 2n_i kT_e = 2n_i m_H c_1^{\,2}$
At the shock, $P_s = P_i = \varepsilon \rho_0 V_s^{\,2} = 4\varepsilon n_0 m_H \dot{R}^2$, where $0.75 < \varepsilon < 1$ (adiabatic $< \varepsilon <$ isothermal) – take $\varepsilon = 1$.
→ $\dot{R}^2 = \left(\frac{n_i}{2n_0}\right) c_i^{\,2}$
But $S_* = \frac{4\pi R^3}{3} n_i^{\,2} \beta_2$.
Eliminate n_i :

$$\dot{R}^2 R^{3/2} = \left(\frac{3S_*}{4\pi\beta_2}\right)^{1/2} \frac{c_i^{\,2}}{2n_0}$$

But initial Strongren radius

$$R_s = \left(\frac{3S_*}{4\pi\beta_2 (2n_0)^2}\right)^{1/3}$$

as ionized $n_p = 2n_0$, where n_0 is the number of H_2 molecules.

→ $\dot{R}^2 R^{3/2} = c_i^{\,2} R_s^{\,3/2}$

If we define a dimensionless length variable $\lambda = \dfrac{R}{R_s}$, and a dimensionless time variable $N = \dfrac{t}{\left(R_s/c_i\right)}$, which includes the sound crossing time for R_s, we get:

$$\lambda^{3/4} \frac{d\lambda}{dN} = 1$$

Take $N \approx 0$ at $\lambda = 1$, i.e. ignoring the time taken by the first stage, which is short.

Then $\int_1^{\lambda} \lambda'^{3/4} d\lambda' = \int_0^N dN' = N$.

$$\frac{4}{7}\left(\lambda^{7/4} - 1\right) = N$$

or $\lambda = \left(1 + \dfrac{7}{4}N\right)^{4/7}$

$$\frac{d\lambda}{dN} = \left(\frac{4}{7}\frac{7}{4}\right)\left(1 + \frac{7}{4}N\right)^{-3/7}$$

At $N = 0$, $\dfrac{d\lambda}{dN} = 1$, i.e. $\dot{R} = c_i$ (as stated earlier).

6.2.3 Final Stage

When $P_i \to P_0$, shock weakens.

Final equilibrium when $P_i = P_0$ → $2n_f kT_i = n_0 kT_n$.

$$n_f = n_0\left(\frac{T_n}{2T_i}\right)$$

As usual $S_* = \dfrac{4\pi}{3} R_f^{\,3} n_f^{\,2} \beta_2$.

→ $\dfrac{R_f}{R_s} = \left(\dfrac{2n_0}{n_f}\right)^{2/3} = \left(\dfrac{4T_i}{T_n}\right)^{2/3}$

increase in mass of molecule:

$$\frac{M_f}{M_i} = \left(\frac{R_f}{R_i}\right)^3 \frac{n_f}{n_i} = \frac{8T_i}{T_n} \approx 6400 \text{ for } T_n = 10k.$$

Plausible?

To reach equilibrium, t is longer than the lifetime of an OB star.

Find mass found above > typical mass of cold cloud. → nebula will start eating into warmer inter-cloud gas.

6.3 Real Nebulae

Very "lumpy" nature of molecular clouds make expansion very irregular.
Visible H*II* regions must have burst out of the dust cloud in which the stars were formed, on at least one side. So we use the same equations, but no longer assuming symmetry.

(6.3-1)

High pressure gas escapes into low density region.
IF = surface of cold cloud.
Shock:
Low V_s if high ρ_0.
High V_s if low ρ_0.
Dense clumps impede the progress of the IF → column of protected neutral material. "Elephant's trunk".