(Lectures 9-10)

Last time: $\rho_{crit}(t) = \dfrac{3H^2(t)}{8\pi G}$

$\rho_{crit}(t_0) = 1.99h^2 \times 10^{-26} kgm^{-3}$ if $H = 100 kms^{-1}Mpc^{-1}$ = Hubble's Constant.

Rough guess at current density from counting galaxies locally.
'Typical" galaxy $\sim 10^4$ stars $\sim 2\times10^{41} kg$.
'Typical" separation $\sim 1Mpc \sim 3\times10^{22} m$.
→ crude guess at local density of matter $\sim 6\times10^{-27} kgm^{-3}$ (order of magnitude).
Looks less, but not too far off. It could have been many orders of magnitude different.

ρ_{crit} provides a useful marker comparison, define a "density parameter" $\Omega(t)$.

$$\Omega(t) = \frac{\rho(t)}{\rho_{crit}(t)}$$

with a present value Ω_0.

So, rewrite Friedman equation.

$$\left(\frac{\dot a}{a}\right)^2 \equiv H^2 = \frac{8\pi G}{3}\underbrace{\rho_{crit}\Omega}_{\rho} - \frac{kc^2}{a^2}$$

Substituting for ρ_{crit} (as above)

→ $H^2 = H^2\Omega - \dfrac{kc^2}{a^2}$

i.e. $\Omega - 1 = \dfrac{kc^2}{a^2H^2}$

This is just another way of writing the Friedman equation, which is a useful form for later on about inflationary universe.

$\Omega = 1$ → $k = 0$ the critical mass.
Once you have a value for k (i.e. U_{total} in Newtonian approach) it stays the same.

Deceleration Parameter

Parameterization of the fact that the universe is slowing due to gravitational attraction – look back at earlier times: was expanding faster. (picture 29).
Expand $a(t)$ as a Taylor series.

$$a(t) = a(t_0) + \dot a(t_0)[t - t_0] + \frac{1}{2}\ddot a(t_0)[t - t_0]^2 + ...$$

$$\frac{a(t)}{\dot a(t_0)} = 1 + H_0[t - t_0] - \frac{1}{2}q_0 H_0^{\ 2}[t - t_0]^2 + ...$$

where $q_0 = -\dfrac{\ddot a(t_0)}{a(t_0)}\dfrac{1}{H^2}$, i.e. $q_0 = -\dfrac{\ddot a(t_0)a(t_0)}{\dot a(t_0)^2}$, a dimensionless parameter.

A large q_0 value → large deceleration. E.g. universe is dominated by "cold" matter i.e. non-relativistic → pressure = 0.

Therefore acceleration equation (differential form of Friedman equation, i.e. no force version)

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4}{3}\pi G\rho$$

And we know ρ_{crit} (see above)

