6. Gamma Decay

A large part of our understanding about the structure of nuclei comes from looking at electromagnetic transitions.

- i. Easy to measure
- ii. Easy to calculate

EM interactions are the best-known field theory. Interactions of charges and currents can be exactly calculated in principle. If we take two states with a gamma decay, then we need to know something about the wavefunctions of the original and final states. If we know the wavefunctions, then we can calculate the gamma decay (its' probability, angular distribution, ...). If we measure the gamma decay, then we can compare the results with the calculations. This means that we can infer things about the real wavefunctions.

What are gamma rays? They are electromagnetic waves or photons emitted by atomic nuclei. X-rays and gamma rays differ in source. In general, X-rays have a lower energy than gamma rays. Generally, $E_{X-ray} < 100 keV$, while $E_{\gamma-ray} > 50 keV$, but this is not necessarily so.

What is gamma decay sensitive to? It depends on spin-parity changes (multipolarity); the energy difference between the states; nuclear shape and size; and structure of participating states (e.g. rotational, vibrational, single-particle-like).

Gamma ray techniques measure:

- i. Level energies
- ii. Nuclear spin parities
- iii. Lifetimes of excited states
- iv. Magnetic moments

What states can gamma decay?

In principle anything but the ground state, i.e. all excited states. However, excited states can decay three ways – either gamma decay, beta decay, or alpha decay. These are examples of EM decay, weak decay, and strong decay respectively.

 $\lambda = \sum \lambda_i$ decay probability sum of all individual decay probabilities.

Timescales: strong interactions ~ $10^{-22} s$, EM interactions $10^{-13} \rightarrow 10^{-15} s$ and weak interactions $\ge 1s$ (is this correct? See particle physics – $10^{-10} s$...).

In general, if there is sufficient energy then the states will decay though particle decay. Below a threshold energy (particle threshold), there is not enough energy to loose a particle, hence it will decay through gamma decay.

Examples:

- 1) Beta decay down to one state, and then a gamma ray decay down to a lower state. Hence the gamma ray will be emitted almost simultaneously with the beta decay.
- 2) H1 reactions. Projectile hits target. Forms a highly excited compound nucleus. This gives out neutrons first; then continues to collapse through emission of gamma particles.

6.2 Angular Momentum considerations

Photons carry intrinsic spin of $1\hbar$. They only exist in $s_{z} = \pm 1$.

Like beta particles, gamma rays have difficulty in carrying away orbital angular momentum $\,\ell$.

 $\underline{p} = \hbar \underline{k}$

Angular momentum carried off $\ell \hbar = \underline{r} \times p$.

Energy of the gamma ray is $E_{\gamma} = pc$

$$\Rightarrow r \approx \frac{\ell \hbar c}{E_{\gamma}}$$

This gives a radius of around $100\ell fm$ for $E_{\gamma} \sim 2MeV$.

i.e. gamma rays are emitted far from the centre of the nucleus. i.e. by the tails of the nuclear wavefunction.

From $\ell \hbar = \underline{r} \times \underline{p}$, $kr \approx \ell$ in order to remove angular momentum $\ell \hbar$, but $kR \ll 1$, where *R* is the nuclear radius.

So gamma ray transitions with $\ell \neq 0$ are inhibited. (actually by a factor $\propto (kR)^{2\ell}$).

The total angular momentum carried away is $\underline{L} = \underline{\ell} + \underline{s}$.

Rule: L = 0 is forbidden. So monopole radiation is not allowed since $\underline{\ell} = \underline{r} \times \underline{p}$, ℓ is perpendicular to p and can't cancel out the $s_z = \pm 1$.

The photons emitted are characterised by σL . $\sigma = E$ if the transition is electric, or $\sigma = M$ if the transition is magnetic. These have different parity changes.

