

3. The Cool ISM

Interstellar gas is dominated by Hydrogen. This can exist in three forms:

- Molecules (molecular hydrogen H_2)
- Atoms (HI , or “Neutral hydrogen”)
- Ionized (III)

In order of increasing temperature T .

Focus on H_2 and HI .

3.1 HI (Neutral Atomic Hydrogen)

Ground state ($1s$) of Hydrogen.

Split into 2 levels, magnetic interaction between p and e .

Either anti-parallel (i.e. dipoles are opposite), or parallel (dipoles are in the same direction). Parallel has a higher energy. This is called hyperfine splitting.

For the anti-parallel case, the total angular momentum $F = 0$. There is only one state.

$g_1 = 1$.

For the parallel case, $F = \frac{1}{2} + \frac{1}{2} = 1$. There are now 3 degenerate states, $m_z = -1, 0, 1$.

So $g_2 = 3$.

$$\Delta E = 5.9 \times 10^{-6} \text{ eV}.$$

$$\rightarrow \nu = 1.4204 \text{ GHz}, \lambda = 21.105 \text{ cm}.$$

Coollest temperature possible in space $T \sim 2.7 \text{ K}$, $kT = 2.3 \times 10^{-4} \text{ eV}$.

Boltzmann factor $e^{-\frac{\Delta E}{kT}} \approx 1$ (or $1 - \frac{\Delta E}{kT}$)

Transition probability is very low $A \approx 2.9 \times 10^{-15} \text{ s}^{-1}$ (how often until a transition takes place).

Lifetime in the upper level $\sim 10 \text{ Myr}$.

Natural width $\sim \frac{A}{2} \ll$ Doppler width $\sim \sqrt{T} \text{ kHz}$ (random thermal motion).

Number of hydrogen atoms capable of absorbing a photon of frequency $(\nu, \nu + d\nu)$:

$$n_H^{(\nu)} d\nu = n_H f(\nu) d\nu$$

(Careful of ν (nu) and v (velocity)).

$f(\nu) d\nu$ is the fraction of atoms along a line of sight that have velocities in the range $(v, v + dv)$.

$$n_H = \int_{all \nu} n_H^{(\nu)} d\nu$$

$$\int f(v) dv = 1$$

Now substitute the previous equation with $|dv| = \frac{v_0}{c} |d\nu|$

$$n_H^{(\nu)} = n_H f(v) \frac{c}{v_0}$$

Occupancy of the hyperfine levels is set by the “spin temperature” $T_s \approx$ kinetic temperature of atoms.

$$\text{Boltzmann } \frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-\frac{h\nu}{kT}} \approx \frac{g_2}{g_1} = 3$$

$$\rightarrow n_H = n_1 + n_2 = 4n_1.$$

$$\text{Absorption coefficient } \kappa(\nu) = \frac{h\nu B_{12}}{c} n_1^{(\nu)} \left(\frac{h\nu}{kT_s} \right) = \underbrace{\left[\frac{h^2 \nu_0 B_{12}}{4k} \right]}_{C = \frac{1}{1.822 \times 10^{19} \text{ sm}^{-3} \text{ K}^{-1}}} \frac{n_H}{T_s} f(\nu)$$

$$\text{In LTE, } \frac{dT_s(\nu)}{ds} = [T_s - T_b(\nu)] \kappa(\nu)$$

$$\rightarrow T_b(\nu) = C \int_{s_0}^{s_1} \left[\frac{T_s - T_b(\nu)}{T_s} \right] n_H f(\nu) ds$$

along line-of-sight, s

At low τ_ν (i.e. optically thin), $T_b \ll T_s$.

$\rightarrow T_b(\nu)$ is directly proportional to the total column density of atoms with $\nu = \frac{\Delta\nu}{\nu_0} c$,

independent of T.

The total column density

$$N_H = \int_{s_0}^{s_1} n_H ds = 1.822 \times 10^{19} \int T_b(\nu) d\nu$$

This equation is used to get H I mass of other galaxies – see B&M.

