4. Alpha decay

This is a process through which a heavy nucleus can reduce its charge, and thus reduce the disruptive Coulomb force. The emission of the alpha particle is particularly favoured because it is tightly bound, i.e. it has a large E_B.

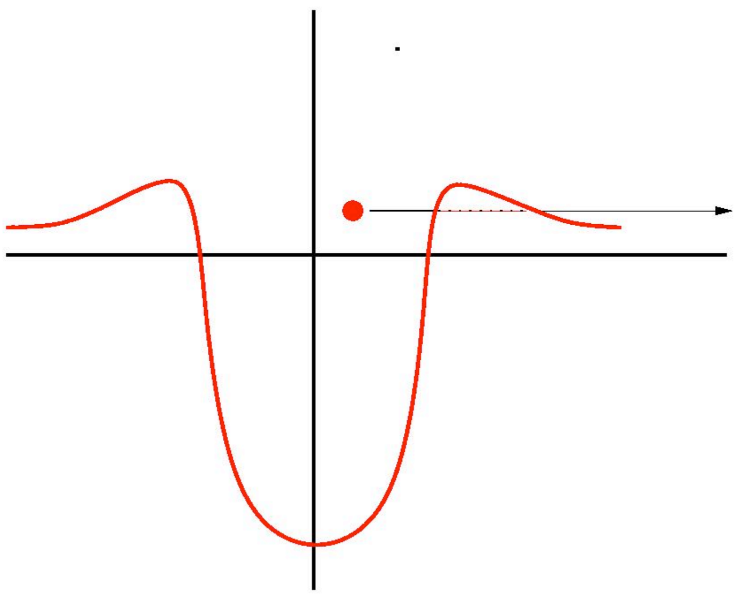
We have already seen that the energy release is

$$Q = E_B\left({}^4He\right) + E_B\left(daughter\right) - E_B\left(parent\right)$$

This Q value is equal to the kinetic energy of the alpha particle, plus the kinetic energy of the daughter nucleus.

Theory of Alpha Emission (1928)

We assume that the alpha particle is pre-formed and moving in a spherical potential of what will be the daughter nucleus. There is two parts to the potential well: the nuclear attraction due to the strong nuclear force, plus the Coulomb repulsion between the alpha particle and the daughter nucleus charge. (Outside the nucleus, it will be $eV = \frac{(Z-2)2e^2}{4\pi\varepsilon_0 r}$. Inside, it will flatten out. It must end up flat in the centre.)



The red curve represents the coulomb force. The red dot is the alpha particle. The difference between the arrow and the line is the Q-value. The alpha particle spends a lot of time bouncing between the barriers before it manages to get through it.

a is the point where the coulomb force becomes above 0. b is the point where the Q-value exceeds the coulomb force. B is the height of the Coulomb barrier above the zero point, at its' highest point. $V_0 \approx -35MeV$ is the maximum depth of the well. The velocity of the alpha particle in the nucleus will depend on its kinetic energy, given by $V(\alpha) = \sqrt{\frac{2E}{m_\alpha}} = \sqrt{\frac{2(Q+V_0)}{m_\alpha}}$.

Classically, the alpha particle moves with kinetic energy equaling $Q+V_0$ inside the well, but cannot escape because the region between a and b forms a potential (Coulomb) barrier. The height and width of the barrier determine the probability of tunneling. In ^{238}U, the alpha particle hits the barrier on average 10^{38} times before it escapes. (10^{21} hits per second, for 10^{9} years).

Very crudely, the barrier penetration probability is given by an exponential factor.

$$P \approx e^{-k_2(b-a)}$$

where $b-a$ is the width of the barrier, and $k_2=\sqrt{\frac{2m}{\hbar^2}\frac{1}{2}(B-Q)}$, where $\frac{1}{2}(B-Q)$ is the average height of the barrier, with $B=\frac{(Z-2)2e^2}{4\pi\varepsilon_0 a}$.

The radioactive decay constant λ (as in $N(t)=N_e e^{-\lambda t}$) is $\lambda = fP$, where P is the probability of getting through the barrier, and f the number of tries the alpha particle makes to get through the barrier, normally $f=v/a$. f is called the "assault frequency". This simple theory is able to account for the very rapid Q-dependence on half-life as illustrated in the Geiger-Nuttall plots.