$$q_0 = -\frac{\ddot{a}}{a}\frac{1}{H_0^{\ 2}}$$

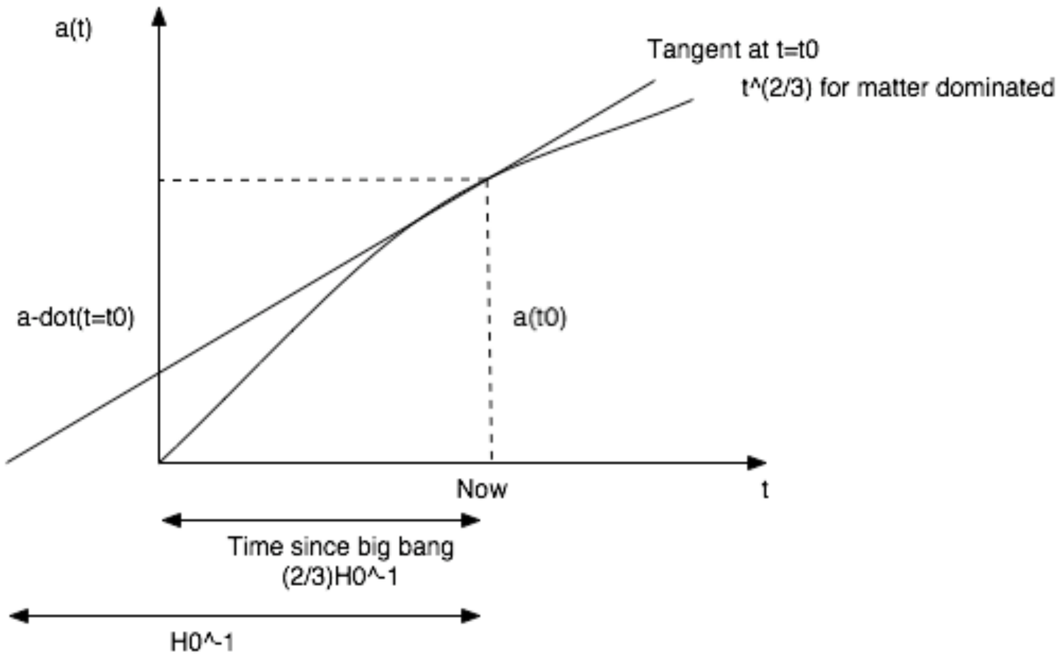
$$q_0 = \frac{4\pi}{3}G\frac{3}{8\pi G}\frac{\rho}{\rho_{crit}} = \frac{\Omega_0}{2}$$

$$\boxed{q_0 = \frac{\Omega_0}{2}}$$

for cold matter-dominated universe (us now).
If we can measure the rate of deceleration → measure density parameter.
q_0 is just a parameterization of H_0 (KE term in Friedman) and Ω_0 (PE term in Friedman).
Balance between KE and PE → measuring U_{total} or k (GR)
Directly measurable from observations of distant galaxies *(Picture 26)* i.e. how Hubble's "Law" varies as $f(t)$ (will come back to recent surprise in this).

The age of the universe

1. From the Hubble parameter



$$H(t_0) \equiv H_0 = \frac{\dot{a}(t_0)}{a(t_0)}$$

has units of 1/time (velocity per distance). Turn into SI units (Problems 2):

$$\frac{1}{H_0} \approx 10h^{-1} \times 10^9\, years$$

where h is the fudge factor as we don't know the number exactly. So for $H_0 = 100kms^{-1}Mpc^{-1}$, we get an age of 10^{10} years. For $H_0 = 50kms^{-1}Mpc^{-1}$ we get 2×10^{10} years. This is basically turning the universe's clock back to its starting point, when things were close together.
→ H_0 larger implies that the universe is younger.
$1/H_0$ is sometimes called the "Hubble Time". It is a characteristic age for the universe.

We proved earlier on that for $k=0$, $E_k = E_p$, and $\rho = \rho_{crit}$, we get that the age is $\frac{2}{3}H_0^{-1}$, as in the diagram above.

2. Radioactive Timescale
From knowledge of production of uranium isotopes in massive stars (the initial abundance), which are released through supernovae explosions and taken into other stars. Can look at the spectrum of these stars → get the abundance now. Knowing the half life for the decay process, we can get the age.

This was done in 2001 using U^{238}, where the age was obtained to be $(12.5\pm 3)\times 10^9$ years. You have to add $\sim 1\times 10^9$ years for the stars to start after the big bang.

3. Stellar evolution calculations for oldest stars in globular clusters
After they have finished nuclear burning, a white dwarf is left behind which then cools to a black dwarf through simple radiative cooling, which is well understood. So if you find the coolest (therefore faintest) stars in big clusters of stars, you can get an age through physical modeling. *(picture 31)*

The latest results indicate that the age of the oldest stars are $\sim (12 \rightarrow 13)\times 10^9\, years$.

