Circuits

Current: $I = \frac{dq}{dt} = nqv_d \ell = \frac{P}{V} = \int_A \underline{j} \cdot d\underline{A}$

AC current: $I = I_0 e^{i(\omega t + \phi)}$

Current density: $J = nqv_d = \frac{I}{A}$

Electron gun equation: $\frac{1}{2} mv^2 = eV$

Ohm's Law: $V = IR = \frac{i\rho\ell}{A}$,

$j = \sigma E \; V_0 e^{i\omega t} = I_0 e^{i(\omega t + \phi)} Z$,

$\sigma =$ conductivity

Power: $P = \frac{dU}{dt} = VI = I^2 R$

Charge: $Q = \int I \, dt$

Complex impedance of a resistor: $Z_R = R$

AC voltage: $V = V_0 e^{iwt}$, $V_{act} = \mathrm{Re}\{V\}$

Complex impedance: $Z = \frac{V_0}{I_0} = \frac{1}{\omega C} = \left| \frac{1}{j\omega C} \right|$

Impedance in series: $Z_T = Z_1 + Z_2$

Impedance in parallel:

$\frac{1}{Z_T} = \frac{1}{Z_1} + \frac{1}{Z_2}$, $Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}$

Natural frequency of a circuit: $\omega_o = \frac{1}{\sqrt{LC}}$

Quality factor: $Q = \frac{\omega_0}{\gamma}$, $\gamma = \frac{R}{L}$

Capacitance

Capacitance (Parallel plates): $C = \frac{Q}{V} = \varepsilon_0 \frac{A}{d}$

Combining in series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Combining in parallel: $C_{eq} = C_1 + C_2$

Displacement current: $I_D = \frac{dq}{dt} = \varepsilon_o \frac{d\Phi_E}{dt} = i_C$

Displacement current density: $J_D = \frac{I_D}{A} = \varepsilon_o \frac{d\underline{E}}{dt}$

Potential Energy: $E_P = \int V \, dq = \frac{1}{2} CV^2$

Energy density: $u = \frac{U}{volume} = qV$

Dielectric: $C = \varepsilon C_0$

ε is the dielectric constant, or relative permeability.

C_0 is the capacitance in a vacuum.

Inductance

Mutual inductance:

$\varepsilon = -N \frac{d\Phi_B}{dt} = -M \frac{dI}{dt} = NA \cos\phi \frac{dB}{dt}$,

$M = \frac{\Phi_{B2}}{I_1} = \frac{\Phi_{B1}}{I_2}$

Self inductance: $\varepsilon = -\frac{d\Phi_B}{dt} = -L \frac{dI}{dt}$,

$L = \frac{total\ flux}{current} = \frac{N\Phi_B}{I}$

Voltage: $V = L \frac{dI}{dt}$

Energy: $E = \frac{1}{2} LI^2$

Width of power resonance curve: $\gamma = \frac{R}{L}$

Complex resistance: $Z_L = i\omega L$

Inductance with magnetic material: $L_m = \mu L_0$

Kirchoff's Law:

The sum of all voltages around a circuit is 0

Electric

Coulomb's Law: $|F| = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} = |E_1||q_2|$

Electric field:

$E = \frac{1}{4\pi\varepsilon_0} \sum_{i \neq j} \frac{q_i}{r_{ij}^2} \hat{\underline{r}}_{ij} = \frac{F_2}{q_2} = \frac{\sigma}{\varepsilon_0} = -\underline{\nabla} V$

Electric potential: $\phi = \underline{E} \cdot d\underline{\ell}$,

$V = \frac{U}{q_0} = \frac{1}{4\pi\varepsilon_0} \sum_j \frac{q_j}{r_i} = -\int \underline{E} \cdot \underline{dl}$

Dipole moment: $\underline{p} = q\underline{d} = \alpha \underline{E}$, where α is the atomic polarizability

Polarization (number times dipole moment):

$\underline{P} = N \underline{p} = \chi_E \varepsilon_0 \underline{E}$, $\chi_E = \varepsilon - 1$

Polarization charge per unit volume $\rho = -\underline{\nabla} \cdot \underline{P}$

Energy: $U = -\underline{p} \cdot \underline{E} = \frac{1}{2} \int_V \underline{D} \cdot \underline{E} \, dV$

Electric torque: $\underline{\tau} = \underline{p} \times \underline{E}$

Energy density:

$\langle u \rangle = \frac{\varepsilon_0 \underline{E}^2}{2} = \frac{\underline{E} \cdot \underline{D}}{2} = \varepsilon_0 E_0^{\,2} \cos^2(kz - \omega t)$