3.1.2 H I in the Galaxy

Here, we mean our galaxy – the Milky Way.

In our galaxy, $f(\nu)$ is mostly fixed by the geometry of large-scale rotation.

MORE OF THIS LECTURE HERE

3.1.3 Cold and Warm H I

H I optical depth $\propto \frac{1}{T_s} \rightarrow$ cold gas absorbs best.

Get T_s by comparing (absorption line against bright radio source) vs (surrounding emission).

On source measurement (i.e. looking directly at the actual source, not its' surroundings), then $T_{bc}(\nu) \gg T_s$ (T_{bc} depends on frequency, but not by much).

$$T_b(\nu) = T_{bc} e^{-\tau_\nu}$$

$$\rightarrow e^{-\tau_\nu} = \frac{T_b(\nu)}{T_{bc}}$$

Look at a patch of sky very close to the source, but not on the source (off source), you're looking at the same ISM but no background radiation. Hence $T_{bc}(\nu) = 0$

(subscript: bc, bo, bg, ...?)

$$T_b(\nu) \rightarrow N_H f(\nu)$$

$$\text{Now } \tau_\nu = C \frac{f(\nu) N_\mu}{T_s}$$

Hence we get the spin temperature.

Results emission and absorption profiles are very different. There is a smooth component of neutral hydrogen which shows no absorption. $\rightarrow T_s \geq 1000k$. Probably $\sim 5000k$.

“Warm neutral medium” (WNM). Seems to fill a large fraction of space, $20 \rightarrow 60\%$.

Narrow features seen in both absorption and emission. $20 \leq T_s \leq 110k$.

If you weight it by the mass of gas, then a typical temperature might be around $80k$. This may be a little on the high side... This is called the cold neutral medium (CNM). It is also known as HI clouds. This is also seen in absorption against emission from the WNM. In this case it is called “HI self-absorption” (HISA).

Shows very filamentary structure. This is the strongest absorbing gas, i.e. the lowest T_s . Sometimes the strands seen in self-absorption are clearly on top of emission typical of very cold, dense gas which can be identified by dust or molecular emission (see later). Other types of features are not associated with these molecular clouds.

3.1.4 Interstellar “Turbulent” Motions

Note that this is not the same turbulent as in fluid dynamics.

RMS thermal line of sight speed $\sigma_\nu = \sqrt{\frac{kT}{m}}$ (through the Boltzmann distribution)

where m is the “mean molecular mass”, which is an average of hydrogen and helium atoms in the ISM. A typical mass is $m = 1.4 M_H$.

$\rightarrow \sigma_\nu = 0.7 km s^{-1}$ for a temperature of $80k$, and $\sigma_\nu = 5 km s^{-1}$ for $5000k$.

When you look at the gas, you get a larger number than this. The individual CNM components in the absorption spectra are well fitted by Gaussians, but these Gaussians have widths several times larger than the thermal σ_ν , typically

$2 \rightarrow 6 km s^{-1}$.

So there is some sort of bulk motion within the “cloud” at several times the sound speed. This is called interstellar turbulence, “and is quite a mystery”. This is very unlike terrestrial turbulence as in the interstellar case the speed of the turbulence is around 3 times the sound speed, so they are supersonic. Any terrestrial turbulence is very much smaller than the sound speed.

$\rightarrow$ compression, shock waves, instead of the typical vortices or turbulent eddies.

3.2 Interstellar Dust

(Dyson and Williams chapter 4, B&M 3.7)

3.2.1 Why do we think interstellar extinction is due to dust?

All we see is the absence of starlight. Why do we think this is due to dust?

- Small solid particles with size $\leq \lambda$ are much more efficient at scattering than atoms.

Thomson cross-section (e^-) $\sigma_T = 6.65 \times 10^{-29} m^2$

Cross-section of H atom is $\sim 500nm \rightarrow \sim 10^{-28} m^2$.