Summary of Ages

H_0 $kms^{-1}Mpc^{-1}$	Closed $\Omega_0 > 1$ $t < \frac{2}{3}H_0^{-1}$	Critical $\Omega_0 = 1$ $t = \frac{2}{3}H_0^{-1}$	Open ("empty") $\Omega_0 << 1$ $t = H_0^{-1}$
50 $(h=0.5)$	$<13\times 10^9 y$ Could possibly be this.	$13\times 10^9 y$ OK…	$20\times 10^9 y$ OK..
100 $(h=1)$	$<6.5\times 10^9 y$ Can't be this	$6.5\times 10^9 y$ Can't be this	$10\times 10^9 y$ Could possibly be this, but the universe is not very empty.

Age estimates favour:
1. $h<0.75$ $\left(H < 75kms^{-1}Mpc^{-1}\right)$
2. Not closed universes

We will come back to this in the era of the cosmological constant.

Density of the Universe and Dark Matter
This is from direct mass measurements.

We know that

$$\rho_{crit} \sim 1.88h^2 \times 10^{-26}\,kg\,m^{-3}.$$

This is uncertain to at least 20%, as h is uncertain to at least 10%. This is from:

1. "Counting" stars via the sum of the starlight.
   - Very understood relations between $\sum_{stars} mass$ and $\sum_{stars} starlight$ (we know how stars radiate [black bodies] as a function of mass).
   - If you take an inventory of nearby galaxies, you get:

     $$\Omega_{stars} \equiv \frac{\rho_{stars}}{\rho_{crit}} \approx 0.005 \to 0.01$$

     i.e. $0.5\% \to 0.1\%$ of ρ_{crit}. So there isn't much stuff in stars cf. the size of the universe.
2. "Dark" Matter

- Has gravity but emits little EM radiation.
Effects of gravity can be seen using test masses e.g. hydrogen gas – watch the gas moving in the gravitational potential.
e.g. galaxy rotation curves.
(Picture 33)
Consider a spherical galaxy. Inside the galaxy,

$$\frac{GM(r)m}{r^2} = \frac{mv^2(r)}{r}$$

$$v(r) = \sqrt{\frac{GM(r)}{r}}$$

$$M(r) = \frac{4}{3}\pi\rho r^3$$

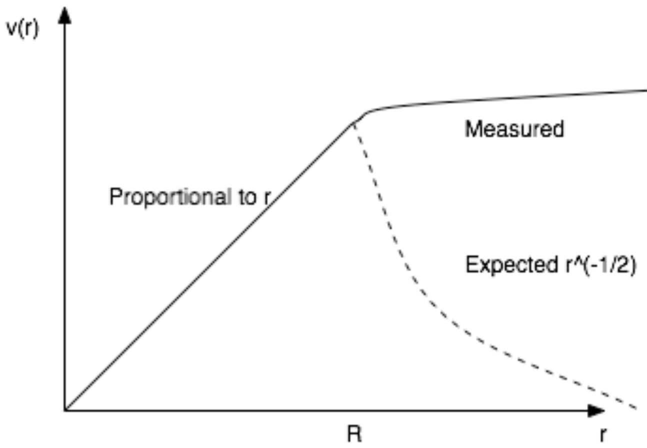
assuming constant density.

$$\to v(r) \propto r$$

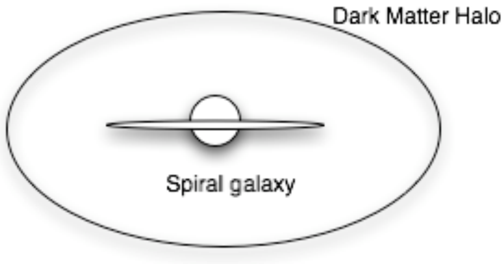
Outside the galaxy, there is no more mass.

$$\to v(r) \propto r^{-1/2}$$

(Keplerian motion)



Measure rotation curve with 21cm line of atomic hydrogen (Doppler → velocity as function of r).
Goes flat outside the "visible" galaxy.
→ faster than expected.
→ more mass present than is visible.



