Quark Model (Continued)

Hadrons:
Baryons qqq Fermions.
Mesons $q\bar{q}$ Bosons.
→ quarks have ½ integral spin. Fermions.
→ Simplest – each quark has spin ½.

Initially only consider u, d and s (not c, t, b yet). Start by accounting for only lowest mass / energy contributions.
→ lowest mass hadrons.
→ only consider no relative angular momentum between quarks.
"Ground state" (Fred's notation).
i.e. $\underline{L} = 0$.
(if $L > 0$, then we have higher mass states.)
Total $J = L + S = S$
where L is orbital angular momentum, and S is spin angular momentum.
(NB: $Y = B + S + C + \tilde{B} + T$, in M&S it's outdated $Y + B + S$.)
Can represent quark in S vs. I_3 plot. See handout.

Baryons
qqq can have angular momentum spin $\uparrow\uparrow\downarrow$ → spin $J = \frac{1}{2}$, or $\uparrow\uparrow\uparrow$ (that's equivalent to $\downarrow\downarrow\downarrow$) → spin $J = \frac{3}{2}$. i.e. two distinguishable states.
(By "spin" it doesn't actually spin. Just called spin to differentiate it from Isospin. … can't just spin a point …)
If q_i, q_j, q_k where i, j and k are any of u, d or s.
See handout – diagram 2.
So what you get is 10 possible combinations with $J = \frac{1}{2}$, and 10 possible combinations with $J = \frac{3}{2}$.

$\underline{J = \frac{1}{2}}$
QM: for Fermions.
Pauli → total wave function must be anti-symmetric.
$\Psi \sim \Psi_{space} \Psi_{spin} \Psi_{colour}$.
Space wf symmetric for lowest energy state.
Colour wave function is anti-symmetric for all hadrons (all quarks in hadrons have different colours.)
So:
$\Psi \sim \Psi_{space} \Psi_{spin} \Psi_{colour}$
SYM ~ SYM ??? ANTI-SYM.
Therefore I_{spin} must be symmetric (or spin wavefunction).
→ all <u>like</u> quarks in composite hadrons must have parallel angular momentum spin for lowest mass states.

Each quark has 3 colour states: Red (r), Blue (b), Green (g).
Composite hadrons have no <u>net</u> colour.
Baryons qqq must be $q_r q_b q_g$.
Meson $q\overline{q}$ must be $q_r\overline{q}_r$, $q_s\overline{q}_s$, etc.

For $J = \frac{1}{2}$, we cannot have uuu, ddd, sss → they would all have spin $\uparrow\uparrow\uparrow$, i.e. $J = \frac{3}{2}$.
Therefore:
uud must have $\uparrow\uparrow\downarrow\,(=\downarrow\downarrow\uparrow)$
dds must have $\uparrow\uparrow\downarrow\,(=\downarrow\downarrow\uparrow)$.
Etc.
Only have one possible combination for all q: $q_i q_j$, $i \neq j$.

→ left with uds. Can have angular momentum spins in 2 independent different combinations.
→ uds: $\uparrow\uparrow\downarrow$ or $\downarrow\downarrow\uparrow$
All other combinations are equivalent, because u and d treated together with the same I and S $\left(I = \frac{1}{2}, S = 0\right)$.

Have two combinations of uds with $I_3 = 0$, $S = -1$:

$dds \quad (u \to d)$	uds	$(d \to u) \quad uus$
$\uparrow\uparrow\downarrow$	$\uparrow\uparrow\downarrow$	$\uparrow\uparrow\downarrow$
$I_3 = -1$	$I_3 = 0$	$I_3 = +1$

This is a triplet of state with the same $S(=-1)$ and $I = 1$.
From uds $\uparrow\downarrow\uparrow$, a u can't change to a d, and vice-versa due so parallel spin of like quarks constraint.
This becomes a single state with $S = -1$, $I = 1$, $I_3 = 0$.

Parity:
$\left(J = \frac{1}{2}\right)$
Operator reflects states through origin.
(multiplicative, not additive)
Conserved in SI and EMI, but NOT WI.
We know that a photon can decay to a quark and an anti-quark. These have intrinsic parity. From experiments,
$P_\gamma = -1$
Since parity is conserved by EM interaction,
$P(q) = -P(\overline{q})$
By convention, the parity of a quark is $+1$, and the parity of an anti-quark is -1.
Parity of a Baryon $(qqq) = (+1)^3 = +1$
Anti-baryon: $(\overline{qqq}) = (-1)^3 = -1$

Parity of a Meson: $(q\bar{q}) = (+1)(-1) = -1$
For lowest energy states where L (angular momentum) is 0.

If $L \neq 0$, there is an extra parity factor which goes as $(-1)^L$ (this is multiplied onto the normal parity).

For $J = \frac{3}{2}$, all 10 combinations are possible. All spins are parallel, therefore no additional spin constraints.
→ the Decuplet $\frac{3}{2}^+$.
Supermultiplets.
All particles within have the same B, the same J, the same P. These are made up of Isospin multiplets (parallel horizontal lines).