Small sphere of radius a ($\ll \lambda$) with a dielectric constant m :

Total cross-section is the extinction efficiency. $Q_{ext}\pi a^2$ with

$$Q_{scattering} = \frac{8}{3} \left(\frac{2\pi a}{\lambda} \right)^4 \left| \frac{m^2 - 1}{m^2 + 1} \right|^2 \quad \text{and} \quad Q_{absorption} = \frac{8\pi a}{\lambda} \text{Im} \left(\frac{m^2 - 1}{m^2 + 1} \right)$$

Note that $Q_{scattering}$ is Rayleigh scattering.

NB: $\sigma_{scattering} \propto a^6$.

e.g. for $m = 1.5$, sphere radius 100nm , we have:

$$Q_{ext}(\lambda)\pi a^2 = 1.8 \times 10^{-14} m^2 \quad \text{at} \quad \lambda = 500\text{nm}.$$

If this sphere has the density of silicate rock ($\rho_s = 2500\text{kg m}^{-3}$), then the mass of the sphere is 6×10^9 H atoms.

→ absorption per unit mass is around 3×10^4 times larger for small particles than for atoms.

- Interstellar extinction is frequently associated with reflection nebulae. So it is not an absorption process, but there's a lot of scattering happening (scattering dominates) in the optical, as expected for particles with a size $a < \lambda$.
- Extinction correlates with increasing polarization of starlight, as if the ISM is acting like a Polaroid filter. The directions of these polarizations is organized over the sky. The only explanation that anyone's come up with is that it can only be explained if there is a weak Galactic magnetic field which is aligning non-spherical grains in space.
- Extinction correlated with a specific frequency absorption in narrow bands due to
 - Silicate $9.7, 18.5\mu\text{m}$
 - Graphite (probably) $0.22\mu\text{m}$.
- The estimated mass of dust is approximately equal to the estimated mass of heavy elements which are apparently “depleted” from the gas.

3.2.2 Extinction Curves

These describe how the extinction varies with wavelengths.

Working in magnitudes.

Extinction $A(\lambda) = m(\lambda) - m_0(\lambda)$, i.e. the observed minus the original.

e.g. $A_v = V - V_0$.

The problem is: how do you know what the original magnitude was?

$m_0 = M + 5 \log d - 5$. But the distance (in parsecs) is usually not known.

So work with “colours”, which are ratios of the brightnesses in different bands (blue and visual, for example).

e.g. $B - V$ ($\propto \log \frac{F(\lambda_v)}{F(\lambda_B)}$), where F is the flux at said wavelength).

$B_0 - V_0 = M_B - M_V$, i.e. no distance uncertainty.

So, instead of getting the extinction, we define the “reddening” e.g. for blue and visual bands: $E(B - V) = B - V - (B - V)_0$, where $(B - V)_0$ is the intrinsic colour for a star of the same spectral type and $B - V$ is the colour of the target star.

$$E(B - V) = A_B - A_V$$

This can easily be measured.

To get the total extinction, we need $R_V = \frac{A_V}{E(B-V)}$. Note that this subscript V is misleading, as it depends on B too. This is the “ratio of total to selective extinction”.

Since $A_V = 1.086\tau_V \propto \int_{s_0}^{s_1} \kappa(\nu) ds \sim \kappa_V(s_1 - s_0)$.

Similarly: $A_B - A_V \propto (\kappa_B - \kappa_V)(s_1 - s_0)$.

So $R_V = \frac{\kappa_V}{\kappa_B - \kappa_V}$, i.e. it depends on the micro properties and shouldn't get bigger for

more distance stars. We can expect it to be roughly independent of distance.

Conventionally we normalise the extinction curve by plotting

$$\frac{E(\lambda - V)}{E(B - V)} = R_V \left(\frac{A(\lambda)}{A_V} - 1 \right) \text{ vs. } \lambda^{-1} \text{ (i.e. } \frac{\nu}{c} \text{)}.$$

A detailed curve with fine resolution in λ can be obtained by taking ratios of spectra of a reddened star, as well as an unreddened star of the same type.

