Radio Astronomy Lecture Notes

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Contents

1 Introduction

Production – Propagation – Processing

1.1 Radio Window

Radio waves have a big window for seeing through the atmosphere – from circa λ 10mm to 10m. Radio astronomy is astronomy done through this window.

The long wavelength end is produced by the Earths ionosphere. Radio waves will not propagate if their frequency is less than the plasma frequency ν_p, which is around 10MHz (30m) in the Earths ionosphere. This boundary does change with time and location (as the ionosphere does).

The plasma frequency is:

$$\nu_p = \frac{e}{2\pi}\sqrt{\frac{n_e}{\varepsilon_0 m_e}} \approx 9\sqrt{n_e}Hz \tag{1}$$

NB: the Interstellar Medium (ISM) has $n_e \sim 3 \times 10^4 m^{-3} \rightarrow \nu_p \sim 2kHz$, which defines the absolute boundary.

The short wavelength end is produced by molecules in the atmosphere absorption lines due to water vapour at 22GHz, oxygen at 60–70GHz, etc., which give $\sim 60dB$ attenuation.

1.2 Decibels

Definition of decibels:

$$10\log_{10}\left(\frac{I}{I_0}\right) \tag{2}$$

gives ratio in db, where I_0 is the initial signal strength, and I is the new one. For example:

$$100dB = 10^{10} \tag{3}$$

$$3dB = 2 \tag{4}$$

1.3 Collision Broadening

Absorption lines in the atmosphere arent thin due to collisions occurring in the atmosphere (it's very dense) – so any excited electrons quickly get knocked back down to a ground state by colliding atoms. The uncertainty principle kicks in because of the uncertainty of the energy of the system.

$$\Delta E \Delta t \geq \hbar \tag{5}$$

Δt is the time the system is in an excited state. In atmosphere, $\nu \sim 450ms^{-1} \rightarrow$ mean free path of $\sim 6 \times 10^{-8}m \rightarrow \tau \sim 10^{-10}s$

$$\Delta E = \frac{\hbar}{\Delta t} \tag{6}$$

$E = h\nu$, so $\Delta E = h\Delta\nu$

$$\Delta\nu = \frac{\hbar}{h\Delta t} = \frac{1}{2\pi\Delta t} \tag{7}$$

For $\tau \sim 10^{-10}s$, $\Delta\nu \sim 1GHz$.

Radio astronomy is photon rich – unlike IR and shorter wavelengths, where some of the error comes from counting the number of photons detected, there are lots of photons so the related error is very small. This defines the techniques that can be used. IR and shorter wavelengths have their techniques limited by photon count statistics.

1.4 Hot and Cold Radio Astronomy

Cold:

- Planets
- Neutral Hydrogen
- Molecules
- CMB

Hot:

- Pulsars
- Supernovae
- AGN – Active Galactic Nuclei
- GRBs – Gamma Ray Bursts

1.5 Sensitivity

Minimum strength that can be detected:

$$\Delta S_{min} = \frac{2k}{A_{eff}} \frac{T_{sys}}{\sqrt{B\tau}} \tag{8}$$

where k is Boltzmans constant, A_{eff} is the effective collecting area of the telescope, T_{sys} is a measure of how noisy the system is (in Kelvin), B is the bandwidth available and τ is the integration time.

1.6 Resolution

$$\Theta \approx \frac{\lambda}{d} \tag{9}$$

where Θ is in radians, λ is the wavelength and d is a characteristic length. For the Lovell telescope at $\lambda = 1m$, $\Theta \sim 1^\circ$.

1.7 Solid Angles

- Solid angle of sphere is 4π steradians (sr).
- The solid angle of the area subtended by A at radius R is $\Omega = \frac{A}{R^2}$ sr.
- $1Sr \equiv 3283$ square degrees.
- E.g. the Sun diameter $\approx 0.5°$, subtends a solid angle of 6×10^{-5} sr.

2 Fundamentals

2.1 Brightness

Brightness (specific intensity):

- We measure the energy dE per unit area, per unit bandwidth, per steradian, per unit time.
- For a detector element dA, then $dE = I_\nu dA dt d\nu d\Omega$
- The brightness I_ν has units of $Wm^{-2}Hz^{-1}Sr^{-1}$.

Sample brightness is a property of the source.

Want to measure $I_\nu(\theta, \phi)$, the brightness distribution on the sky.

Flux density S_ν of a source is the brightness integrated over the solid angle.

$$S_\nu = \int_n I_\nu(\Omega) d\Omega \tag{10}$$

which is measured in $Wm^{-2}Hz^{-1}$. Define 1 Jansky (Jy) = $10^{-26}Wm^{-2}Hz^{-1}$.