E1: electric dipole radiation. L = 1; changes parity. M1 magnetic dipole radiation. L = 1; won't change parity. E2 Electric quadrupole radiation L = 2; won't change parity. M2 Magnetic quadrupole radiation L = 2; changes parity. E3 changes parity M3 won't change parity

Radiation Pattern

("angular distribution")

If a sample of excited nuclei are aligned along a lab axis, then the decay radiation will be emitted anisotropically. The pattern is exactly understood, and is entirely determined by the angular momentum involved (I_i, I_f, L) . This pattern may be used to determine initial and final spins of the nucleus and L of the radiation. This is independent of E or M type.

Example

If we have $I_i = 1$ and $I_f = 0$, and we align the upper state in the $m_z = 0$ substate only (the bottom state has m = 0). This means that all the spins will be in the x-y plane. The gamma ray distribution is exactly the Hertzian dipole pattern.

FIGURE 9-3. Radiation pattern of an accelerated electric dipole.

 $W(\theta)$, which is a vector which starts on (0,0,0), and ends on a point on the surface of the donut, is proportional to the probability of emitting a photon at angle θ .

NB: photons can't be emitted along the z-axis (in this case), because they carry $m_z = \pm 1$, but $\Delta m = m_i - m_f = 0$, hence angular momentum would not be conserved.

Parity Rules

Think of a transition going from energy level E_i , with I_i, π_i , and E_f with I_f, π_f , emitting a photon with energy E_{γ} .

EL	Parity change?	Example	ML	Parity change?	Example
E1	Yes	$1^- \rightarrow 0^+$	M1	No	$1^+ \rightarrow 0^+$
E2	No	$2^+ \rightarrow 0^+$	M2	Yes	$2^- \rightarrow 0^+$
E3	Yes	$3^- \rightarrow 0^+$	M3	No	$3^+ \rightarrow 0^+$
E4	No	$4^+ \rightarrow 0^+$	M4	Yes	$4^- \rightarrow 0^+$

E4 transitions are extremely rare.

(Remember one of these, and everything else alternates.)

Angular Momentum Conservation

Radiation carries away $L\hbar$ of angular momentum.

$$\underline{I_i} = \underline{I_f} + \underline{L}$$

This can be represented by a vector triangle. Therefore the possible L-values are: $|I_i - I_f| \le L \le I_i + I_f$, but L = 0 is not possible, and transitions proceed by the least-hindered process. This is normally the lowest allowed L-value. There is a common

9.2 ELECTRIC DIPOLE RADIATION

exception to this: "enhanced" E2 can often compete with magnetic dipole M1. These are called "mixed transitions" when more than one L-value contributes to the transition.

Example

Consider a vibrational nucleus. We have states 0^+ (ground), 2^+ (1 phonon), and 4^+ 2^+ and 0^+ (2 phonon states at around twice the energy of the first).

 $2^+ \rightarrow 0^+$ (1 phonon to ground) has to take away L = 2 only. Pure E2.

 $0^+ \rightarrow 0^+$ (1 phonon to ground) can't happen.

 $0^+ \rightarrow 2^+$ (2-1 phonon). L = 2 only. Pure E2.

 $2^+ \rightarrow 0^+$ (2 photon to ground state) is pure E2.

 $2^+ \rightarrow 2^+$ (2-1 phonon) L = 1, 2, 3, 4. $\sigma L = \underbrace{M1, E2}_{possibly mixed}, \underbrace{M3, E4}_{negligible}$. The final two are

negligible as they are very hindered.

 $4^+ \rightarrow 2^+, 0^+$ (2-2 phonon). These have very small energy differences, so in general we don't see (and don't consider) these transitions.

 $4^+ \rightarrow 0^+$ (2-ground). L = 4 only. E4: very slow, so not observable.

 $4^+ \rightarrow 2^+$ (2-1 phonon). L = 2, 3, 4, 5, 6. No parity change.

 $\sigma L = E2, \ \underline{M3, E4, M5, E6}$

very hindered, so not observed

Transition rates

These are determined by the Fermi Golden Rule.

$$\lambda = \frac{2\pi}{\hbar} \left| \int \Psi_f * \hat{M}(\sigma L) \Psi_i dV \right|^2 \frac{dN}{dE}$$

dN/dE is the density of states of the phonon, which gives a E_{γ}^{3} dependence.