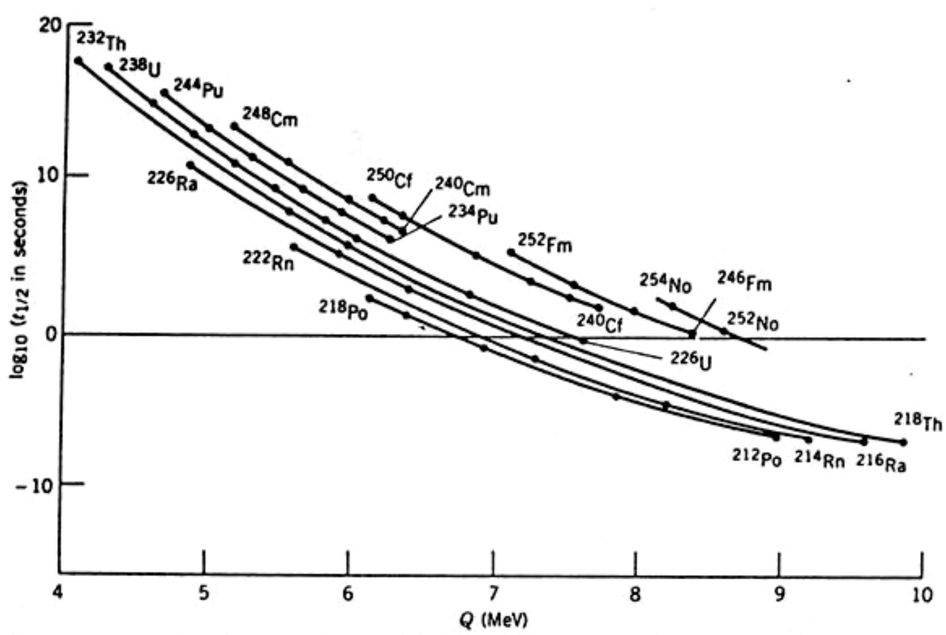


Figure 8.1 The inverse relationship between α-decay half-life and decay energy, called the Geiger-Nuttall rule. Only even-Z, even-N nuclei are shown. The solid lines connect the data points.

NB: the time scale is very large on this, ranging from microseconds to the age of the universe.

Example:

^{232}Th has $Q = 4.08 MeV$ and $T_{1/2} = 1.4 \times 10^{10}$ years.

^{218}Th has $Q = 9.85 MeV$ and $T_{1/2} = 1 \times 10^{-7}$ seconds.

So a factor of 2.4 in Q leads to a factor of 4×10^{24} in $T_{1/2}$.

Note:

- All commercial alpha sources have E_α in the range $5 \to 8 MeV$.
- The $T_{1/2}$ sensitivity to radius parameter a (4% charge in a gives a factor of 5 in $T_{1/2}$) may be used to measure the radius of these isotopes.