Result: extinction between $30\mu m$ (mid-infrared) down to $100nm$ (ultraviolet) are pretty similar in most but not all directions.

It is roughly fit by theoretical $Q_{ext}(\lambda)$ for silicate spheres with radius $a \sim 100nm$ (for IR-optical).

Because the size of these spheres approach λ in the optical region of the spectrum, we don't get simple Rayleigh scattering ($Q_{scat} \propto \lambda^{-4}$). Instead, the curve is $\sim \lambda^{-1}$.

For a typical line of sight through the ISM: $R_V = 3.05$. $A_V = 1.8mag / kpc$.

Both these are larger in dense regions. As R_V is different, the particles must be a different size...

In UV, absorption shows two features: “Bump” at $217.5nm$, which is usually attributed to Graphite absorption, as well as a rise towards the far UV, which might also be graphite or PAH's – see later.

Some variability of these from place to place. E.g. the bump just about vanishes if you look at the small Magellanic cloud. So the ISM in that galaxy is different from ours – presumably lacking in Carbon.

3.2.3 Composition of Dust

Abundances:

Abundances of elements in the Sun are roughly equal to the abundances in primitive meteorites. → composition of the “proto-solar nebula”. Similar to abundances in other stars, which are not too far off from the solar abundances. So take them as roughly typical.

Hydrogen and Helium = 98% by mass

C + N + O = 1.36% - N is not important.

Mg + Si + Fe = 0.24%

Ne = 0.17% - but doesn't chemically react, so can be ignored.

All the other elements are essentially minor contaminants.

Depletion:

We can measure the ISM abundance via absorption lines in optical and UV spectra.

Result: heavier elements are under-abundant – “depleted” relative to the solar system.

Depletion increases with condensation temperature. Most of Si, Fe, and Mg plus some of the carbon and oxygen is missing from the gas phase (along with many other trace elements).

If condensed into grains ~ 0.4 to 0.9% of the mass of the ISM it tied up in these grains.

Emission and absorption features:

IR absorption spectra of reddened stars show:

- Silicate absorption SiO_2 at $9.7, 18.5\mu m$
Strength implies nearly all the silicon is in silicate form, which is odd.
- Hydrocarbon (carbon-hydrogen bond) feature at $3.4\mu m$
→ “hydrogenated Amorphous Carbon” HAC, i.e. a carbon powder with some hydrogen bonded to it.
- In dense, cold “dark” clouds we also see
 - o H_2O ice ($3\mu m$)
 - o CO_2 ice ($4.3\mu m$)
 - o CO ice ($4.7\mu m$)
- IR spectra of regions containing dust heated by nearby stars show a series of molecular emission lines due to “Polycyclic Aromatic Hydrocarbons” (PAH’s).
e.g. 4 overlapping rings of Carbon surrounding Oxygen, with Hydrogen coming off the edges. (Pyrene).

3.2.4 Infrared Continuous Emission

(D&S 4.4.1, B&M 9.3.1)

Dust grains are heated by absorption of starlight (mainly UV) and so must cool by emitting photons (mostly IR) to stay at constant T.

Roughly half of all starlight is absorbed by dust → for IR illuminosity of ISM $\sim$ optical / UV luminosity of stars in spiral galaxies.

(see flux incident on a surface on web).

In detail:

$$\text{Energy absorbed} = \int Q_{abs}(\nu) \sigma_0 \pi I(\nu) d\nu$$

$\pi I(\nu)$ accounts for photons traveling in all directions.

$$\text{Energy emitted} = \pi \int Q_{em}(\nu) \sigma_0 B(\nu, T_{dust}) d\nu$$

But $Q_{abs} = Q_{em}$ from Kichoff’s law (scattering does not contribute to heating or cooling).

$$\text{At equilibrium, these are equal with } u(\nu) = \frac{4\pi}{c} I(\nu)$$

$$\int Q_{abs}(\nu) u(\nu) d\nu = \frac{4\pi}{c} \int Q_{abs}(\nu) B(\nu, T_{dust}) d\nu$$

This fixes T_{dust} .