Energy collected by Lovell telescope in 50 years?

- Lovell telescope is 76m in diameter, with a bandwidth of $\sim 20MHz = 2 \times 10^7 Hz$.
- Flux density at $21cm$ from Cassiopeia $A \approx 2000Jy$.
- Power $= 2 \times 10^{-23} \times A_e \times 2 \times 10^7 = 10^{-12}W$. Note: the effective area $A_e \approx 0.6 A_{geometric}$
- Energy collected = power $\times$ time $\approx 1.7 \times 10^{-3} J$
- Typical sensitivity is about $10\mu Jy$ atm.

2.2 Black Body Radiation

Planck radiation formula:

$$I_\nu = \frac{2\nu^2}{c^2} \frac{h\nu}{\left(e^{\frac{h\nu}{kT}} - 1\right)} Wm^{-2}Hz^{-1}Sr^{-1} \tag{11}$$

In radio astronomy, nearly always $h\nu << kT$. Using this, we can make the approximation:

$$I_\nu = \frac{2\nu^2}{c^2}\frac{h\nu}{1+\frac{h\nu}{kT}-1} = \frac{2kT}{\lambda^2} = \frac{2\nu^2 kT}{c^2} \tag{12}$$

This is the Rayleigh-Jeans approximation. In the RJ regime, $I_\nu \propto T$.

We can define a brightness temperature T_b. This is the temperature of a hypothetical black body that would produce the same brightness as the source at a frequency ν.

For sources which are emitting by virtue of their temperature, i.e. thermal sources, then the brightness temperature of the source is approximately equal to their physical temperature. For non-thermal sources, their brightness temperature is not equal to their thermal temperature, but nevertheless you can still define a brightness temperature for them.

For a thermal source, the brightness temperature $T_b \approx T_{phys}$ the physical temperature. For a non-thermal source, $T_b \neq T_{phys}$.

Example: What is the brightness temperature of the Milky Way at $\lambda = 1m$?

$$I_\nu \sim 3\times 10^{-21} Wm^{-2}Hz^{-1}Sr \tag{13}$$

$$I_\nu = \frac{2kT_b}{\lambda^2} \to T_b \approx 109k \tag{14}$$

2.3 Nyquist Theorem and Noise Temperature

Due to the random motions of electrons, resistors produce a random fluctuating current/voltage. Nyquist showed that the average noise voltage $\langle V_n^2\rangle = 4RkT\Delta\nu$, where R is the resistance, T is the temperature of the resistor and $\Delta\nu$ is the range of frequencies that the amplifier detects.

If we have a resistor, and want to measure the power we need to use a device with resistance R_s. For max power transfer we need $R_s = R$.

$$P = \langle IV\rangle = \frac{\langle V_x^2\rangle}{R} = \frac{\langle V_n^2\rangle}{4R} \tag{15}$$

where V_n is the open circuit voltage. Combined with the first equation, this gives

$$P = kT\Delta\nu \tag{16}$$

This gives the definition of Noise Temperature for a circuit.

2.4 Radiation Transfer

Look at a part of a medium between the radio receiver and the source, length ds (along the line of sight). Input brightness is I_ν, output is $I_\nu \sim dI_\nu$. The medium can emit and absorb.

$$dI_\nu = dI_{\nu-} + dI_{\nu+} \tag{17}$$

where $dI_{\nu-}$ is absorption, $dI_{\nu+}$ is emission.

$$dI_{\nu-} = -\kappa_\nu I_\nu ds \tag{18}$$

$$dI_{\nu +} = \epsilon_\nu ds \tag{19}$$

$$dI_\nu = (-\kappa_\nu I_\nu + \epsilon_\nu) ds \tag{20}$$

This gives the equation of radiation transfer:

$$\frac{dI_\nu}{ds} = \epsilon_\nu - \kappa_\nu I_\nu \tag{21}$$

2.4.1 Special Cases

- Emission only: $\kappa_\nu = 0$, $\frac{dI_\nu}{ds} = \epsilon_\nu$

  $\rightarrow I_\nu(s) = I_\nu(s_0) + \int_{s_0}^{s} \epsilon_\nu ds$

- Absorption only: $\epsilon_\nu = 0$, $\frac{dI_\nu}{ds} = \kappa_\nu I_\nu$

  $\rightarrow I_\nu(s) = I_\nu(s_0) exp\left(-\int_{s_0}^{s} \kappa_\nu(s) ds\right)$

- If κ_ν is independent of s:

  $I_\nu(s) = I_\nu(s_0) exp\left(-\kappa_\nu(s - s_0)\right)$

- Thermodynamic Equilibrium (TE):

  Absorption balances emission at all frequencies.

