PC 3101 – Quantum Mechanics

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0.1 Books:

- Rae, A.I.M – Quantum Mechanics (IoP)
- Mandl, I. – Quantum Mechanics (Wiley)
- Gasiorowicz – Quantum Physics (Wiley) ("G") (*)

0.2 Key experiments

- Photoelectric effect (Hertz 1897, Einstein 1905).
  Electromagnetic radiation is quantized.
- Compton effect (Compton 1923)
  Photons have both energy and momentum
  $\underline{p} = \hbar \underline{k}$
- Diffraction of electrons (De Broglie 1923)
  The Wave-Particle duality
  $E = h\nu \quad \underline{p} = \hbar \underline{k}$

0.3 Probabilistic description

The classical concept of trajectory is replaced by a time varying complex function $\Psi(\underline{r},t)$, the wave function, which contains all the information about the system.

$$P(\underline{r},t) = c\left|\Psi(\underline{r},t)\right|^2 d^3r$$

is the probability of finding the particle at time t between $\underline{r}$ and $\underline{r}+\underline{dr}$. c is the normalization constant. We require that

$$\int_{all\,space} P(\underline{r},t)\,d^3r = 1.$$

The evolution of a particle of mass m subject to a potential energy field $V(\underline{r},t)$ is described by the Schrödinger equation:

$$ih\frac{\partial\Psi}{\partial t}(\underline{r},t) = -\frac{\hbar^2}{2m}\nabla^2\Psi(\underline{r},t) + V(\underline{r},t)\Psi(\underline{r},t).$$

Observables are what we can measure. To each observable we associate an operator. For example, the position of the particle in 1D is obtained by the operator $\hat{x}$.

$$\hat{x}\Psi(x,t) = x\Psi(x,t)$$

For momentum in 1D:

$$\hat{p}_x\Psi(x,t) = -i\hbar\frac{\partial\Psi(x,t)}{\partial x}.$$

0.4 Expectation values

The average value from many measurements of certain observables is called the expectation value for that observable. For the observable A, the expectation value

$$\langle A\rangle = \int \psi^*(\underline{r})\hat{A}\psi(\underline{r})\,d^3r.$$

For example,

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^*(x)\hat{x}\psi(x)\,dx = \int_{-\infty}^{\infty} \psi^*(x)x\psi(x)\,dx .$$

The root mean square or uncertainty for an observable A is

$$\begin{aligned}
\Delta A &= \sqrt{\left\langle \left(A - \langle A \rangle\right)^2 \right\rangle} \\
&= \sqrt{\left\langle A^2 - 2\langle A \rangle A + \langle A \rangle^2 \right\rangle} \\
&= \sqrt{\langle A^2 \rangle - 2\langle A \rangle^2 + \langle A \rangle^2} \\
&= \sqrt{\langle A^2 \rangle - \langle A \rangle^2}
\end{aligned}$$