"Fine structure" of the alpha spectrum

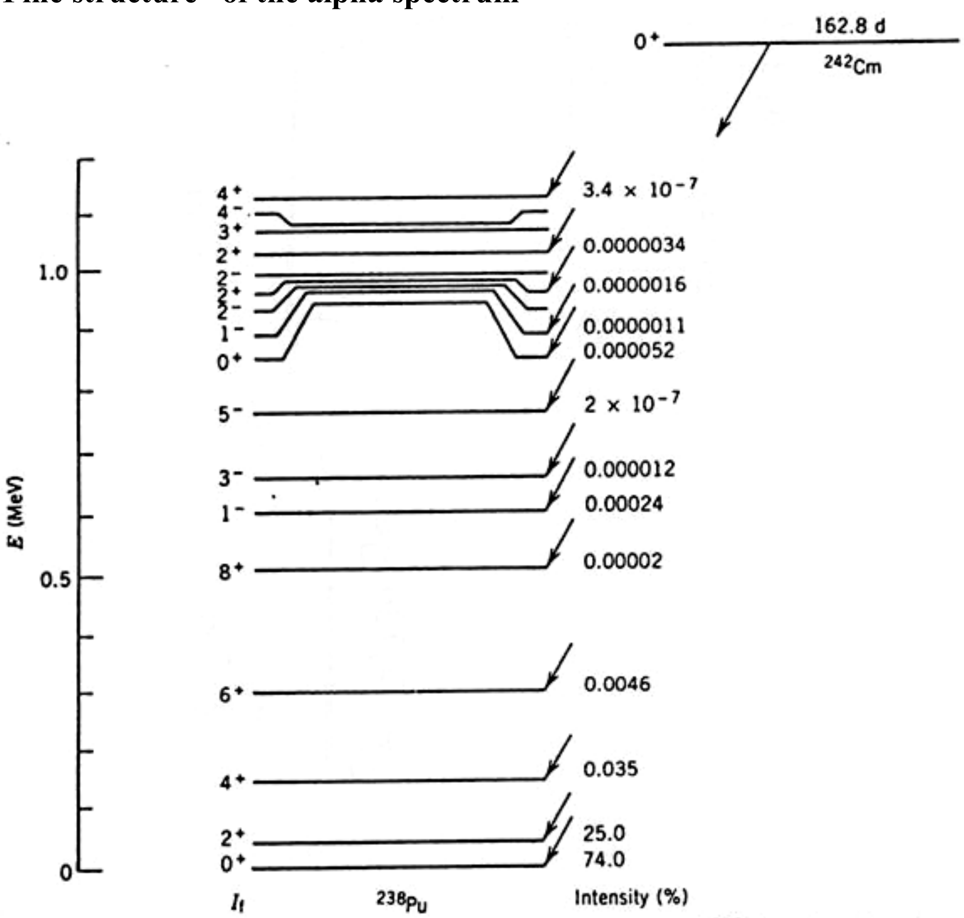


Figure 8.7 α decay of ^{242}Cm to different excited states of ^{238}Pu. The intensity of each α-decay branch is given to the right of the level.

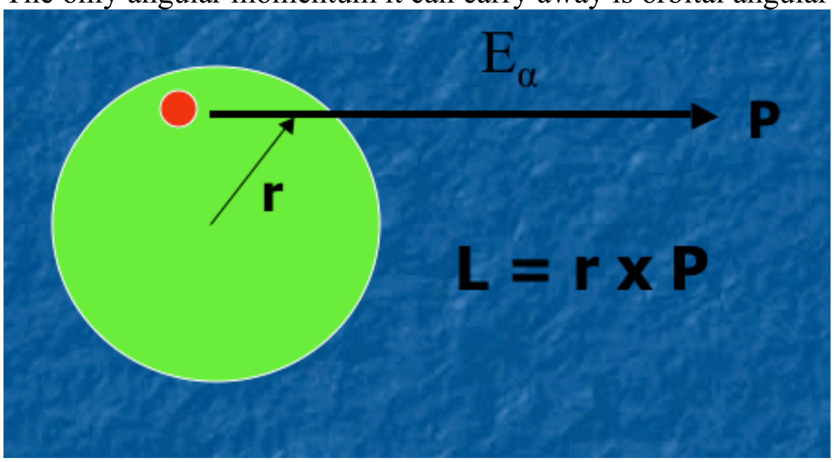
Using ^{242}Cm as an example, we note that:

1. In general higher Q transitions have higher decay rates (this is due to the Q-dependence on P, the probability).

Call the transition to the 0^+ state α_0, and that to the 2^+ state α_1. On a graph of the number events vs. the alpha energy, there will be a large peak at α_0 (around $6MeV$), a smaller peak by 1/3 at the lower energy of α_1, and then a small background of the other possible states.

2. Decays to states of higher spin are more desired (compare 8^+ with 1^-).
3. Decays to some states are not observed at all (2^-, 3^+, 4^- are forbidden by parity conservation).

Alpha particle has spin-parity $= 0^+$.
The only angular momentum it can carry away is orbital angular momentum.



$\underline{L} = \underline{r} \times \underline{P}$; $\underline{L} = \ell\hbar$ where $\ell = 0,1,2,...$

Rough estimate of r:

$$r \approx \frac{\ell\hbar}{p} = \frac{\ell\hbar}{\sqrt{2m_\alpha E_\alpha}} = \frac{\ell\hbar c}{\sqrt{2m_\alpha c^2 E_\alpha}} = \frac{\ell\ 197MeV\ fm}{\sqrt{8\times 931MeV\times(\sim 6MeV)}} \approx \ell fm$$

Remember nuclear radius for $A \approx 240$ is $\sim 9\,fm$.
There are two reasons why high ℓ is not favoured:

1. The alpha particle must originate from a more restricted region of the nucleus (nearer the edge)
2. There is an additional contribution to the barrier height and width called the "angular momentum barrier" which comes naturally out of the Schrödinger equation (from the angular part of the ∇^2 operator).

Therefore the spin between the parent $\left(\underline{I_i}\right)$ and daughter $\left(\underline{I_f}\right)$ can change by $\ell\hbar$ where $\underline{I_i} = \underline{I_f} + \underline{\ell}$, and the parity change is $(-1)^\ell$.

If $\underline{I_i^+} = 0^+$, then $\underline{I_f} = 0^+(\ell=0), 1^-(\ell=1), 2^+(\ell=2),...$, and $0^-,1^+,2^-,3^+,4^-,...$ are forbidden by parity conservation.