In practice, $Q_{abs}(UV) \approx 1$. $u(\nu)$ peaks in UV / optical.

But total energy density of the starlight $u_{\text{photon}} = \int u(\nu) d\nu$ is equivalent to a black body with $T \approx 3k \rightarrow$ suggests $T_{\text{dust}} \approx 3k$, giving peak at $\lambda \approx 1nm$. But $Q_{\text{abs}}(\nu) \ll 1$ in the far IR / nm band.

$\rightarrow T_{\text{dust}}$ must be higher to compensate.

We expect $Q_{\text{abs}}(\nu) \propto \nu^\beta$ with $1 \leq \beta \leq 2$ depending on the material.

Detail calculation $\rightarrow T \approx 15 \rightarrow 45k$.

The observed IR continuum:

- Good match to H1 – definitely from ISM.
- $T_{\text{dust}} \approx 17k$, $\beta = 1.8$. Peak emission at $\lambda = 150\mu m$.
NB: collisions with atoms too slow to make $T_{\text{dust}} = T_{\text{gas}}$
- Excess emission in IR $5 \rightarrow 30\mu m$

$B(\nu, T)$ falls very rapidly at $h\nu > kT$ (Wien)

$\rightarrow$ mid IR emission impossible from grains with $T < 50k$

$\rightarrow$ Not all grains are in steady state thermal equilibrium.

Solution: Very Small Grains (VSG's)

(Polycyclic Aromatic Hydrocarbons)

Size ~ a few nm. (roughly the size of PAHs) can be heated to $> 50k$ by absorption of a single UV photon.

Radiate very efficiently in MIR when hot, but rapidly cool in ~ 1 hour.

$\rightarrow$ MIR emission is from small fraction of VSGs recently heated.

3.2.5 Formation of Grains

Grains must form in very dense regions: growth is too slow in the normal ISM (cf. example 1.6).

Radius at time t:

$$r(t) = r(0) + \frac{\epsilon n_i m_i \bar{v}_i}{4\rho_s} t$$

where n_i , m_i and $\bar{v}_i$ are the number density, mass and mean speed of molecules of type i. ρ_s is density of solid phase.

ϵ is “sticking coefficient” < 1 .

Best sites are:

- Winds from stars (dense, relatively cool, $n_h \approx 10^{-19} m^{-3}$, $T \sim 1000k$)
- Supernova explosions (rich in heavy elements)

Of these major elements, the first molecule to form is CO. Stable below 2000k.

For normal abundance $C < O$, this uses up almost all the carbon. Hence remaining heavy elements form oxygen compounds, especially silicates e.g. $MgSiO_3$,

$MgSiO_4$ FeO etc.

For carbon stars, where $C > O$, i.e. more carbon than oxygen, in the wind CO uses up all the oxygen. Hence Fe_3C , SiC , graphite, amorphous, presumably PAH's.

Mystery: SiC is not found in the dust in the ISM.

3.2.6 Summary of Dust Models

Large grains: typical size $a \sim 100nm$

But range of sizes $n(a) \propto a^{-q}$, $q \sim 3 \rightarrow 3.5$.

Mixture of both silicates and carbon-rich grains.

Very small grains: separate population with $a \sim 1nm$. Includes PAHs and other carbon-rich forms.

In dark clouds:

- CO mantles condense onto grains
- H_2O and other ices form by chemical reactions on the grain surface
- Grains may coagulate to form amorphous clusters (see handout 2)

Outside dark clouds, mantle evaporated by UV. Grains probably never spherical (they tend to go spherical-ish).

