Lectures 21-22

Dark Energy and Acceleration of the Universe (c/ntd)

Hubble plot (revisited).
$v = Hd$ (velocity = Hubble parameter x distance).
The distance is the "proper distance", i.e. the physical distance. So at high redshift, i.e. large distance, we are looking at the universe when it was younger, smaller and expanding more quickly. So the relationship in practice ceases to be linear (as above).

Now conventional to plot the Hubble diagram with the axis swapped over. We used to plot it with D along the x-axis, and v along the y-axis. Now plot it with v along the x-axis, and D along the y-axis.

(Larger H (faster) implies a smaller, younger universe which is expanding more quickly in the past due to gravity decelerating now.)

Revisit stretching as a function of time. *(Picture 29)*. All topologies would have the same Hubble constant now, $H(t) = \frac{\dot{a}(t)}{a(t)}$. H was higher in the past for a decelerating universe. (See examples 3, question 2.2)
For larger D and Z, we are looking at the universe when it was stretching faster. For $Z = 1$, the universe was a factor 2 $(= 1 + Z)$ smaller. *(Picture 26)*. These models are (still) plotted in terms of the proper distance, i.e. $a_0 x_0$. As we can't measure this, we have to use a practical distance measure, via the apparent brightness, i.e. the luminosity distance. They plot this in terms of Z not "velocity", i.e. in terms of the stretch factor $\left(a \propto (1+Z)^{-1}\right)$. The apparent brightness (see earlier lecture) = luminosity / $4\pi a_0^2 x_0^2 (1+Z)^2$, where the $(1+Z)^2$ is from the photons having less energy, and arriving less often, due to stretching. The effect is that even for an empty universe, the "practical" Hubble plot is curved. (Plotting brightness of galaxies (y) [less bright upwards, more bright downwards] vs. their redshift (x) will curve the line upwards, i.e. making the galaxies at higher redshift fainter.) So a flat line here would represent a faster expansion at higher Z, therefore the universe would be decelerating (see above). If the line were more curved, then the universe would be accelerating, i.e. the universe expanded slower in the past than now. It turned out that the line was more curved, i.e. our universe is accelerating.

The Hubble plot is a direct diagnostic of the stretching of the universe (Z) as a function of time, i.e. distance (apparent brightness, i.e. luminosity distance).

Latest results: type 1 supernovae either have lower redshift (less stretched space) than expected, or they are less bright than expected compared with a constant expansion rate.
→ the universe is accelerating. *(Pictures 48, 49)*
Therefore if it is accelerating, it was stretching more slowly in the past, therefore expanding for longer, therefore older.
(Picture 51)

Recap on the Basic Equations

Friedman

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G\rho}{3}$$

ρ is the total gravitating matter density ($k = 0$ case; no Λ. $E_k = E_p$)

Fluid (density behaviour):

$$\dot{\rho} + 3\left(\frac{\dot{a}}{a}\right)\left[\rho + \frac{P}{c^2}\right] = 0$$

Pc^{-2} is the effect of pressure relativistic effect. $E = mc^2$, therefore any energy is equivalent to mass for normal matter (inc. dark matter).

Acceleration equation:
(Derived from the other two; force equation)

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left[\rho + \frac{3P}{c^2}\right]$$

$\left[\rho + \frac{3P}{c^2}\right]$ is the effective energy density of everything including the pressure.

NB:
$\rho > 0$ for any type of matter.
$P = 0$ for any slowly moving "normal" matter (including dark matter).
$P = \frac{\rho c^2}{3}$ for highly relativistic matter, e.g. radiation. Therefore $\rho + \frac{3P}{c^2} \equiv 2\rho$, therefore more gravity from radiation than from slow matter.
But if negative pressure, then $\rho + \frac{3P}{c^2}$ could be less than 0, and could provide acceleration $\frac{\ddot{a}}{a} > 0$. Einstein invented a "fudge factor" in his equations effectively antigravity to balance positive gravity to make a static universe (he did this on philosophical grounds), $\frac{\ddot{a}}{a} = 0$. He had a term called Λ, the cosmological constant, in his equations. However:

i. A balance of large forces is not stable to small perturbations (changes get amplified)
ii. The universe was then found to be expanding, not static.

