4. Photo-Ionized Gas

Stars with $T_* \geq 2 \times 10^4 k$, i.e. O and B stars aka OB Stars) produce photons with $h\nu > 13.6eV = I_H$ (ionization potential of H) allows ionization
$H + h\nu \to p + e$.
Regions near OB stars are in photo-ionization equilibrium
$H + h\nu \rightleftharpoons p + e$.
Balance determined by photon density and density of H, p.

4.1 Individual processes

Typically "recombined" atom is in an excited state n.
→ photon produced has energy

$$h\nu = \frac{I_H}{n^2} + \left(E_k \; of \; e^-\right)$$

Then atom drops down to ground state, emitting a series of "recombination" lines e.g. $H\alpha$, $H\beta$, $Ly\alpha$, etc.
For a typical nebula, the electron $E_k << I_H$.
→ only photons emitted by recombinations to ground state $(n = 1)$ have enough energy to ionize another atom.
"on-the-spot" approximation: these photons very quickly ionize another atom.
→ recombinations to ground state have no net effect.
Photons emitted in recombinations to $n > 1$, or in the recombination lines are very unlikely to be re-absorbed.
e.g. H_β $n = 4 \to n = 2$ can only be absorbed by atoms in state $n = 2$. There aren't very many of these – transitions to ground state are so rapid (few microseconds) that to a very good approximation, all the hygdrogen atoms are in $n = 1$.

Recombination rate

$\propto n_e$ electron number density
$\propto n_p$ proton number density
But $n_e \approx n_p$ since the gas is mostly H.
Rate to level n:

$$\dot{N}_n = n_e n_p \beta_n \left(T_e\right) \; m^{-3} s^{-1}$$

$\beta_n\left(T_e\right)$ = recombination coefficient to level n.
Function of T_e = electron temperature (and protons, and H).
(e-p collisions rapidly equalize kinetic temperature of e and p. Also, p-H collisions make $T_H \sim T_e$.)
For total recombination state we need:

$$\beta_2\left(T_e\right) = \sum_{n=2}^{\infty} \beta_n\left(T_c\right) = 2 \times 10^{-16} T_e^{-3/4} \; m^3 s^{-1}$$

→ total recombination rate $\dot{N}_R = n_e^{\,2} \beta_2\left(T_e\right)$.

Ionization rate

$\propto n_H$
$\propto J$ the flux of stellar photons with $h\nu > I_H$ crossing per unit area.

When the flux enters the H, then some of it will be absorbed – at some small time dt after entering, it will be $J \to J + dJ$.

$\dot{N}_I = \alpha_0 n_H J \ m^{-3}$

α_0 is the photoionization cross section.

Strictly depends on the photon energy, but we will be assuming it doesn't. Most ionizing photons are not much above I_H, in which case $\alpha_0 = 6.8 \times 10^{-22} m^2$.

4.2 Photoionization Equilibrium

Define $n = n_p + n_H$ i.e. the total number density of the H nuclei.

Then:

$n_e = n_p = xn$

$n_H = (1-x)n$

We have $\dot{N}_R = \dot{N}_I$

$x^2 n^2 \beta_2(T_e) = \alpha(1-x) nJ$

$$\frac{x^2}{1-x} = \frac{J}{n}\frac{\alpha_0}{\beta_2(T_e)} \text{ --(A)}$$

For 50% ionized:

$$\frac{\left(\frac{1}{2}\right)^2}{\frac{1}{2}} = \frac{J_{\frac{1}{2}}}{n}\frac{\alpha_0}{\beta_2}$$

i.e. $J_{\frac{1}{2}} \equiv \dfrac{n\beta_2}{2\alpha_0}$

For $J \ll J_{\frac{1}{2}}$, then $x \approx \sqrt{\dfrac{J}{2J_{\frac{1}{2}}}}$, $1 - x \approx 1$

For $J \gg J_{\frac{1}{2}}$, $1 - x \approx \dfrac{2J_{\frac{1}{2}}}{J}$, $x \approx 1$.

Usually we are in one or other of these regimes, so gas is either nearly neutral or nearly fully ionized.