Potential energy:

$dW = -dU = \underline{F} \cdot \underline{dl} = \underline{p} \cdot \underline{E} = \tau d\phi$

Potential difference: $\underline{E} = -\nabla\phi$

Electric flux: $\int_S \underline{E}\cdot\underline{ds}$

Laplace's Equation (no charge): $\nabla^2\phi = 0$

Poisson's equation: $\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$

Induced surface charge: $\sigma_p = \underline{P}\cdot\underline{\hat{n}}$

$\rho = \rho_P + \rho_F$

$\underline{D} = \varepsilon_0\underline{E} + \underline{P} = \varepsilon_0\varepsilon\underline{E}$

$\oint_S \underline{D}\cdot\underline{ds} = \int_V \rho_F dV$

Magnetism

Biot-Savart: $B = \left(\frac{\mu_o}{4\pi}\right)\int I\frac{\underline{d\ell}\times\underline{B}}{r^2}$

Dipole moment: $\underline{m} = I\underline{A}$

Energy density: $u = \frac{U}{Volume} = \frac{B^2}{2\mu_o}$

Field energy: $U = \frac{LI^2}{2} = \int P\,dt$

Torque: $\underline{\tau} = \underline{m}\times\underline{B}$

Potential energy: $U = -\underline{m}\cdot\underline{B} = \frac{\underline{B}\cdot\underline{H}}{2}$

$dE = \pm\mu_B B$

$\underline{B} = \underline{\nabla}\times\underline{A}$

$\underline{E} = -\frac{\partial\underline{A}}{\partial t} - \nabla\phi$

$\nabla^2 A = -\mu_o\underline{j}$

Potential energy: $U(\theta) = -\underline{m}\cdot\underline{B}$

Magntism; χ_B is magnetic susceptibility.

$\underline{M} = N\underline{m} = \frac{\chi_B}{\mu_o}\underline{B}$; $\chi_B = \frac{\mu_0 M}{B}$

Current per unit length of a cylinder; $M \equiv i_s$

Overall Magnetism

$\underline{B}_0$ is magnetism in air

(magnetic cylinder inside solenoid):

$\underline{B} = \underline{B}_0 + \mu_o\underline{M}$

$j_B = \nabla\times\underline{M}$

"Magnetic Field" or "Magnetic Intensity":

$\underline{H} = \frac{\underline{B}}{\mu_0} - \underline{M}$

$\oint \underline{H}\cdot\underline{dl} = i_f$

$B = \mu\mu_0\underline{H}$

$\mu = \frac{1}{1-\chi_B}$

≈ 1 for most materials

≈ 1000 in magnetic materials.

Diamagnetism:

$\underline{B}\uparrow, v_e\downarrow, m\uparrow$

$\underline{m} = -\underline{B}$ → field down overall.

Magnitude: $\sim 10^{-5}$

Paramagnetism

Orbital and spin components of the atom don't cancel out. Randomly orientated → $\underline{B}$ field applied → alignment → $\underline{B}\uparrow$. $\sim 10^{-3}$

Ferromagnetism:

Materials contain "domains" where dipoles are aligned. Applying a magnetic field aligns all the domain's fields. Magnetism remains after B is removed.

$B\uparrow \sim 1000$.

Hysterisis (history dependant)

Lenz's Law

"The direction of any magnetic induction effect is to oppose the cause of the effect"

ElectroMagnetism

Force (plus force on a conductor):

$\underline{F} = q(\underline{E} + \underline{v}\times\underline{B}) = q\underline{E} + \int I\underline{d\ell}\times\underline{B}$

Speed of Light: $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{E}{B}$

Waves in a vacuum (transverse):

$\frac{\partial^2 E}{\partial z^2} = -\frac{\partial^2 B}{\partial z\partial t} = \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2}$

$\frac{\partial E}{\partial z} = -\frac{\partial B}{\partial t}$

Electro-: $E = E_0\cos(kz - \omega t + \alpha)$

-Magnetism: $B = \frac{B_0}{c}\cos(kz - \omega t)$

Relationship between $\underline{E}$ and $\underline{B}$: $\underline{B} = \frac{1}{c}(\underline{\hat{k}}\times\underline{E})$

$\nabla^2\underline{E} = \varepsilon_0\mu_0\frac{\partial^2\underline{E}}{\partial t^2}$

Poynting Vector (energy flux per second):

$\underline{N} = \frac{1}{\mu_0}(\underline{E}\times\underline{B})$, $N = cU\,\Delta A\,\Delta t$,

