Energies

$E = hf = \hbar\omega$

$E^2 = p^2c^2 + m^2c^4$

Potential energy between two charges:

$$V(r) = -\frac{q_1 q_2}{4\pi\varepsilon_o r}$$

Potential due to Angular Momentum:

$$V_\ell(r) = \frac{\ell(\ell+1)\hbar^2}{2m_r r^2}$$

Rydberg Energy:

$$E_R = \frac{e^2}{8\pi\varepsilon_o a_o'} = \left(\frac{e^2}{4\pi\varepsilon_o}\right)^2 \frac{m_r}{2\hbar^2}$$

Photoelectric effect ($W =$ Work function):

$E_{k,\max} = E - W$

Energy of electron level: $E_n = \dfrac{n^2h^2}{8mL^2}$

Operators

Operator x Eigenfunction = Eigenvalue x Eigenfunction

$\hat{O}\psi = O\psi$

Expectation value: $\langle O \rangle = \int_{-\infty}^{\infty} \Psi * \hat{O}\Psi dx$

Commutator: $\left[\hat{O}_1, \hat{O}_2\right] = \hat{O}_1\hat{O}_2 - \hat{O}_2\hat{O}_1$

X direction: $\hat{x} = x$

Momentum (x-direction):

$$\hat{P}_x = -i\hbar\frac{\partial}{\partial x}$$

$$\hat{P}_x^2 = -\hbar^2\frac{\partial^2}{\partial x^2}$$

Momentum (y-direction):

$$\hat{P}_y = -i\hbar\frac{\partial}{\partial y}$$

$$\hat{P}_y^2 = -\hbar^2\frac{\partial^2}{\partial y^2}$$

Momentum (z-direction):

$$P_z = -i\hbar\frac{\partial}{\partial z}$$

$$P_z^2 = -\hbar^2\frac{\partial^2}{\partial z^2}$$

Angular momentum (x direction):

$$L_x = i\hbar\left(\sin\phi\frac{\partial}{\partial\theta} + \cot\theta\cos\phi\frac{\partial}{\partial\phi}\right)$$

Spin $\hat{S}\psi = s(s+1)\psi$

Angular momentum (y-direction):

$$L_y = i\hbar\left(-\cos\phi\frac{\partial}{\partial\theta} + \cot\theta\sin\phi\frac{\partial}{\partial\phi}\right)$$

Angular momentum (z-direction):

$$L_z = -i\hbar\frac{\partial}{\partial\phi}$$

Energy: $\hat{E} = -\dfrac{\hbar^2}{2m}\nabla^2 + v(r)$

Quantum Mechanics

Schrödinger's Equation (TDSE 1D):

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\Psi = Ae^{i(kz-\omega t)}$$

Schrödinger's Equation (TISE, 1D):

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

$$i\hbar\frac{dT}{dt} = ET$$

$$\Psi = \psi T$$

Solution for TISE 1D:

$$\psi_n(x) = \underbrace{\left(\frac{1}{n!2^n a\sqrt{\pi}}\right)^{1/2}}_{\text{Normalization}} \underbrace{H_n\left(\frac{x}{a}\right)}_{\substack{\text{Hermite}\\ \text{Polynomial}}} \underbrace{e^{-\frac{x^2}{2a^2}}}_{\substack{\text{Gaussian}\\ \text{Exponential}}}$$

$$a = \sqrt{\frac{\hbar}{m\omega}}$$

Schrödinger's Equation (TISE, 2D):

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + V(x,y)\psi = E\psi$$

$$E = E_{n,x} + E_{n,y} = \left(n_x + n_y + 1\right)\hbar\omega$$

Solution for TISE 2D:

$$\psi_{n_x,n_y}(x,y) = H_{n_x}\left(\frac{x}{a}\right)H_{n_y}\left(\frac{y}{a}\right)e^{\frac{\left(x^2+y^2\right)}{2a^2}}$$

Schrödinger's Equation (TISE, 3D):

$$\nabla^2\Psi(r,\theta,\phi) + \frac{2m_r}{\hbar^2}\left(E - V(x)\right)\psi(r,\theta,\phi) = 0$$

Normalization of Waveform.