Transitions between neutral and ionized gas:

Mean free path of the ionizing photon in neutral gas:

$\Lambda_0 = (\alpha_0 n)^{-1}$

For $n = 10^6 m^{-3}$, $\Lambda \approx 0.48 pc$

Mean free path increases as gas becomes ionized, but transition from $x = 0.9$ to $x = 0.1$ occurs over less than $\approx 18\Lambda_0$. This is generally small compared to the size of the ionized region.

(see D&W sect. 5.2.7 for details).

4.3 The Stromgren Sphere

This is an idealized nebula in which you have a single ionizing star in homogenous gas. Size of the nebula is set by the ionization equilibrium.

The number of recombinations within the nebula per second is equal to the number of ionizing photons the star emits per second = S_*.

Assume a sharp transition.

→ $x=1$ inside, $x=0$ outside.

$$S_* = \frac{4\pi}{3} R_s^{\,3} n^2 \beta_2$$

so the "Stromgren radius":

$$R_s = \left(\frac{3}{4\pi} \frac{S_*}{n^2 \beta_2} \right)^{1/3}$$

For an O-star, $S_* = 10^{49} s^{-1}$. For a nebula with $T_e = 10^4 k$, $R_s = 7 \times 10^5 n^{-2/3} pc$.
e.g. 3 parsecs for a typical $n = 10^8 m^{-3}$.

Note $\frac{R_s}{\Lambda_0} = \alpha_0 n \left(\frac{3}{4\pi} \frac{S_*}{n^2 \beta_2} \right)^{1/3} \propto n^{1/3} = 1400$ for low density $10^6 m^{-3}$.

→ sharp transition is reasonable.

4.4 Temperature of Nebula

(DW 5.2.8)

4.4.1 Heating and Cooling

Dominated by absorption and emission of photons.

Heating via Ionization

$h\nu + H \rightarrow p + e^-$

KE of $e^- = h\nu - I_H = Q$ (i.e. that energy not needed to liberate it). $I_H = 13.6 eV$
On average (prob 2.3c) $\langle Q \rangle = kT_*$ which is generally $2-5eV$.

NB: $\frac{1eV}{k_B} \sim 10^4 k$

e^- shares energy by collisions → general heating of the gas.

Heat loss via recombination

The electron comes back together with a proton to re-form a H atom. The energy given off is a photon with energy $h\nu$ - this is not a line, as the energy incoming (i.e. the kinetic energy plus the energy from the energy level the electron is now in) is not quantized.

$e^- + p \rightarrow H^* + h\nu_e$

Atom is now in an excited state, so the electron drops down to the ground level emitting photons (order of a microsecond). These photons are quantized.
H^* → ground state plus recombination lines.

The first photon emitted is the "recombination continuum" for the initial level. It has energy $h\nu_c = \frac{I_H}{n^2} + KE$.

→ heat loss from gas = KE.
Note that the lower energy electrons are more readily absorbed than the higher energy, so the distribution of electron energy is no longer the usual. Instead of energy $\frac{3}{2}kT$, we now have an average of kT_e.
Recombination lines remove no kinetic energy; they simply balance the initial ionization.

Since $\dot{N}_I = \dot{N}_R$, we expect T_e for a pure H nebula is $T_* \approx few \times 10^4 k$.
In fact, $T_e \le 10^4 k$, so we must have more absorption processes.

4.2.2 Lines from Heavy elements (metals)
Note that in astronomy, generally any atom heavier than H is referred to as a metal. Chemists generally disagree…

Common elements are C, N, O, Si, S. These have an unfilled p shell.
e.g.:
OI: $1s^2 2s^2 2p^4$ "electron configuration".
OII : $1s^2 2s^2 2p^3$

