Linear Motion

Force $\underline{F} = \frac{d\underline{p}}{dt} = m\frac{d\underline{v}}{dt}$

Linear momentum $\underline{p} = m\underline{v}$

$v = u + at$

$v^2 = u^2 + 2as$

$s = ut + \frac{1}{2}at^2$

Pressure: $P = \frac{F}{A}$

Friction

N is the normal force, i.e. the force applying between surfaces, e.g. weight

$F = \mu N$

Power: $P = Fv$

Work: $W = \int_a^b \underline{F} \cdot \underline{dx}$

Circular Motion

Centripetal force: $F = \frac{mv^2}{r} = mr\omega$

Angular velocity: $v = r\omega$

$\omega = 2\pi f = \frac{2\pi}{T}$

Angular momentum:

$\underline{L} = \underline{r} \times \underline{p}$

$L = i\omega = mvr$

Torque: $\underline{\tau} = \underline{r} \times \underline{F} = \frac{d\underline{L}}{dt}$

Energy $E = \frac{1}{2}I\omega^2$

Inertia of a sphere: $I = \frac{2}{5}mr^2$

Gravitational force: $F_{grav} = -\frac{GMm}{r^2}$

$F = -\frac{d\phi}{dr}$

(As a rule, $r \to \omega$, $m \to I$, $p \to L$)

Energy

Kinetic energy: $E_K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Potential energy: $E_P = \int \underline{F} \cdot \underline{dl} = mg\Delta h$

Reduced mass: $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Viral theorem: $E_k = \frac{1}{2}E_p$

Diffusion equation $\nabla^2\phi(\underline{r},t) = \frac{1}{D}\frac{\partial\phi}{\partial t}$

Young's Modulus

$$Y = \frac{F_\perp / A}{\Delta I / I_0} = \frac{tensile\,stress}{tensile\,strain} = elastic\,modulus$$

Lorentz Transformations

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1}{\sqrt{1-\beta^2}}$$

Position

$x = \gamma(x' + vt')$

$y = y'$

$z = z'$

$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$

$x' = \gamma(x - vt)$

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$

Velocity

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}}$$

$$u_y = \frac{u_y'}{\gamma\left(1 + \frac{u_x' v}{c^2}\right)}$$

$$u_z = \frac{u_z'}{\gamma\left(1 + \frac{u_x' v}{c^2}\right)}$$

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u_y' = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

$$u_z' = \frac{u_z}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

Momentum

$$p_x = \gamma\left(p_x' + \frac{vE'}{c^2}\right)$$
$$P_y = P_y'$$
$$P_z = P_z'$$
$$P_x' = \gamma\left(p_x - \frac{vE}{c^2}\right)$$

Energy

$$E = \gamma\left(E' + vp_x'\right)$$
$$E' = \gamma\left(E - vp_x\right)$$
$$E^2 = p^2c^2 + m_0^2c^4$$
$$E'^2 = \left(p'c\right)^2 + \left(m_oc^2\right)^2$$
$$E_k = \left(\gamma - 1\right)m_oc^2$$

Simple Harmonic Motion

Physical Pendulum

Angular momentum: $I = \frac{1}{2}m\ell^2$

Torque: $\tau = \frac{1}{2}mg\sin\theta$

Angular frequency: $\omega_o = \sqrt{\frac{\tau}{I}}$

Spring

Force: $F = -kx$, $k = m\omega^2$
Position: $x = A\cos\left(\omega t + \phi\right)$

Acceleration: $\frac{\partial^2 x}{\partial t^2} = -\omega^2 x$

Angular frequency: $\omega_0 = \sqrt{\frac{k}{m}}$

Potential energy: $E_p = \int kx\,dx = \frac{1}{2}kx^2$

Damping factor; $\gamma = \frac{b}{m}$

Velocity: $v = \frac{1}{2}kx^2$

Energy: $E_k = \frac{1}{2}mv^2$

Waves

See Optics
Velocity $v = f\lambda$

Damping factor $\gamma = \frac{b}{m}$

$$Q = \frac{\omega_0}{\gamma}$$

Wave number: $k = \frac{2\pi}{\lambda} = \frac{n\pi}{\lambda} = \frac{v}{c}$

Wave equation: $\mu = \frac{mass}{unit\,length}$,

$$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}, \quad v^2 = \frac{T}{\mu}$$

Phase velocity: $v_p = \frac{c}{n} = \frac{\omega}{k}$

$$v_p = \left(\frac{g\lambda}{2\pi}\right)^{1/2} \text{ in water}$$

Group velocity: $v_g = \left|\frac{d\omega}{dk}\right|$

Refractive index: $n = \frac{c}{v} = \frac{ck}{\omega}$

Oscillator

$$\frac{d^2x}{dt^2} + \omega_o^2 x = 0$$

Energy $E = \alpha\dot{x}^2 + \beta x^2$
Solution: $x_c\left(t\right) = B_1\cos\omega_o t + B_2\sin\omega_o t$

Damped Oscillator

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_o^2 x = 0$$
$$x\left(t\right) = A_oe^{-\gamma t}\cos\left(\omega' t + \phi\right)$$
$$\omega'^2 = \omega_o^2 + \left(\frac{\gamma}{2}\right)^2$$
$$E\left(t\right) = E_oe^{-\gamma t}$$

Forced Oscillator

$$\frac{d^2x}{dt^2} + \omega_o^2 x = F\left(t\right) = F_0\cos\omega t$$

Solution:
$\omega \neq \omega_0$:

$$x = A\cos\omega_0 t + B\sin\omega_0 t + \frac{F_0}{\left(\omega_0^2 - \omega^2\right)}\cos\omega t$$

(Using CF solution to oscillator & PI = $a\cos wt + b\sin\omega t$)

$\omega = \omega_0$

$$x = A\cos\omega_0 t + B\sin\omega_0 t + \frac{F_0}{2\omega_0}\sin\omega_0 t$$

(PI = PI before $\times t$)

Damped Forced Oscillator

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + {\omega_0}^2 x = F(t) = F_0\cos\omega t$$

Trial solution: $x = ce^{i\omega t}$

Resonance condition: $\omega = \sqrt{{\omega_0}^2 - 2\gamma^2}$