Overall probability = 1

$$\sum_{all\, n,\theta,\phi}\left|\Psi_{n,\ell,m}(r,\theta,\phi)\right|^2 = \sum_{all\, n,\theta,\phi}\left|\Psi * \Psi\right|^2 = 1$$

Orthonormal waveforms:

$$\int_{-\infty}^{\infty}\Psi_m * \Psi_n dx = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

Linear superposition of two waveforms

$\Psi = C_1\Psi_1 + C_2\Psi_2$

Normalized if $C_1^{\,2} + C_2^{\,2} = 1$

Represents 2 energy states.

Probability of E_1 is $C_1^{\,2}$, E_2 is $C_2^{\,2}$

Quantized energy: $E = \left(n + \frac{1}{2}\right)\hbar\omega$

Quantized energy difference:

$$\Delta E = E_R Z^2 \left(\frac{1}{n_1^{\,2}} - \frac{1}{n_2^{\,2}}\right)$$

Quantized angular momentum:

$L^2 = \ell(\ell+1)\hbar^2$

Quantized spin: $S^2 = s(s+1)\hbar^2$

Degeneracy of an atom: $\sum_{\ell=0}^{n-1}(2\ell-1) = n^2$

Magnetic Moment of particle in atom

$$\underline{\mu} = \frac{e}{2m}\underline{L} = \pm g_\ell \mu_M \frac{\underline{L}}{\hbar}$$

g_ℓ and μ_M depend on the particle being looked at.

Rotational Energy:

$$E_{rot} = \frac{I\omega^2}{2} = \frac{L^2}{2m} = \frac{n(n+1)}{2\ell}$$

$$\Delta E = \frac{(n+1)\hbar^2}{I}$$

Spin of a fermion: $s = \frac{1}{2}\hbar = \frac{\mu}{g_s}$

Heisenberg Uncertainty Principle: $\Delta E \Delta t \geq \frac{\hbar}{2}$

Uncertainty: $(\Delta y)^2 = \langle y^2 \rangle - \langle y \rangle^2$

Identical particles:

$$\Psi(x_1, x_2, t) = \Psi_A(x_1)\Psi_B(x_2)e^{-iEt/\hbar}$$

$$E = E_A + E_B$$

Indistinguishable particles:

$$|\Psi(x_1, x_2)|^2 = |\Psi(x_2, x_1)|^2$$

De Broglie Wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE_k}}$$

Compton wavelength if $v \approx c$

Vibrational energy: $E_{vib} = \left(n + \frac{1}{2}\right)\hbar\omega$

Bosons:

$$\Psi(x_1, x_2) = \Psi(x_2, x_1)$$

$$\Psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\phi_a(x_1)\phi_b(x_2) + \phi_a(x_2)\phi_b(x_1)\right)$$

Fermions:

$$\Psi(x_1, x_2) = -\Psi(x_2, x_1)$$

$$\Psi(x_1, x_2) = \frac{1}{\sqrt{2}}\left(\phi_a(x_1)\phi_b(x_2) - \phi_a(x_2)\phi_b(x_1)\right)$$

Sizes

Bohr Radius

$$a_o = \frac{4\pi\varepsilon_o\hbar^2}{me^2}$$

a_o' when reduced mass is used

Pauli exclusion principle

No two electrons in the same atom can have all quantum numbers the same

Quantum Numbers

Principle Quantum Number: $n = 1, 2, 3, \ldots$

Orbital angular momentum quantum number:

$\ell = 0, 1, \ldots, n-1$

Azimuthal / Magnetic Quantum Number:

$m = -\ell, \ldots, 0, \ldots, \ell$

Spin Quantum Number: $m_s = \pm\frac{1}{2}$

$\underline{j} = \underline{\ell} + \underline{s}$