Predicted properties of the Ω^- (sss):

1) Baryon number $= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = +1$
2) Change $= -\frac{1}{3} - \frac{1}{3} - \frac{1}{3} = -1$
3) Strangeness $= -1 - 1 - 1 = -3$
4) Mass $1676 MeV$ (actually $1672 MeV$)

Know the mass up the up quark is the same as the down quark (Isospin disabled.)
Δ (u's and d's) qqq $(q = u\, or\, d)$
Σ is qqs with mass increase of 147.
Cascade Ξ qss, with mass increase of 145.
So mass of the strange quark minus the mass of an up or down is $\sim 146 MeV$
(Mass: 1238 for the top row, 1385 for the second (difference 147). Third have 1580, with the difference being 145. So the mass for the last row is 1580-146…)

5) Lifetime: long (weak, $\sim 10^{-10} s$) (actually $0.8 \times 10^{-10} s$)

Ω^- has $S = -3$. There is no other particle in the lowest energy state with $S < -2$.
If Ω^- were to decay by the Strong interaction, then it would have to conserve S.
Possible decay would be:
$\Omega^- \to K^- + \Xi^0$
$-3 \to -1 + -2$
So strangeness is conserved.
Mass $\sim 1676 \to \underbrace{494 + 1315}_{1809}$
No particle can decay at rest to states with higher rest mass, so from kinematics this is not allowed – energies are not conserved.
It turns out that there are no other strong interactions that have a lower mass than this while conserving S, so there is no way to conserve S through the strong interaction.
Therefore S must be violated → decay must be weak.
Therefore lifetime must be long.

6) Name: Ω. The first radiation is α, which is the first letter of the greek alphabet. This particle was seen to be the last particle, so they used the last letter of the greek alphabet - Ω.

Ω^- discovered in K^-p (bubble chamber) interactions.

$K^-p \to \Omega^- K^+ K^0$

$-1,0 \to -3,+1,+1$

Change in strangeness is 0, so strong interaction production.

$\Omega^- \to \Xi^0 \pi^-$

$-3 \to -2,0$

So $\Delta S = +1$, so it has to be a weak decay.

$\Xi^0 \to \Lambda^0 \pi^0$

$-2 \to -1,0$

So $\Delta S = +1$, so again a weak decay.

$\Lambda^0 \to p\pi^-$

$-1 \to 0,0$

So $\Delta S = +1$, weak again.

Mass relationship for the Baryon $\frac{1}{2}$ octet:

Quarks do not have all same parallel spin, so direct interchange between u/d and s quarks is not as straight forward. You need to look at all the possible combinations, of which there are many. You have mixed symmetries, and you get Clebsch-Gordon coefficients.

→ expect a "weighted mean" of the masses which says that

$$\underbrace{2(M_n + M_\Xi)}_{4508\,MeV} = \underbrace{M_\Sigma + 3M_\Lambda}_{4535\,MeV}$$

(this is better than 1%)

Mesons

$q\bar{q}$

Consider first the lowest energy states, with no relative angular orbital momentum.

$q\bar{q}$ can have the spins of the two quarks antiparallel $\uparrow\downarrow$ → spin $J = 0$, or paraller $\uparrow\uparrow$ → spin $J = 1$.

Expect 9 combinations: nonets in each case.

There is no parallel spin constraints because $q \neq \bar{q}$.

But: you get a complication at the center of this plot, with $I_3 = 0$ and $S = 0$. There are three combinations here - $u\bar{u}$, $d\bar{d}$, $s\bar{s}$.

Consider the $J = 1$ nonet $\uparrow\uparrow$.

ρ^0 is part of the ρ^- ρ^0 ρ^+ isospin triplet.

$\rho^- = \bar{u}d$, $\rho^+ = u\bar{d}$. So ρ^0 has no s quarks, and is $\rho^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$.

Masses of 1^- states:

$M_\rho \sim 770 MeV$. Number of strange quarks = 0.

$M_{k^*} = 890MeV$, number of s quarks = 1.
Mass difference due to one s quark is $\sim 120MeV$.
Difference between this number, and the previous one, is due to noise and binding energies.
$M_\omega = 782MeV$ - similar to M_ρ → infer that it has 0 strange quarks.
$M_\phi = 1020MeV \approx m_\rho + 240MeV$ → infer that it has 2 strange quarks.
So:

$$\rho = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$$

$$\omega = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$$

$$\phi = s\bar{s}$$

This is harder for the $J = 0$ case…
NB: opposite corners on the diagram give you the antiparticle. The central particles are their own antiparticles.

Quark model so far:
Baryons qqq
- all observed (once we include the c, b and t quarks)
- States not expected don't exist.

Mesons $q\bar{q}$
- all observed (once we include the c, b and t quarks)
- States not expected don't exist.

So we have 8 + 10 Baryons, 9+9 mesons = 36 particles.
So far, have discovered > 300 hadrons.
Extensions:
1. Add c, b and t quarks → gives us more combinations.
2. Must add orbital angular momentum to the quarks
   → higher mass supermultiplets.

Start off with #2…

Resonances
Definition: short-lived particle decaying via the strong interaction with a lifetime $\tau \sim 10^{-23} s$. E.g. Baryons: $\Delta^-, \Delta^0, \Delta^+, \Delta^{++}$ Mesons: ρ^-, ρ^0, ρ^+ .
Problem: as the lifetime is so short, we can't actually see them – once generated, they go too quickly to be seen… Uncertainty principle.
To investigate the properties of resonances, we need to generate them either by formation, or production.