$\langle\underline{N}\rangle = \frac{1}{2\mu_0}\frac{E_0^{\,2}}{c}$

Radiation pressure:

$$P_{absorbed} = \frac{\varepsilon_o E_o^{\ 2}}{2} = \frac{N}{c}$$

$$P_{reflected} = \varepsilon_o E_o^{\ 2} = \frac{2N}{c}$$

Reflection (perfect conductor):

$$\underline{E}_{incident} = \hat{\underline{x}} E_o \cos(kz - \omega t),$$

$$\underline{E}_{reflected} = -\hat{\underline{x}} E_o \cos(kz + \omega t)$$

Linear polarization:

$$E(z,t) = \hat{x}E_{ox}\cos(kz - \omega t) + \hat{y}E_{ox}\cos(kz - \omega t)$$

Circular polarization:

$$E(z,t) = \hat{x}E_{ox}\cos(kz - \omega t) + \hat{y}E_{ox}\sin(kz - \omega t)$$

Photon momentum: $p = \frac{E}{c}$

$$E = hf$$

$$\frac{F_B}{F_E} = \varepsilon_o \mu_o v^2 = \frac{v^2}{c^2}$$

Maxwell Equations

1) Gauss's Law (Electric flux):

$$\phi_E = \oint_s \underline{E} \cdot \underline{ds} = \frac{\sum_i q_i}{\varepsilon_o} = \frac{Q}{\varepsilon_o}$$

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\varepsilon_o}$$

$$\underline{\nabla} \cdot \underline{D} = \rho_F$$

2) Gauss's Law (Magnetic Flux):

$$\phi_B = \int \underline{B} \cdot \underline{dA} = BA\cos\phi$$

$$\underline{\nabla} \cdot \underline{B} = 0$$

3) Ampere's (corrected) law

$$\oint_{loop} \underline{B} \cdot \underline{d\ell} = \mu_o I_{enclosed}$$

$$= \mu_o (I_c + I_d)$$

$$= \mu_o \left(I_c + \varepsilon_o \frac{d\Phi_E}{dt} \right)$$

$$\underline{\nabla} \times \underline{B} = \mu_o \underline{J} + \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t}$$

$$\underline{\nabla} \times H = j_f + \frac{\partial D}{\partial t}$$

4)

$$\int \underline{E} \cdot \underline{dl} = \frac{d\Phi_B}{d\ell}$$

$$\underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

In free space, $\rho = j = 0$.

EM in Materials

$$\nabla^2 \underline{B} = \varepsilon\varepsilon_o \mu\mu_o \frac{\partial^2 \underline{B}}{\partial t^2}$$

$$\nabla^2 \underline{E} = \varepsilon\varepsilon_o \mu\mu_o \frac{\partial^2 \underline{E}}{\partial t^2}$$

$$\varepsilon\varepsilon_o \mu\mu_o = \frac{1}{v^2} = \frac{\varepsilon\mu}{c^2} = \frac{n^2}{c^2}$$

$$n = \frac{c}{v} = \frac{ck}{\omega} = \sqrt{\varepsilon\mu}$$

$\varepsilon\mu \approx 1$ for most materials

$$\nabla \cdot \underline{D} = \rho_f$$

$$\underline{\nabla} \cdot \underline{B} = 0$$

$$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\underline{\nabla} \times \underline{B} = \mu\mu_o \varepsilon\varepsilon_o \frac{\partial \underline{E}}{\partial t}$$

Snell's Law: $\frac{\sin\theta}{\sin\phi} = \frac{c}{v} = n = \sqrt{\varepsilon}$

Incident:

$$\underline{E}_1 = \hat{x}E_{0I}e^{i(\omega t - kz)}$$

$$\underline{B}_1 = \hat{y}B_{0I}e^{i(\omega t - kz)}$$

$$k_1 = \omega\sqrt{\varepsilon_o \mu_o}$$

Transmitted:

$$\underline{E}_1 = \hat{x}E_{0T}e^{i(\omega t - kz)}$$

$$\underline{B}_1 = \hat{y}B_{0T}e^{i(\omega t - kz)}$$

$$k_1 = \omega\sqrt{\varepsilon\varepsilon_o \mu\mu_o}$$

Reflected:

$$\underline{E}_1 = \hat{x}E_{0R}e^{i(\omega t + kz)}$$

$$\underline{B}_1 = -\hat{y}B_{0R}e^{i(\omega t + kz)}$$

$$\underline{E}_I + \underline{E}_R = \underline{E}_T \to E_{0I} + E_{0R} = E_{0T}$$

