Lectures 18-19
Summary of typical values:

|  | Nucleosynthesis | Atomic synthesis <br> (decoupling) |
| :--- | :--- | :--- |
| $t$ | Few minutes | 350,000 years |
| $T$ | $\sim 10^{10} \rightarrow 10^{9} \mathrm{k}$ | 3000 k |
| $E$ | MeV | eV |
| What? | $p+n \rightarrow$ nuclei <br> $e^{-}$remain free <br> Photons interact with both <br> nuclei but particularly the <br> $e^{-}$ | Nuclei $+e^{-} \rightarrow$ atoms <br> Photons cease to interact <br> and form CMBR ("free <br> flying") |

## Distances in Cosmology

(Part of Liddle Advanced Topic 2)
See http://www.anzwers.org/free/universe.html for a short, non-mathematical introduction, and the cartoon.
Problem: universe is a) big, b) expanding (at different rates as $f(t)$ ), and c ) information travels at a final speed.

2 galaxies close ( 2 billion light years) when universe was 1 billion years old. First galaxy emits a pulse of photons. The second galaxy does not receive these photons until 13 billion years have passed, i.e. the universe is then 14 billion years old. This is the "light travel time". By this time, the universe has (let's say - it depends on the model used) expanded to 26 billion light years. This is the "co-moving distance" or "coordinate distance". We see an image of the $1^{\text {st }}$ galaxy when it was only 1 billion years old and 2 billion light years away (arbitrary choices of numbers). This applies to the "angular diameter distance".

More formally, the options for distance measures are:

1. "Proper" distance, which is the physical distance "now". "tape measure distance". This is impossible to measure in practice due to the finite speed of light.
2. Co-moving, or coordinate, distance: associated with the reference frame which expands along with the universe. This tells us where galaxies are now although we view the universe at a much earlier phase when it was younger and smaller.
(The very edge of the universe now is in fact 47 billion light years away, cf. the 26 billion years in the cartoon on the website. - these galaxies are only half way out.)
The geometry of the universe ( $k>0, k<0$, etc.) is encoded in the commoving coordinate system. A 2D analogy is a Cartesian set of coordinates on an expanding rubber sheet which may not be flat $(k=0)$.
3. Light travel time distance

This is just the time taken for light from distant galaxies to reach us. This is what is meant when we say that the universe has radius 14 billion years, i.e. it
is 14 billion years old - light from more distant sources has not had time to reach us. Problem: we don't know when light started out.

None of these are very useful.
We want to use distances based on observables such as apparent brightness, apparent angular size, and the redshift (stretch of the universe).
4. Luminosity distance: related to the apparent brightness vs. $z$, the redshift. It is the distance a galaxy appears to have assuming a perfect inverse square law.

Imagine a distant galaxy. Coordinate distance is $x_{0} a_{0}$, which is the physical distance (tape measure distance) when we observe the light. $a_{0}$ is the stretch factor (See earlier in notes). Flux at the earth is $\frac{L}{4 \pi\left(x_{0} a_{0}\right)^{2}} W m^{-2}$ (flux measured in (the equivalent of) a 1 square metre telescope).

Note: since space can be non-Euclidean, the inverse square law may not be true since $x_{0}$ encodes the fact that the rubber sheet can be not flat in the 2D analogy, i.e. areas of spheres do not grow proportional to $r^{2}$ in 3D case. Luckily it looks like $k=0$, so we don't have to worry too much about this.

NB: Liddle changes from $x$ for co-moving coordinates to $r$.
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## Universe is expanding

This has 2 effects on the photons, since there are two lengths involved. It will affect both the wavelength, and the separation, of the photons - both of these will be stretched by $a\left(t_{0}\right) / a\left(t_{\text {enited }}\right)=1+Z$ (see earlier lectures). Hence the light will be dimmed by $1 /(1+Z)^{2}$ (lower energy, and less frequent).

So the cosmological flux $=f=\frac{L}{4 \pi\left(a_{0} x_{0}\right)^{2}} \frac{1}{(1+Z)^{2}}$. Therefore, compared with
Euclidian $f=\frac{L}{4 \pi D_{L}{ }^{2}}$, where $D_{L}$ is the luminosity distance, we have
$D_{L}=\left(a_{0} x_{0}\right)(1+Z)$, where $a_{0} x_{0}$ is the "proper", "physical", or "tape measure" distance (three names for the same thing).
Therefore distant objects look further away than they are due to redshift reducing the apparent luminosity. They can be much dimmer than they would have been otherwise.
5. Angular Size Distance: this is a measure of how large objects appear to be (referring to a Euclidian universe as for $D_{L}$ ).

Take an object of fixed physical size $L$ (i.e. it does not expand with the universe), which emits a light when it is at a physical distance $a\left(t_{\text {emit }}\right) x_{0}$ at $t=t_{\text {enit }}$. We can measure the angular size of the object $d \theta$ when the object emitted the light. As the universe is expanding, this angular distance will decrease with time as the object is "carried" to distance $a\left(t_{\text {obs }}\right) x_{0}$, which is the physical distance when the light arrives at the observer.

Note that $d \theta$ is small, and can be around 1 degree in the case of the CMBR "peak" the biggest thing we see. $d \theta$ does not change as the universe does not expand since expansion is self-similar, and there is no differential stretching to distort angles.
$\rightarrow$ angular size we perceive $d \theta=\frac{L}{x_{0} a\left(t_{\text {emit }}\right)}$, but since $a\left(t_{0}\right) / a\left(t_{\text {emit }}\right)=1+Z$, $d \theta=\frac{L}{x_{0} a\left(t_{0}\right)}(1+Z)$, where $x_{0} a\left(t_{0}\right)$ is the physical distance at reception time. $D_{A}=$ angular size distance $=\frac{x_{0} a_{0}}{1+Z}$.
Therefore $D_{A}=D_{L} /(1+Z)^{2}$.
Note "funny behaviour" of $D_{A}$ for objects at very great distances since we see them when they were very much closer to us (see the cartoon again). This means that they can appear to be larger than objects less far away (apparently paradoxical, but true). The problem is that we don't have "standard rods" (or "tape measures") perpendicular to the line of sight stretched out across the universe.

But we do have the favoured "blob sizes" in the CMBR: we know their physical dimension from physical arguments (see earlier lecture on CMBR). Therefore we can calculate the angular diameter distance and can hence demonstrate that the coordinates of the universe are flat.
(Picture 45)

## Dark Energy and the Acceleration of the Universe

How does the geometry become flat ( $k=0, \rho=\rho_{\text {crit }}, \Omega_{\text {star }}=1$ ) when $\Omega_{\text {baryons }} \sim 0.04$, i.e. $4 \%$ of $\rho_{\text {crit }}$ ), $\Omega_{\text {non-baryonic, } D M} \sim 0.26$, and hence we're missing $\Omega=0.7$ ?

Evidence from several different experiments:
i. Measurement of high $Z$ supernovae, which are "standard candles". We don't have any objects of absolutely known luminosity. We do have objects of approximately the same luminosity. These are the type 1a supernovae. (Pictures 46, 47)
The thermonuclear explosions caused by accretion onto the white dwarf star in a binary system give out approximately the same amount of energy at all redshifts. They are not affected by their environment, only the physics of the star.
We can use them to extend the Hubble plot (picture 26) to very large redshifts; can see out to $z \sim 1$.