Formation
The incident particles "fuse" to form resonance, which subsequently decays very quickly either to the original particles, or something else.
e.g. $\pi^+ p \to \pi^+ p$

This will only happen if you have exactly the correct energy to obtain the Δ^{++} - too low, and it doesn't happen. Too high, and it's not energetically practical.
So if it is a resonance, then you can look for it by altering the E_k of the π^+. You can then get a resonance curve when plotting the $E_k(\pi^+)$ against the $\sigma_{tot}(mb)$. At the correct E_k, then you get a peak in the curve – this is the resonance.

Lab system:
(2)
Peak in σ occurs at $E_{k_{\pi^+}} = 180MeV$

$E = m + E_k$

$E^2 = m^2 + p^2$

${m_{inv}}^2 = (\Sigma E)^2 - (\Sigma p)^2$

$M_{inv}(p\pi) = M_{inv}(\Delta^{++})$

→ $\Delta^{++} \approx 1230MeV$

Same procedure with the width to get the natural width Γ of the Δ^{++}.
→ center of mass system:
(3)
The peak is the mass of the particle, produced with no KE in CM frame. Γ is the natural width of the particle, assuming little experimental uncertainty. This is caused by the uncertaintly principle.

$\Delta E \Delta t \sim \hbar$

ΔE is the natural width Γ, Δt is the lifetime τ.
So:

$$\tau \sim \frac{\hbar}{\Gamma} \sim 10^{-23}s$$

Δ is a generic name for (qqq) bosons where q is only up or down, and $I = 3/2$ (Isospin quartet).
→ $\Delta(1230)$, $\Delta(1900)$, $\Delta(2420)$, … 22 Δ's have been discovered.
N is a generic name for (qqq) bosons where q is only up or down, and $I = 1/2$.
→ $N(980)$, $N(1440)$, $N(2250)$, … 22 N's discovered.
Resonances once formed needn't decay to original particles.

$\pi^- p \to \Delta^0 \to \pi^- p$

$\pi^- p \to \Delta^0 \to \pi^0 n$

etc.

Production ("Brute force")
Have more than enough incoming energy and resonance is produced with other final state particles.

$\pi^- + p \to \Delta^0 + \ldots \to \pi^- + p + \ldots$

Look at the effective invariant mass for all combinations of produced particles.

$M^2 = \left(\sum E\right)^2 - \left(\sum p\right)^2$

You will find peaks superimposed on the background of the invariant masses, where the peak lies at the mass of the produced resonance.(after this experiment has been done many times).

Meson Resonances

Formation: e.g. $e^+e^- \to \rho^0 \to \pi^+\pi^-$

This uses Electron-Positron colliders where you can tune the energy of the beam and scan the total center of mass energy available. $E_{cm} = E_{e^-} + E_{e^+}$

The nice thing about this is that the CM system is the lab system.

$$\rho^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} / d\bar{d} \right)$$

Production:

$\pi^- p \to \rho n \to \pi^+\pi^- n$

Here there is one specific case where the extra junk of particles is a neutron only.

This is part of a series of meson resonances where $q\bar{q}$ have higher relative angular momentum.

More "unexpected" decay rates

Consider $J = 1$ neutral mesons (π, ω, ϕ)

e.g. $\omega = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$

Predominant decay (Branching ratia BR) = 90%:

$\omega(783) \to \pi^+\pi^-\pi^0$

$$\underbrace{u\bar{u}}_{\omega} \to \underbrace{\left(u\bar{d}\right)}_{\pi^+} + \underbrace{\left(d\bar{d}\right)}_{\pi^0} + \underbrace{\left(d\bar{u}\right)}_{\pi^-}$$

Q value $= m_\omega - 3m_\pi = 783 - 3 \times 140 = 363 MeV$. Hence large Q value.

Similarily $\phi(1020) = (s\bar{s})$.

The strange quarks annihilate, and the energy is then used to make the needed particles. This goes through 3 gluons to conserve the charge (change conjugation).

$\phi \to \pi^+\pi^-\pi^0$

Q-value $= m_\phi - 3m_\pi = 1020 - 420 = 600 MeV$. OK.

Branching ratio: 15%.

ϕ can also decay to Kaons.

$\phi(s\bar{s}) \to K^-(s\bar{u}) K^+(u\bar{s})$

Q-value $= m_\phi - 2m_K = 1020 - (2 \times 495) = 30 MeV$. This is OK, but only just. So it is much likely to go the first way (you'd think…).

Branching ratio: 84%

The reason: the gluons.

At each quark / gluon vertex, you get a factor of α_s in the amplitude. $\alpha_s < 1$. So with the minimum of 3 gluons, you get a minimum of $(\alpha_s)^6$ in the amplitude for

$\phi \to \pi^+\pi^-\pi^0$. There is no such factor for $\phi \to k^+k^-$ So this wins in cross-section, despite the Q-value being much smaller.

ϕ can also decay to e^+e^-.

$\phi(s\bar{s}) \to \gamma \to e^+e^-$

This is now an EM process, which is weaker than the other two interactions, so much lower cross-section than the strong.

$$\frac{\phi \to e^+e^-}{\phi \to k\bar{k}} = 4\times10^{-4}$$

ϕ decays via strong to $k\bar{k}$, strong but repressed to $\pi^+\pi^-\pi^0$, (Zweig suppressed) and EM to e^+e^-.