3.3 Molecular Clouds

3.3.1 Energetic States of Molecules

Rotation:

$$E = \frac{J^2}{2I} = \frac{J(J+1)\hbar^2}{2I}$$

Typical $\frac{E}{k}$ is a few Kelvin $\rightarrow \nu \sim 10^{11} Hz$, $\lambda \sim 1mm$

Electric dipole transitions:

Selection rule $\Delta J = \pm 1$

$$\rightarrow \Delta E = [J(J+1) - J(J-1)] \frac{\hbar^2}{2I} = \frac{J\hbar^2}{I}$$

$\rightarrow$ this gives a set of lines at roughly equal intervals.

(for large J, centrifugal forces stretch molecule, changing I slightly).

Vibration:

$$E \approx \left(n + \frac{1}{2}\right) \hbar \omega_0$$

$$\frac{E}{k} \sim \text{few} \times 100k.$$

$$\nu \sim 10^{13} Hz \text{ (MIR)}$$

Excitation:

(change of electron configuration)

No simple formula for E, but $\frac{E}{k} \sim 10^5 k$.

$$\nu \sim 10^{15} Hz \text{ (optical or UV)}$$

3.3.2 Molecular H_2

As H_2 is a “homonuclear” molecule, i.e. 2 identical atoms, by symmetry it has zero electric dipole in ground state.

$\rightarrow$ only slow quadrupole transition, i.e. need $\Delta J = \pm 2$.

Otherwise it shows only pure rotation, and/or vibration states.

$\rightarrow H_2$ usually detected by absorption of UV photons.

In this case, $h\nu > 11.2eV \rightarrow$ get into excited state.

$\rightarrow$ cluster of lines (“band”) for rotation / vibration states.

Pure rotation and vibration lines are only seen from gas with local heat source.

Comparing $N(H_2)$ from UV absorption line strengths with $N(H)$ from Ly absorption:

$$\frac{N(H_2)}{N(H)} \text{ increases strongly with } N(H_{tot}) = N(H) + 2N(H_2)$$

→ gas is distributed in “clouds”. So high column density $N(H_{tot}) > 10^{23} m^{-2}$.

Implies that there is a dense cloud between us and the star.

→ large H_2 fraction is dense clouds.

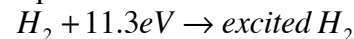
Why?

H_2 is energetically preferred: $4.48 eV$ binding energy per molecule.

$4.48/k = 5 \times 10^4 K$, so expect H_2 to be stable in CNM ($\sim 80K$).

But H_2 is vulnerable to UV photons.

Would expect $H_2 + 4.48 eV \rightarrow H + H$. But this doesn't happen because of zero dipole moment. Instead:



This then leads to a decay into H_2 90% of the time, or $H + H$ 10% of the time.

NB: once H-H bond is broken, it's very hard for it to be reconstructed.

But H_2 is also very slow to form, since $H + H \rightarrow H_2 + 4.48 eV$ doesn't happen for some reason.

3-Particle collisions form H_2 in the lab, but in ISM the density is too low. Instead, H atoms are absorbed onto the surface of dust grains (either van der Waals or chemical reactions). Binding energy $\sim k_B 300K$ lost via lattice vibrations.

Barrier to moving to next lattice site $\approx k_B \times 50K \rightarrow$ thermally allowed.

Absorbed atoms randomly wander over the surface. Gamma ray next partner, then $H + H \rightarrow H_2 +$ lattice vibrations. Binding energy may release H_2 from surface.

→ grains acting as catalysts. Similarly for H_2O .

Handout 2 typo's:

P1: Q2, “Homopolar” should be “homonuclear”.

P11 Q4(c): $h\nu \ll kT_{XS} \rightarrow h\nu \ll kT$

P13: 3rd of RHS: $\sin(\ell + \phi) \rightarrow \sin(\ell + \alpha)$, 8th equation: $\frac{\sin \alpha}{\sin(\ell + \alpha)} \rightarrow \frac{\sin \ell}{\sin(\ell + \alpha)}$

H_2 fraction based on equilibrium between creation and destruction. Favoured by:

- dense environments → lots of absorption events, meeting of two atoms is likely.
- Low T_{dust} to minimize evaporation
- Large column density of H_2 gas and dust grains to absorb UV photons.