(continuous at surface)

$$\underline{H}_I + \underline{H}_R = \underline{H}_T$$

$$\to \frac{B_{0I}}{\mu_o} - \frac{B_{0R}}{\mu_o} = \frac{B_{0T}}{\mu\mu_o}$$

$$\to E_{0I} - E_{0R} = E_{0T}$$

Reflection coefficient: $R = \frac{{E_{0R}}^2}{{E_{0I}}^2} = \left(\frac{1-n}{1+n}\right)^2$

Transmission coefficient:

$$T = \frac{{E_{0T}}^2 v}{{E_{0I}}^2 c} = \frac{{E_{0T}}^2}{{E_{0I}}^2} n = \frac{4n}{(1+n)^2} \quad T = \frac{{E_{0T}}^2 v}{{E_{0I}}^2 c}$$

EM and conducting media

$$\mu_0 J >> \mu_0 \varepsilon_0 \frac{\partial \underline{E}}{\partial t}$$

$$\underline{j} = \sigma \underline{E}$$

$$\frac{\partial^2 E}{\partial z^2} = \mu_o \sigma \frac{\partial \underline{E}}{\partial t}$$

$$E = E_0 e^{i(kz-\omega t)}$$

$$k = \pm(1+i)\sqrt{\frac{\mu_0 \sigma \omega}{2}} = \frac{\pm(1+i)}{\delta}$$

Skin depth

Distance over which amplitude decreases by e (2.72): $\delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}}$

EM and Plasma

Electron density n_e, temperature T_e

$\rho = 0$, $j \neq 0$

$$\mu_0 j \approx \varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

$F_e = m_e \ddot{r} = e\underline{E}$ (collisionless)

$$j = ne\dot{r}$$

$$\frac{dj}{dt} = ne\ddot{r} = \frac{ne^2 E}{m_e}$$

$$\nabla^2 E = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2} + \mu_0 n_e \frac{e^2 E}{m_e}$$

$$k^2 = \varepsilon_0 \mu_0 \omega^2 - \frac{\mu_0 n_e e^2}{m_e}$$

Plasma frequency

$$\omega_p = \sqrt{\frac{n_e e^2}{\varepsilon_0 m_e}}$$

$\omega < \omega_p$, waves are attenuated.

$$v_p = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \frac{{\omega_p}^2}{\omega^2}}}$$

$$v_g = \frac{d\omega}{dk} = \frac{c}{n + \omega \frac{dn}{d\omega}}$$

Refractive index in a plasma:

$$n = \left(1 - \frac{{\omega_p}^2}{\omega^2}\right) \approx 1 - \frac{{\omega_p}^2}{2\omega^2}$$

If the pulse is observed at two different frequencies, then Dn_e can be measured.

$$t = \frac{D}{v} = \frac{Dn}{c}$$

Optics

Brewster's Angle: $\tan\theta_i = \frac{n_r}{n_i}$

Lensmaker's Equation: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Snell's Law: $n_1 \sin\theta_i = n_2 \sin\theta_r$

Lens Magnification: $\frac{h_v}{h_u} = \frac{v-f}{f}$

Gravitational Lenses: $\alpha = \frac{4GM}{bc^2}$, $\theta - \beta = \alpha \frac{D_{ls}}{D_s}$

Stoke's Parameters:

$$I = \langle {E_{0x}}^2 \rangle + \langle {E_{0y}}^2 \rangle$$

$$Q = \langle {E_{0x}}^2 \rangle - \langle {E_{0y}}^2 \rangle$$

$$U = \langle 2E_{0k} E_{0y} \cos\varepsilon \rangle$$

$$V = \langle 2E_{0x} E_{0y} \sin\varepsilon \rangle$$

Young's Double Slit $d\sin\theta = n\lambda$

Resolution $\theta = \frac{\lambda}{d}$ (due to diffraction)

Refractive index $n = \frac{c}{v} = \frac{ck}{\omega}$

Units

Flux (Weber): $1Wb = 1Tm^2 = 1NmA^{-1}$

Inductance (Henry):

$$1H = 1Wb\,A^{-1} = 1V\,s\,A^{-1} = 1\Omega\,s$$

Magnetism (Gauss): $1G = 10^{-4}\,T$

Magnetism (Tesla):

$$1T = 1N\,s\,C^{-1}\,m^{-1} = 1N\,A^{-1}\,m^{-1}$$

Magnetic Dipole Moment μ:

$$1N\,s^2\,C^{-2} = 1N\,A^{-2}\,Wb\,A^{-1}\,m^{-1} = 1T\,m\,A^{-1} = 1H\,m^{-1}$$

Volt: $1V = 1J\,C^{-1}$