NB: $\phi(s\bar{s}) \to \overline{k^0}(s\bar{d}) + k^0(d\bar{s})$ is also possible. This has 50% of the probability of $\phi \to k^-k^+$.

"Hidden strangeness".

$\tau \sim 10^{-22}s$ → broad resonance $\Gamma \sim 4MeV$

$\Gamma\tau \sim \hbar$

Charm

11 Nov. 1974

1. SLAC (California) (Richter). e^+e^- colliding beams. $E_{cm} = E_{e^-} + E_{e^+}$ is tumable.

Scanned E_{cm}, they looked for the production of hadrons, e^+e^- pairs, $\mu^+\mu^-$ pairs.

Found a resonance peak at $E_{cm} = 3.1GeV$. This peak was very narrow, $\Gamma \sim 88keV$ → long lifetime.

Resonance formation:

$$e^+e^- \to \Psi \to \begin{cases} hadrons \\ e^+e^- \; only \\ \mu^+\mu^- \; only \end{cases}$$

Problems:

1) Quark model couldn't explain
2) Narrow width

2. Brookhaven (New York) (Ting)

Proton beam into Be (n, p) target.

Looked for production of e^+e^- pairs.

$p + Be \to e^{+e^- +\ldots}$

Looked at the invariant mass.

$p + {}^{p}\!/_{n} \to J \to resonance + junk \to e^+e^- + junk$

Ended up calling it the J/ψ particle…

21 Nov 1974 – SLAC increased E_{cm} → another, new narrow resonance at $3.7GeV$ (ψ').

New particles $(J/\psi, \psi', ...)$ were heavy $(\sim 3m_p)$ produced in the same way as mesons of type ρ, ω, ϕ - would expect them to decay similarity.
But lifetimes of the $\rho = 0.5\times10^{-23}s$, $\omega = 10\times10^{-23}s$, $\phi = 20\times10^{-23}s$. These have normal strong interaction lifetimes.
$J/\psi(3.1) \sim 1000\times10^{-23}s$, $\psi'(3.7) \sim 300\times10^{-23}s$. These have longer strong interaction lifetimes. Definitely not EM or weak (too short).

Theory extended quark model by the addition of the Charm quark (c).
Charm quark had $B = \frac{1}{3}$, $Q = +\frac{2}{3}$, $S = 0$, $C = +1$ where C is Charm.
Then J/ψ was $(c\bar{c})$. This has hidden charm.
Similar to the ϕ $(s\bar{s})$.

ϕ decays	J/ψ decays
$\phi(s\bar{s}) \to K(s\bar{q}) + \bar{K}(q\bar{s})$ Normal strong interaction decay. Allowed $m_\phi > 2m_k$ Dominant $BK \sim 84\%$	$J/\psi(c\bar{c}) \to D(c\bar{q}) + \bar{D}(q\bar{c})$ Would give a normal strong interaction decay with $t \sim 10^{-23}s$. NOT observed. Therefore $M_{J/\psi} < 2M_D$. NOT allowed → kinematics.
$\phi(s\bar{s}) \to 3\times gluons \to \pi^+\pi^0\pi^-$ Zweig-suppressed strong interaction decay Allowed but suppressed. $BR \sim 15\%$	$J/\psi(c\bar{c}) \to 3\times gluons \to \pi\pi\pi$ Allowed even though it is also Zweig-suppressed. High BR $(\sim 80\%)$ as no normal strong interaction competition. Longer lifetime and narrower width. (this is the way Richter discovered ψ)
$\phi(s\bar{s}) \to \gamma \to e^+e^-$ Allowed but swamped by the strong interactions. $BR \sim 3\times10^{-4}$. EM decay.	$J/\psi(c\bar{c}) \to \gamma \to e^+e^-$ Allowed. More competitive because only competing with the Zweig-suppressed strong interaction. $BR \sim 6\times10^{-2}$. (this is the way Ting discovered the J)

In total, there are 10 $c\bar{c}$ states, with increasing orbital angular momentum, therefore higher mass. “Charmonium”. The spacing of mass and decays between them are exactly as expected by quantum mechanics.
$\psi(3770)$ has a lifetime of $2\times10^{-23}s$, i.e. normal strong interaction decay time.
$\psi(3770)(c\bar{c}) \to D(1865)(c\bar{q}) + \bar{D}(1865)(q\bar{c})$ where q is another quark type. This is the first state where the energy is big enough that you can have this normal quark flow rather than the Zweig-suppresssed one. All subsequent ones from this can all go through this route.
So from the c quark, you can generate a whole new series of new mesons where you have $c\bar{q}$ and $\bar{c}q$, as well as a whole series of new Baryons where you have cqq etc.