Several energy levels are possible for different arrangements of the electrons within the p shell.
e.g. for $3e^-$, the lowest energy state has all the spins parallel.
From Pauli, then $1e^-$ in each state $(-1,0,1)$
Net effect spin $s = \frac{3}{2}$, the multiplicity $2s+1=4$.
Orbital angular $L=0$ → s shell.
So we have ${}^4s_{3/2}$ (multiplicity at top, then the letter of the shell, then the spin).
Alternatively, pair two e^- $(\uparrow\downarrow)\uparrow$
→ $s = 1/2$ $(2s+1=2)$
$L = 2,1$
$J = \underline{L} + \underline{S} = L \pm 1/2$
${}^2D_{5/2}$, ${}^2D_{3/2}$, ${}^2P_{3/2}$, ${}^2P_{1/2}$
Applies OII, NI, SII, etc.
Since all states are in the same level n, energy differences between states are fairly small. Typically ~ few eV for different L, ~ few meV for different J.
Gap between ${}^4S_{3/2}$ and ${}^2D$ is $3.34eV$, which is equivalent to $\lambda = 372.7nm$ (just to the ultraviolet from optical). Actually a doublet - $372.6nm$ and $372.9nm$, due to the two ${}^2D$ states.
e^--ion collisions may have enough energy to excite ion to higher state. (probability $P \propto e^{-h\nu/kT} \sim e^{-3} \sim 5\%$. Compare this to that of hydrogen, which is near enough negligible, as $n=2$ level is $10.2eV$ above ground state.
Radiative transitions between states in a given electron configuration are forbidden. (electric dipole transitions require change of parity: $parity = (-1)^{\Sigma \ell_i}$ i.e. depends on the ℓ quantum number.
→ selection rule is that you must have a change of $\Delta\ell = \pm 1$ to change between parity.)
But electric quadrapole and magnetic dipole transitions can happen from these states. However the probability of radiation is much lower, i.e. a relatively low A_{21} Einstein coefficient. $A_{21} \sim 1s^{-1}$. Cf. $A_{21} \sim 10^8 s^{-1}$ for electric dipole.

Not observed in the lab because in a typical gas tube, the pressure is low but is still vastly higher than that of a nebula. So you'll get radiation by collisions rather than transitions. In a typical nebula, $e^{-}-ion$ collision times are of order of a week → photon emission almost certain.
Ions nearly always in the ground state.

4.4.3 Forbidden-line Thermostat

Rate of excitation to level j from level i:

$\dot{N}_{ij} = n_e n_{Ions} C_{ij}\left(T_e\right)\ m^{-3}s^{-1}$

(Collisions with protons are much slower due to lower mean speed.)

$$C_{ij}\left(T\right) \propto \frac{e^{-hv_{ij}/k_B T_e}}{\sqrt{T_e}}$$

e.g. for [OII] (note that the square brackets show that it is forbidden line):

Cooling rate $4\pi j_{[OII]} = hv \times \dot{N}_{ij}$

Where here j in the LHS is the emissivity per steradian.

Write $n_s = y_{OII} a_0 n_H$ where y_{0II} is the ionization fraction, a_0 the oxgygen abundance, and n_H the total H number density.

1st ionization potential of O is 13.6eV.

→ O is ionized when H is ionized.

Let $y_{OII} \approx 1$, $a_0 \approx 6\times10^{-4}$, $n_e \approx n$:

$$4\pi j_{[OII]} = 1.1\times10^{-33} y_{OII} \frac{n_e^{\ 2}}{\sqrt{T_e}} e^{-3.34eV/kT_e}\ Jm^{-3}s^{-1}$$

Note that this is the most important line, hence the main cooling mechanism. Other lines are less by a factor of around ½, at least.

→ $4\pi j_{[OII]} = \dot{N}_R Q = n_e^{\ 2}\beta_2\left(T_e\right)kT_* \propto n_e^{\ 2}T_e^{-3/4}$

→ T_e is the solution of $T_e^{1/4}e^{-3.34/kT_e} = 2.5\times10^{-6}T_*$

Results:

T_* / k	T_e
2×10^4	7450
4×10^4	8500
6×10^4	9300

i.e. $T_e \sim 8000k$ for a wide range of T_*.

Increasing T_*

- Increases T_e
- Reduces the number of ionizations
- Reduces $\beta_2\left(T_e\right)$, which gives less heating.
- Increases $4\pi j_{OII}$ → more cooling.

Hence "thermostat".

PC 449 (Manchester 4th year), PC 597 (?) (UMIST)

4.5 Ionization Stratification

Ionization potentials in eV:

Element	1st IP	2nd IP
H	13.6	-
He	24.6	54.4
C	11.3	24.4
N	14.5	29.5
O	13.6	35.1
He	21.6	41.1

→ He will absorb photons with $h\nu > 24.6eV$. If He is all HeII, then Ne, N will be at least singly ionized.
O ionized as OII if H is ionized. Likely OIII if He is ionized.
Typical OB star: produce far fewer photons with $E > 24.6eV$: hence "stromgren radius" for He is smaller than for H.
Get layers – HeII, HII, OIII, NIII within one sphere (S), HII, HeI, OIII, CII, NII in the next shell, HI, HeI, HI, CII within the "CII layer" or "Photo-dissociation region" PDR. Beyond this is H_2, CI, CO.
For very hot star, He ionized to the edge of the HII region → surface S vanishes.
In reality, density structure of the nebula is greatly more complicated.