Einstein called it his "greatest blunder". However it is now taken seriously, as:

i. The universe is flat (CMBR), however we are missing ~70% of the energy density required to give ρ_{crit}
ii. Some ideas of the physics of "empty space"

(Liddle Chapter 7: Cosmological Constant)
"Cost" in energy for having a volume of space. Quantum theory tells us that the vacuum is not empty; it consists of virtual particles and antiparticles creating and annihilating themselves, producing photons. $\Delta E \Delta t \le \hbar$; $\Delta E = \Delta mc^2$. This picture has recently been directly verified via a lab experiment, the "Casimir effect".

Basic points:

- More space, more energy of space.
- Vacuum has negative pressure (tension).

Illustration:
Consider a piston in a cylinder. Put in 1) normal gas, 2) a false vacuum.

1) Normal gas:
   Gas has a positive pressure → pushes the piston out. Gas does work $W = pdV$. Internal energy falls, and pressure falls.
2) False vacuum:
   Pulls on the piston because as the piston is moved out, you are creating more space therefore more energy, therefore you have to do work pdV. Hence work has the opposite sign. Therefore this false vacuum would exhert a negative pressure (tension).

Work done $W = -pdV$, where p is the effective pressure of the vacuum. The amount of energy increase for change in volume $dV = \rho_{vac}c^2 dV$ the mass equivalent density. Hence $p_{vac} = -\rho_{vac}c^2$. Cf. normally $p = \frac{1}{3}\rho c^2$ for highly relativistic matter.

Can get the same result from the fluid equation applied to a vacuum.

$$\dot{\rho}_{vac} + 3\left(\frac{\dot{a}}{a}\right)\left(\rho_{vac} + \frac{P_{vac}}{c^2}\right) = 0$$

$\dot{\rho}_{vac} = 0$ i.e. constant energy density for space. Hence $\left(\rho_{vac} + \frac{P_{vac}}{c^2}\right) = 0$, so $P_{vac} = -\rho_{vac}c^2$. i.e. as space is stretched, each new m^{-3} has the same energy density. This is the cosmological constant.

So the cosmological constant has the following properties:

i. "Normal" energy density ρ_{vac} contributing positive gravity
ii. Negative pressure, which in GR corresponds to negative gravity.

So the effective mass density $\left[\rho_{vac} + \frac{3P}{c^2}\right] = -2\rho_{vac}$, where again a negative sign has appeared. So space will accelerate if this overcomes positive gravity from matter.

See problems 3, questions 8 and 2.4 for how the relative contributions of radiation, matter and Λ change as universe expands.

Including these terms elegantly in the standard equations:

$$p \rightarrow p_m + p_\Lambda$$

$$\rho \rightarrow \rho_m + \rho_\Lambda$$

Define $\rho_\Lambda = \rho_{vac} = \frac{\Lambda}{8\pi G}$

- Friedman

  $H^2 = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda)$

  $k = 0$ case. For a flat universe, $\rho_m + \rho_\Lambda = \rho_{crit}$. $\Omega_m + \Omega_\Lambda = 1$
- Acceleration equation

  $\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}\left(\rho_m + \frac{3p_m}{c^2}\right) - \frac{4\pi G}{3}\left(\rho_\Lambda + \frac{3p_\Lambda}{c^2}\right)$

  With $\rho_\Lambda = \frac{\Lambda}{8\pi G}$, $p_\Lambda = -\rho_\Lambda c^2 = -\frac{\Lambda c^2}{8\pi G}$:

  $\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}\left(\rho_m + \frac{3p_m}{c^2}\right) + \frac{\Lambda}{3}$

  With these extra parameters, ρ_Λ and p_Λ (and the negative sign in P_Λ), we can model the behaviour in the Type 1a supernovae Hubble plot. See examples 2, questions 2.1 and 2.2. eventually the cosmological constant's negative gravity overcomes positive gravity, and at late times the universe begins expanding exponentially. *(Picture 50 shows the rate of change of expansion as a function of time – like tree rings.)*

This is not fully understood. It is merely a route to the solution. The problem is the size of the effect: it appears to be too small. 70% of ρ_{crit} is still a tiny energy density. Current ideas based on quantum theory (see earlier) suggest that ρ_{vac} is $10^{55} \to 10^{120}$ times greater than observed. "Greatest fine tuning problem in physics".