We graduate from the 2D plot where you have strangeness vs. I_3, to a 3D plot with C, S and I_3

C quark has $B = 1/3$, $Q = +2/3$, $I_s = 0$, $S = 0$, $C = +1$.
1977, Fermilab (Illinous) (Lederman) did an experiment very similar to Ting's – a beam of protons into a solid target, either Pt or Cu (essentially p and n). Looked for things coming out which included in them μ^+, μ^- - i.e. a resonance going to the two particles. Looked for the invariant mass of the pairs, and then plotted it as a histogram. They found a peak with mass of $9.4GeV$, and it was narrow.
It was an unexpected peak, which is not explained by the udsc quark model. Narrow (therefore long decay) → Zweig suppressed strong interaction decay.
Called $\Upsilon(9.4)$, $\Upsilon'(10.0)$, $\Upsilon''(10.3)$.
These are resonances produced by proton in pt / cu (á la Ting). Also formed by e^+e^- (á la Richter).
Interpretation: Upsilon $\Upsilon = (b\bar{b})$, where the b quark is bottom / beauty.
Narrow resonances could not decay by normal strong to $B\bar{B}$, where $B = b\bar{q}$ and $\bar{B} = \bar{b}q$, as the mass of Υ is less than twice the mass of the B.
$\Upsilon, \Upsilon', \Upsilon''$ decay by the Zweig-suppressed:
$b\bar{b} \to gluons \to 3\,particles$.
Higher mass Υ can decay by normal strong interaction.
$\Upsilon(10.55GeV)(b\bar{b}) \to B(5.2GeV)(b\bar{q}) + \bar{B}(5.2GeV)(\bar{b}q)$
Therefore kinematically OK.
B quark has $B = 1/3$, $Q = -1/3$, $I_3 = S = C = 0$, $\tilde{B} = -1$.
This is similar to the Strange quark with the same charge, and $S = -1$.
→ whole new range of mesons and baryons with various combinations of b inside.
Strangeness, Charm and Bottom all decay (not seen stably in universe).
Only way to loose S, C or $\tilde{B}$ is by $W^\pm$ emission → weak decay.
e.g. $K^- \to \pi^0\pi^-$ where $S = -1 \to 0 + 0$. S changed, therefore $W^\pm$ weak.
$K^-(\bar{u}s) \to \pi^0(\bar{u}u) + W^- \to \pi^0(\bar{u}u) + \pi^-(d\bar{u})$.
For Charm, e.g. $D^0 \to K^-\mu^+\nu_\mu$
$D^0(\bar{u}c) \to K^-(\bar{u}s) + W^+ \to K^-(\bar{u}s) + \mu^+ + \nu_\mu$
b, c, s quarks decay in a cascade way via $W^\pm$ emission, therefore weak.
For Bottom:
$b \to c + W^-$
$c \to s + W^+$
$s \to u + W^-$
(cascade)

Top
We know that leptons come in pairs, e.g. e^- and ν_e. Lepton conservation.
We know up and down quarks are a pair, i.e. $I_{3,u} = +1/2$, $I_{3,d} = -1/2$
We know charm and strange quarks are a pair.

$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} \\ b \end{pmatrix}, \ldots$

$\begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}, \ldots$

These are generational – 1st, 2nd, 3rd from left to right. How many more generations are there…?
We have an obvious gap here, which would be top. Energy worked out through deduction by looking at the other quarks – they found a mass series.

e^-e^+ colliding beam machines failed to find by formation. $e^+e^- \to t\bar{t}$ had insufficient energy.
→ higher energy. Tevatron. Proton and antiproton with 1TeV of enery each.
$p\bar{p} \to q\bar{q} \to gluon \to t\bar{t}$.
Energy high but cross-section low. Run for long period.
April 1994:
$q + \bar{q} \to gluon \to t + \bar{t}$
$\bar{t} \to \bar{b} + W^-$
W^- then decays to leptons or quarks → can then reconstruct the W^-.
$\bar{b}$ decays into a hadron jet. Can detect this by measuring a finite path of the $\bar{b}$
$\sim 400\mu m$.
The energy of these two particles will give you the mass of the $\bar{t}$.
The same happens for t → can get the energy of t trough the same method as above.
Plotting number of occurrences vs. their mass, and you get a peak.
Since then, more experiments have shown that these is no doubt that the Top was found. The mass of the top quark turned out to be $175GeV$ - this is comparable to the rest mass of a gold atom.

Top quark has:
$T = +1$, $Q = +\frac{2}{3}$, $I_3 = S = C = \tilde{B} = 0$.

Note that now, u c and t have $Q = +\frac{2}{3}$ and positive quantum numbers, while d, s and b have $Q = -\frac{1}{3}$ and negative quantum numbers.

The top mass is so large that the top quark decays before it can form $t\bar{t}$ resonance.
Therefore there is no top equivalent to $\phi(s\bar{s})$, $\psi(c\bar{c})$, $\Upsilon b\bar{b}$
$10^{-24}s$ Yocto second
$10^{-21}s$ Zepto second.
$10^{-18}s$ Atto second.
Decay is part of the cascade process.
$t \to b + W^+$
$b \to c \to s \to u$

$$Q = \frac{B + S + C + \tilde{B} + T}{2} + I_3$$

Summary of "Basic" particles so far:

	Quarks	Leptons
Mass	$u \sim 0.005GeV$ $d \sim 0.005GeV$ $s \sim 0.1GeV$ $c \sim 1.5GeV \; \left(M_\psi / 2 \right)$ $b \sim 5GeV \; \left(M_\Upsilon / 2 \right)$ $t \sim 175GeV$	$\nu_e < 3 \times 10^{-6} MeV$ $\nu_\mu < 0.19MeV$ $\nu_\tau < 18MeV$ $e \sim 0.5MeV$ $\mu \sim 110MeV$ $\tau \sim 1800MeV$

NB: all leptons other than the τ are lighter than the lightest hadron, the π with mass $140MeV$. The τ is the only lepton with mass > mass of the lightest hadron. It is the only lepton that can decay to hadrons. $\tau^- \to \nu_\tau + W^- \to \nu_\tau + \pi^-(\bar{u}d)$

Quarks and Leptons

- Both sets are Fermions.
- Both sets are "pointlike" – they have no extension down to $\sim 10^{-19} m$.
- Both sets couple to all γ, $W^\pm$, Z^0.