NB: we only detect H_2 on sightlines with relatively low column density $N(H_{tot})$ - otherwise we can't see the star. But dense clouds show lots of IR dust emission, lots of other molecules, but no 21cm emission from H atoms → all the hydrogen is molecular. “Molecular clouds”.

3.3.3 Carbon Monoxide

(DW 2.2.4, BM 8.1.4)

After H and He, C and O are the most common elements. CO molecule is stable (binding energy 11.1eV - sensitive to UV), and easily formed compared to H_2 . $\rightarrow$ second most common molecule.

“Heteronuclear” so has electric dipole.

$J = 1 \rightarrow J = 0$ rotation line at 115GHz ($\lambda = 2.6\text{mm}$) is mapped all around the galactic plane.

Note CO and H_2 both disassociated by 11eV photons, so should occur in the same regions. Cool dark clouds.

CO distribution very clumpy, fractal.

Individual clumps almost always optically thick to “ordinary” $^{12}\text{C}^{16}\text{O}$ so only see surface of clump. Probe interiors with rare isotopes:

e.g. at $T = 20\text{K}$, CO transitions are optically thick for following column densities

(atoms $m^{-2} / \text{K km s}^{-1}$)

Species	Relative abundance	Transition	$N_{\tau=1}(^{12}\text{CO})$
$^{12}\text{C}^{16}\text{O}$	1	$J = 2 \rightarrow J = 1$ $J = 1 \rightarrow J = 0$	5.5×10^{19} 2.2×10^{20}
$^{13}\text{C}^{16}\text{O}$	$1/65$	$2 \rightarrow 1$ $1 \rightarrow 0$	3.6×10^{21} 1.5×10^{22}
$^{12}\text{C}^{18}\text{O}$	$1/500$	$2 \rightarrow 1$ $1 \rightarrow 0$	2.8×10^{24} 1.1×10^{23}

$$(N_{\tau=1} \propto T^2)$$

$N(H) \sim 10^4 - 10^5 N(CO)$ (depends on how much C and O is in the dust grain).

Generally ^{13}CO , $J = 2 \rightarrow 1$ has $T_b = ^{12}\text{CO}$ $J = 1 \rightarrow 0$

$\rightarrow$ optically thick in both lines.

$$N(CO) > 3 \times 10^{21}$$

Structure of a molecular cloud, e.g. the orion cloud, is “clumps within clumps” i.e. fractal. This allows a rough mass estimate even though each clump is optically thick. Average brightness temperature over the antenna beam is usually called T_A , and is a measure of the number of clumps you have in the beam.

Even though the clumps may overlap in direction, they’re probably going to be separated in velocity and hence frequency by the Doppler effect.

Hence define the CO intensity:

$$I_{CO} = \int dv T_A (115\text{GHz line})$$

Units of I_{CO} are K km s^{-1}

Total column density $N(H_2) = X I_{CO}$. The value of X varies between galaxies, and parts of galaxies. Should depend on the metallicity, depletion (the fraction of C and O tied up in dust grains), T, etc.

$$\text{Roughly } X \approx (2.5 \rightarrow 5) \times 10^{24} m^2 / \text{K km s}^{-1}.$$

3.3.4 Distribution of molecular gas

(BM 9.6.1)

Much H_2 in “Giant Molecular Clouds” (GMC’s) aka “Cloud complexes” of up to $10^6 M_\odot$.

$T \approx 10 \rightarrow 30 K$

Clumps have $\sigma_v \sim \text{few } km s^{-1}$ ($\gg$ sound speed).

They have self-gravity which holds them together.

3.3.5 Other Molecules

Large variety of other molecules detected, with up to 13 atoms ($HC_{11}N$). Rare molecules useful tracers of dense clumps (not optically thick).

Also good diagnostics of the number density of particles, and the temperature in the gas.

→ see “molecular astrophysics” PC 3792