4.6 Radio Free-Free Emission

$e^- - p$ collisions radiate via "free-free" or "bremsstrahlung".
At radio wavelengths:

$$j(\nu) = \kappa(\nu) B(\nu, T) = A\nu^{-2.1} T_e^{-1.35} n_e^{\,2} B(\nu, T)$$

where A is just a known constant.
Total brightness in optically thin regime:

$$I(\nu) = \int_0^L j(\nu)\, ds \propto \int_0^L n_e^{\,2} ds$$

where $\int_0^L n_e^{\,2} ds$ is called the emission measure.
(also relevant for optical emission lines).
Optical depth $\tau_\nu = \int \kappa(\nu)\, ds$. Also $\propto (EM)\nu^{-2.1}$

→ emission is optically thin at high frequencies. $B(\nu, T) \propto 2kT\frac{\nu^2}{c^2}$ → $I(\nu) \propto \nu^{-0.1}$.

At low frequency $\tau_\nu >> 1$, $T_b = const$.
→ $I(\nu) \propto \nu^2$.
If you extrapolate these curves, and find out where they meet, then you get the frequency ν_0 at which $\tau_\nu = 1$.
Intensity at high frequency or intersection frequency ν_0 give EM if T_e is known.

4.7 Photoionized gas in the galaxy

4.7.1 Planetary Nebulae

Ejected envelopes of evolved stars, ionized by naked stellar core ≡ proto-white dwarf.
$T_* = (5 - 10) \times 10^4 k$, so He*III*, N*V*, etc may be found.

Density declines with distance. Typical barrel or hourglass structure because the wind that comes off the star will be denser on the star's equator (or at least, that's one theory…)

4.7.2 "H*II* Regions"

Nebulae ionized by OB stars. $T_* = (2-5)\times 10^4 k$.

OB stars live and die rapidly (few million years).
Environment is molecular cloud in which stars form.
→ lumpy.
Ionized gas flows out of the cloud ("champagne flow"). Most lines from ionized surface of cloud being evaporated.

4.7.3 "Warm Ionized Medium"

(BM 8.1.4)
Ionizing photons can escape from nebulae if they run out of gas before running out of photons.
"Density-bounded" rather than "ionized bounded".
Can iosize large volumes of low-density gas.
The WIM or "Reynolds Layer".
$T \sim 8000k$
Fills significant fraction of volume $(\sim 10-30\%??)$ in the galactic plane.

- Dominant (?) component of ISM at few hundred pc above and below plane.

In plane can be probed by pulsars or pulsar dispersion.
Pulses delayed due to finite refractive index of ionized gas

$$n(\nu) = \sqrt{1-\left(\frac{\nu_p}{\nu}\right)^2}$$

Plasma frequency $\nu_p = \sqrt{\frac{e^2 n_e}{4\pi^2 \varepsilon_0 m_e}} = 9.0\sqrt{n_e}Hz$

Pulses move at group velocity

$$c_g = \frac{d\omega}{dk} = c\sqrt{1-\left(\frac{\nu_p}{\nu}\right)^2}$$

Arrival time $t(\nu) = \int_{s_0}^{s_1} \frac{ds}{c_g} = \frac{1}{c}\int_{s_0}^{s_1}\left(1+\frac{1}{2}\left(\frac{\nu_p}{\nu}\right)^2\right)ds = t_0 + \frac{c^2}{8\pi^2\varepsilon_0 n_e c}\frac{1}{\nu^2}\underbrace{\int_{s_0}^{s_1} n_e ds}_{dispersion\ measure}$

DM is the column density of e^- between pulsar and us.
DM + known distance to pulsars → $\langle n_e \rangle \sim 4\times 10^4 m^{-3}$ in the plane near the sun.
Allowing for neutral regions, $n_e \sim few \times 10^5 m^{-3}$ in the WIM.