But:

- Quarks couple to Gluons; leptons don't.
- Quarks form composite particles; leptons don't.
- Free quarks have never been found; free leptons exist.

→ are quarks real?

Experimental evidence for quarks:

- Get the symmetries (supermultiplets)
  - quark model can explain and predict all particles found.
  - Can also explain the none-existence of those not fund.
- Experimentally measured cross-sections

  $\pi^? \to p$ target. $\sigma(\pi^+ p)$

  $\pi^+(u\bar{d}) \to\leftarrow p(uud)$

  2 particles on 3.

  $p \to p$ target. $\sigma(pp)$

  $p(uud) \to\leftarrow p(uud)$

  3 particles on 3.

  So $\sigma(\pi^+ p) = \frac{2}{3}\sigma(pp)$.

  Measure (at $dE = 100GeV$): $\sigma(\pi^+ p) = 24mb$, $\sigma(pp) = 37mb$.
- Scattering experiments:

  Rutherford: $\alpha \to$ gold foil, and looking at angular distribution of the scattered α-particle → deduces the nuclear structure within atom.

  Can do the same sort of things with particles. Bombard a proton, say, to find out the inner structure.

  $\Delta p \Delta x \sim \hbar$, for $10^{-18} m$ we need $\Delta p \sim 100GeV$. Use probes to investigate structure within a hadron. Looking for quarks, which have strong, EM and weak interactions. So use hadron (strong), electrons or muons (EM), ν (weak). All show that hadrons have point-like constituents with fractional electric charge.

Also show that $\frac{1}{2}$ the energy of a hadron is accounted for by the constituents, while the other half is accounted for by the gluons holding the quarks together.

→ quarks are real. But where are they? Why can't we see free quarks?

Free Quarks

Using fractional changes of the quarks, and pass through detection medium.
Ionization power $\propto Q^2$.
Therefore quarks of $-\frac{1}{3}$ and $\frac{2}{3}$ electric charge, we expect ionization of $\frac{1}{9}$ or $\frac{4}{9}$ of that of a normal charged particle.
Methods to search for free quarks:

1) Knock out of (e.g.) proton. Result: only a jet of hadrons.
2) Try and create free quarks, i.e. $e^- e^+ \to \gamma \to q + \bar{q}$. Result: get two jets of hadrons.
   NB: always tried at new accelerators (higher energy).
   (if energy is too low, you get $e^- e^+ \to e^- e^+$, too high, hoping that $q\bar{q}$ go further…) as soon as q or $\bar{q}$ are produced, they automatically turn to jets.
3) Even higher energies
   Cosmic rays
   Also not seen
   Some Aussie did, "but it was a crap experiment".
4) Even higher energy – Big Band. Highest quarks (u&d). Expect (possibly) to find the remnant quarks in the earth, moon or meteorites.

1977, Fairbank: used Niobium balls. Millikan-type experiment.
Electric & magnetic & gravity. 3 balls with fractional charge.
And in 1981 – new result confirmed the earlier results.
Flawed. Feynmann couldn't find what was flawed with Fairbank experiment.
Rutherford lab offered to do mass-spectrum experiment on Fairbank's balls – nobody else produced the results as yet.

Quarks are real, but free quarks don't exist.

Colour

Need new property of quarks to explain:

i. Source of Strong Interaction (cf. electric charge as source of EM interaction)
ii. Apparently violated Pauli Exclusion Principle.
    $\Omega^-(sss)$, $J = \frac{3}{2}$. All have spins of $\uparrow$. As quarks are Fermions, all of them having parallel spin violates Pauli.
iii. Why no free quarks exist?

These can be sorved by a new property, but it must be such that composite hadrons don't have it.
→ colour.
(in the same way as Isospin has no actual spin, colour doessn't actually mean the colour of a quark. It's just a name.)
All quarks come in 3 coloured varieties:

- Red (R)
- Blue (B)
- Green (G)

All composite hadrons are "white", "colourless", "colour neutral" or colour singlet states.
Therefore do not have RED proton, $u_r u_r d_r$, redish-blue proton, etc…
Must have:
- qqq baryons must have 1R, 1B, 1G quark.
- $\overline{qqq}$ must have $1\bar{R}$, $1\bar{B}$, $1\bar{G}$ quark.
- Mesons $q\bar{q}$ must have $R\bar{R}$, $B\bar{B}$, or $G\bar{G}$.

This new property, Colour, then satisfies Pauli. Composite hadrons show no overall colour.
Source of strong interaction is colour – quantum exchanged gluon. QCD.
c.f. the source of the EM interaction being the electric change, with the photon exchange particle, QED.
Gluons are massless and spin 1 – gluons DO have colour. "Non-Abelian Gauge Theory".
Photons are massless and spin 1 – photons have NO electric charge.