 $\hat{M}(\sigma L)$ is the operator for σL -type transitions. For electric multipole transitions (EL), the form of $M(EL) \sim \frac{1}{k} (k_r)^{\ell} P_L(\cos\theta) \cdot (k_r)^{\ell}$ is where the hindrance factor $(kR)^{2(L-1)}$ comes in.

Transition Operator for Electric Dipole Radiation

Electric dipole is charge x distance = ez (a charge of +e separated from a charge of -e by a distance z).

E1 rate $\propto \left| \int \psi_f * ez \psi_i dV \right|^2$ (Part of Fermi Golden Rule. $z = r \cos \theta = r^1 P_1(\cos \theta)$) Since "z" is an odd function, then the integral is zero if the initial state ψ_i and final state ψ_f have the same parity ($\psi_f * \psi_i = even$).

Rule:

E1 transitions connect states of opposite parity (Similarly, a nuclear state of definite parity cannot have a static electric dipole moment).

6.3 Single-particle "Weisskopf" Estimates

 $\left|\int \psi_{f} * M(\sigma L)\psi_{i}dv\right|^{2}$ is calculated for a single proton making a simple transition between two shell model orbitals. ψ_{f} and ψ_{i} are therefore known. Example: the rate

$\sum_{\gamma} \sum_{\gamma} \sum_{\gamma$				
$\lambda(E1) = 1.0 \times 10^{14} A^{2/3} E^{3}$	$\lambda(M1) = 5.6 \times 10^{13} E^3$			
$\lambda(M1) = 7.3 \times 10^7 A^{4/3} E^5$	$\lambda(M2) = 3.5 \times 10^7 A^{\frac{2}{3}} E^5$			
$\lambda(E3) = 34A^2E^7$	$\lambda(M3) = 16A^{4/3}E^7$			
$\lambda(E4) = 1.1 \times 10^{-5} A^{\frac{8}{3}} E^{9}$	$\lambda(M4) = 4.5 \times 10^{-6} A^2 E^9$			

of E2 transition = $\lambda(E2) = 7.3 \times 10^7 A^{\frac{4}{3}}E_{\gamma}(MeV)^5$.

Lifetime Estimates

If the decay can only go through one process, then the mean lifetime of the initial state will be = $\tau = \frac{1}{\lambda(\sigma L)}$.

If there are two possible transitions (say, $4^+ \to 3^+$ through M1, or $4^+ \to 2^+$ through E2), then $\lambda_{total}(4^+) = \lambda(E2) + \lambda(M1)$, and $\tau(4^+) = \frac{1}{\lambda_{total}}$.

The observed decay rates can be compared alongside the single particle estimates to make judgments like "enhanced rate", or "hindered".

E.g. in rotational nuclei where a large number of nucleons, N, are coherently involved in motion (involving a time-dependent electric quadrupole shape), the electric quadrupole (E2) transition rates are enhanced.

$$\int \psi_f * M(E2) \psi_i dv \sim N \times \text{ single particle.}$$
$$\left| \int ... dv \right|^2 \rightarrow \text{rate of } N^2 .$$
$$N^2 \sim 100 - 150 \text{ is common in rotational nuclei.}$$

From graph:

¹²⁰*Sn* . 2⁺ state (at say 100kev) to 0⁺ state (E2 transition). The internal conversion to gamma decay ratio $\alpha(k - shell) = 1.0$, so $\lambda_e = \lambda_\gamma$, and the decay rate of the 2⁺ state is doubled, and hence τ is halved.

$0^+ \rightarrow 0^+$ transitions

Since L = 0, γ -transitions are not allowed, $0^+ \rightarrow 0^+$ transitions can not proceed by the emission of a gamma ray. They can proceed by internal conversion (usually) although other possible processes are:

- 1. 2-photon emission
- 2. Pair production $(e^+ + e^-)$ if the transition energy is greater than $2m_ec^2$.