Spiral drawn edge on (visible part).
Must be a large halo of unseen or "Dark" matter giving up "additional gravity".

It turns out from computer models that a spiral galaxy is not stable without the dark matter halo.

$\Omega_{DM\ Halo} \approx 0.1$ i.e. much more mass than in stars, but still not enough.

e.g. galaxy clusters
There are several ways – two have been chosen to illustrate ithis.

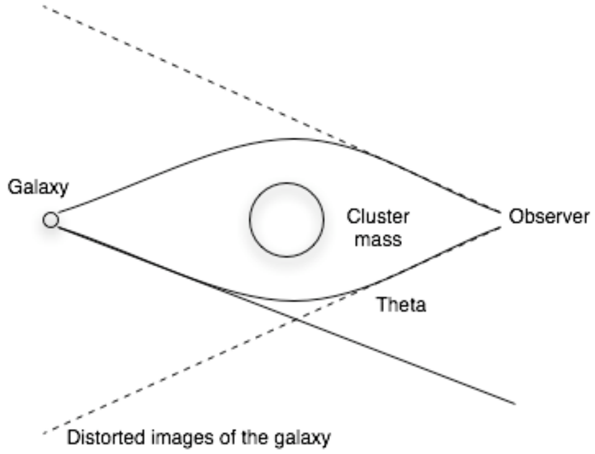
A general truth for any "bound" system of particles is the Virial theorem,

$$\langle E_k \rangle = -\frac{1}{2}\langle E_P \rangle .$$

This is very easy to show for a simple circular orbit with a dominant central mass.
Measure the velocities of the galaxies (Doppler shifts in the spectra) → E_k. Estimate the mass in the galaxies (stars) → E_P. It turns out that the galaxies are moving too fast to be confined by the gravity of visible matter.

This implies that there must be dark matter within and between the galaxies.

Gravitational lensing in clusters
(Picture 35)



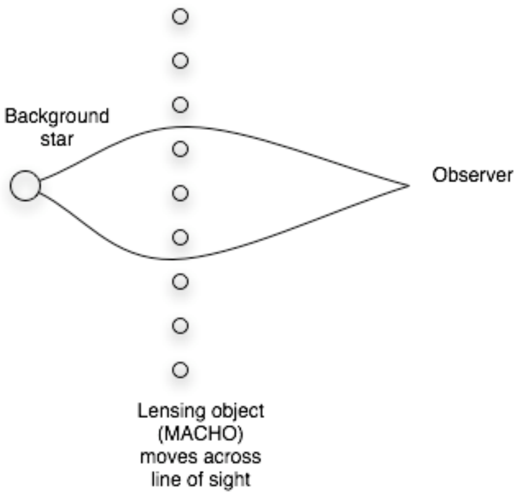
Distorted images of the galaxy are seen.
→ $M(r)$ for cluster. → measure all gravitating matter.

Summary for clusters:
$\Omega_{0,matter} \approx 0.25\Omega_0$ (25% of critical). Note that this is 25x than in stars.

Ultimate expression of the Copernican principle – we are not special – we are not even made of the matter which dominates the universe.

What is the Dark Matter? This is a big research topic. Two possibilities:

1. Baryonic matter (jargon term for ordinary matter). "Almost" stars, "faint" stars (need $\sim 0.08M_{sun}$ to switch on nuclear reactions → light), old stars, cold stars, black holes (worn out or collapsed into black hole). These are all compact masses. Sometimes called candidate MACHOS (Massive Compact Halo ObjectS).
   Can search using gravitational lensing inside our "halo".



Can't see the distorted images since splitting is so small, but you can see the additional light when rays are bent towards us.
Look at missions of star : finding a needle in a haystack : at once. Occasionally, one gets brighter.
→ lots of interesting astronomy, but only $\leq 6\%$ of the inferred halo mass can be due to MACHOS.