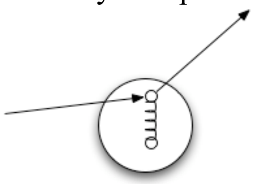
Gluons have the following colours:
$R\bar{B}$, $R\bar{G}$, $B\bar{G}$
$\bar{R}B$, $\bar{R}G$, $\bar{B}G$
plus two linear combinations of $R\bar{R}$, $B\bar{B}$, $G\bar{G}$.
→ $\Sigma = 8$.
DNLT – see also Perkin's Appendix, $\dfrac{R\bar{R} - B\bar{B}}{\sqrt{2}}$, $\dfrac{R\bar{R} + B\bar{B} + G\bar{G}}{\sqrt{6}}$

Quantum Chromo Dynamics

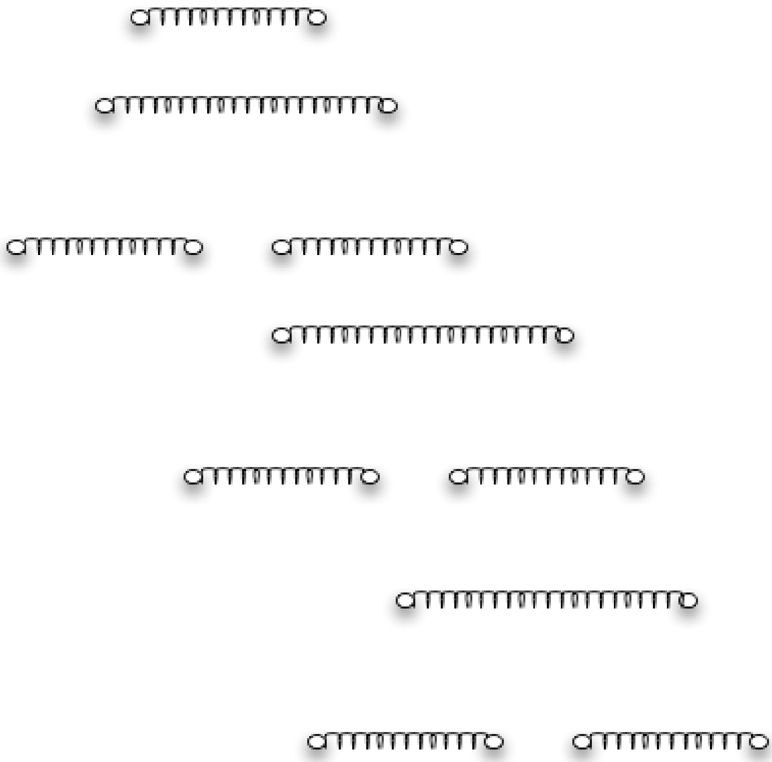
- Basic strong interaction
- Quark confinement

Quark Confinement
(no free quarks)
Take a quark. It has a gluon coming off it in $R\bar{G}$. This then splits, and a virtual gluon pair is created. Here we have two gluons - $R\bar{B}$ and $\bar{G}B$. This is then re-joined, and joined back to the starting quark via $R\bar{G}$.
Let there be a quark X next to the original quark. It sees the "normal" colour charge of Q.
Quark Y sees an enhanced stronger colour field die to Q plus the virtual colour gloud of gluons.
Therefore the attractive force between Y and Q is greater than that between X and Q.

→ attractive strong interaction INCREASES as separation between quarks INCREASES.
Quarks within hadron (separation $< 10^{-15} m$) appear to have very weak interaction (i.e. "free") → asymptotic freedom.
If we try to separate quarks:



In $q\bar{q}$ COM frame you have a quark trying to pull away from the other quark. This is the same case if you do $e^+e^- \to q\bar{q}$.
As we try to separate the q and $\bar{q}$, the colour field increases – connecting bond is called a "colour flux tube". Represented by .
(analogy with coiled spring)
NB: when you stretch a mechanical string too far, it snaps. Something similar happens here – except the two ends of the colour flux tube join on to a $q\bar{q}$ pair that has been created by the extra energy during the snap.



You end up with a large number of hadron pairs, i.e. jets in opposite directions.

As $q\bar{q}$ move apart, colour flux tube extends and then "snaps" to create new $q\bar{q}$ pair. Process continues producing new hadrons until run out of energy. Left with $q \to$ jet of hadrons.
Called "Hadronization".
Analogous to trying to separate N and S poles or dipole magnet → just get smaller dipoles → no free magnetic monopoles → but QED OK.

→ the fact that we have no free quarks shouldn't worry us → QCD OK.

Evidence for Colour

1. Extra distinguishing quantum number to pacify Pauli.
   E.g. $\Omega^{-}\left(s\{r\}s\{g\}s\{b\}\right)$
2. Particle decay rates:
   $\pi^{0} \to 2\gamma$ (need 2 for conservation of momentum) (first photon given off via bremsstrahlung from the d quark, the second by annihilation with the antiparticle).
   Calculated decay rate (pure QED): too slow compared with experiments. If you introduce 3 colour possibilities for the $d\{r/g/b\}\bar{d}\{r/g/b\}$. → increases decay possibilities by 3 → lifetime then agrees with experiment. (number of colours = $2.98 \pm 0.11 = 3$)
3. Ratio $R = \dfrac{\sigma\left(e^{+}e^{-} \to hadrons\right)}{\sigma\left(e^{+}e^{-} \to \mu^{+}\mu^{-}\right)} = \dfrac{\sigma_{had}}{\sigma_{\mu}}$

   Numerator: $e^{+}e^{-} \to hadrons$ is via $q\bar{q}$ production.

   $e^{+}e^{-} \to \gamma \to q\bar{q} \to hadrons$. Two vertexes – between the electrons and the photon, A, and between the photon and the quarks, B.

   $\sigma_{had} \propto$ (coupling constant at A) $\times$ (coupling constant at B).

   $\sigma_{rad} \propto \left(Q_{e}^{2}\right)\left(Q_{q}^{2}\right)$

   Denominator: $e^{+}e^{-} \to \gamma \to \mu^{+}\mu^{-}$.

   $\sigma_{\mu} \propto Q_{e}^{2}Q_{\mu}^{2}$

   Q is always the electric charge.

   So, the ratio $R = \dfrac{\sigma_{had}}{\sigma_{\mu}} = \dfrac{Q_{q}^{2}}{Q_{\mu}^{2}}$, where $Q_{q} = +\frac{2}{3}$ or $-\frac{1}{3}$, and $Q_{\mu} = \pm 1$, in terms of the electric charge.

   The numerator must take account of all possible $q\bar{q}$ states. i.e. only those which are kinematically allowed.

   $R = \dfrac{\sum Q_{i}^{2}}{1}$ where $i = u, d, s, c, b, t, \ldots$ but only up to those that can be produced.

   e.g. if $E_{cm} < 3GeV$ (at $E_{cm} = 3.1GeV$, produce J/ψ → production of $c\bar{c}$):

   $$R = \frac{\overbrace{\left(\frac{2}{3}\right)^{2}}^{u} \pm \overbrace{\left(\frac{1}{3}\right)^{2}}^{d} \pm \overbrace{\left(\frac{1}{3}\right)^{2}}^{s}}{1} = \frac{2}{3}$$

   This is expected. What was measured was 2. The difference of 3 is due to the three different colours that are possible for each $q\bar{q}$.

   If $E_{cm} \sim 11GeV$, so we have c and b too. Expect $R = \dfrac{11}{9}$. Measured, $R = \dfrac{11}{3}$.

   Therefore $R = 3\sum Q_{i}^{2}$, where $i = u, d, s, c, b, t, \ldots$ The factor of 3 is due to colour.

   As the E_{cm} increases, you cross the threshold for new $q\bar{q}$ productions. This

gives a set of new $q\bar{q}$ resonances (e.g. the J/ψ, Υ). You also get a new "plateau" level for R.
This was used to search for higher mass $q\bar{q}$ states, esp. $t\bar{t}$.

Number of Generations
Use the Z^0 and its' decays.
~ 1990. LEP produced $10^7 Z^0$/year.
$e^+e^- \rightarrow Z^0$. $E_{cm} = E_{e^+} + E_{e^-} =\sim 90GeV$.
Z^0 couples to quarks and leptons. Decays:
$Z^0 \rightarrow q\bar{q} \rightarrow hadrons$ (easy to detect)
$Z^0 \rightarrow \ell^+\ell^-$ (easy to detect)
where $\ell = e, \mu, \tau$
$Z^0 \rightarrow \nu\bar{\nu}$ (virtually impossible to detect)
Measure $\sigma\left(e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}\right)$ and $\sigma\left(e^+e^- \rightarrow Z^0 \rightarrow \ell^+\ell^-\right)$ as a function of E_{cm}. You get a distribution centered around m_{Z^0}, and with a characteristic width.
$\sigma_{total} = \sigma_{\rightarrow hadrons} + \sigma_{\rightarrow \ell} + \sigma_{\rightarrow \nu}$. Here, $\sigma_{\nu\bar{\nu}} = \sigma_{\nu_e\bar{\nu}_e} + \sigma_{\nu_\mu\bar{\nu}_\mu} + \sigma_{\nu_\tau\bar{\nu}_\tau}\left(+\sigma_{\nu_4\bar{\nu}_4}\right)$, where ν_4 is a potential fourth generation neutrino.
We have a fixed number of $e^+e^- \rightarrow Z^0$. If ν_4 exists, then more Z^0 will decay to $\nu_4\bar{\nu}_4$, and the remaining number that can decay to hadrons will be reduced.
If ν_4 does not exist, then the number decaying to hadrons is higher.
Plot the σ_{hadron} vs energy for the possibilities of 2, 3 and 4 leptons. The data fits the expectation for 3 neutrinos, $n_\nu = 2.994 \pm 0.012$.
→ 3 neutrino types, so long as the mass of any other neutrino is less that $< 45GeV$.
As the mass of the three neutrinos that we know are "tiny", it is very unlikely for neutrino 4 to have mass $> 45GeV$.
Also assuming that the coupling of neutrinos to the Z^0 is standard.

So, we are saying that there are three neutrinos. These couple with the three leptons. These then couple with the three generations of quarks. If we believe the symmetry (there is good reason to do so), then no additional neutrinos implies no additional leptons implies no additional quarks. So the top quark is the highest, and final, quark – no matter how large the accelerator is, we won't see any more quarks.

So, the total overview of particle physics as it stands is that the following particles are the only one that exist:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$
$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} + \text{Gluon}, W^\pm, Z^0, \gamma$$