  $\frac{dI_\nu}{ds} = 0 \rightarrow I_\nu = \frac{\epsilon_\nu}{\kappa_\nu}$

  (True thermodynamic equilibrium is a black body a system which absorbs at one frequency and emits at another is not a thermodynamic equilibrium).

  The only thing that I_ν depends on is the temperature.

$$I_\nu = \frac{\epsilon_\nu}{\kappa_\nu} = \frac{2h\nu^3}{c^2}\left(\frac{1}{e^{\left(\frac{h\nu}{kT}\right)} - 1}\right) \tag{22}$$

- Local Thermodynamic Equilibrium (LTE):

  Locally $I_\nu = \frac{\epsilon_\nu}{\kappa_\nu}$

2.5 Optical Depth

$$\tau_\nu = \int_{s_0}^{s} \kappa_\nu(s) ds \tag{23}$$

or alternatively,

$$d\tau = -\kappa_\nu ds \tag{24}$$

τ is a measure of how opaque something is. Equation of transfer in terms of optical depth (for LTE):

$$-\frac{1}{\kappa_\nu}\frac{dI_\nu}{ds} = \frac{dI_\nu}{d\tau} = I_\nu - B(T) \tag{25}$$

where $B(T)$ is the brightness of the medium given by the Planck function. Integrated (Rohlfs & Wilson [R & W] p9):

$$I_\nu(s) = I_\nu(s_0)e^{-\tau(s)} + \int_0^{\tau_\nu(s)} B_\nu(T(\tau))e^{-\tau}d\tau \tag{26}$$

Initial signal is attenuated (first term) The medium has a temperature, and the more the optical depth the more it will look like a black body.

Simplify by assuming an isothermal medium. $T(\tau) = T(s) = T = const.$ Equation becomes $I_\nu = I_\nu(0)e^{-\tau_\nu(s)} + B_\nu(T)(1 - e^{-\tau_\nu(s)})$. This can be rewritten as

$$I_\nu = I_\nu(0) + (B_\nu(T) - I_0(0))(1 - e^{-\tau_\nu(s)}) \tag{27}$$

If $I_\nu(0) > B_\nu(T)$, the cloud appears in absorption.
If $I_\nu(0) < B_\nu(T)$, the cloud appears in emission.
Expressing the equation in terms of brightness temperatures:

$$T_b = T_0(0)e^{-\tau_\nu(s)} + T_{b,cloud}\left(1 - e^{-\tau_\nu(s)}\right) \tag{28}$$

where T_b is the final temperature, $T_0(0)$ is the starting temperature, and $T_{b,cloud}$ is the temperature of the cloud.

NB: whenever there is absorption, there is emission.

Special cases:

- $\tau << 1$ "optically thin" $\rightarrow T_b = \tau T_{cloud}$
- $\tau >> 1$ "optically thick" $\rightarrow T_b = T_{cloud}$

2.6 Atmospheric Emission

The atmosphere attenuates all radio signals. Therefore the atmosphere adds noise to the signal. The big problem is that the noise is time dependent.

2.7 Studying the CMB

At its best, the optical depth of the atmosphere is ~ 0.02. Therefore

$$T_{b,atm} = 0.02 \times 270k \approx 5.5k \tag{29}$$

(where k denotes Kelvin.) cf. $T_{CMB} = 2.7k$.

Example: The output of the receiver increases $10k$ when the antenna is tipped from the zenith to 30 degrees elevation. What is the atmospheric emission? Assume a flat earth. a is the distance to the edge of the atmosphere at the zeneth, b is the distance along the line-of-sight at 30 degrees.

$$\sin 30^\circ = \frac{a}{b} \rightarrow b = 2a \tag{30}$$

$\rightarrow$ get plot of atmospheric emission as a function of elevation.

2.8 Antenna Relationships

2.8.1 Effective Area A_{eff}

A source produces a flux density S_ν $WHz^{-1}m^{-2}$. Antenna collects a power P_{rec}.

$$A_{eff} = \frac{P_{rec}}{S_\nu} \tag{31}$$

Sometimes known as the effective aperture, or collecting area.

2.8.2 Aperture efficiency

$$\eta = \frac{A_{eff}}{A_{geometric}} \tag{32}$$

For parabolic telescopes, $\eta = 50 - 80\%$ But efficiency depends on surface accuracy - the reflecting surfaces of the telescope are never perfect.

2.8.3 Ruze formula

The Ruze formula relates the efficiency to the wavelength and the RMS deviations from a perfect surface:

$$\eta_{surface} = e^{-\left(\frac{4\pi\epsilon}{\lambda}\right)^2} \tag{33}$$

where ϵ is the RMS surface error and λ is the wavelength. For example, if $\epsilon = \frac{\lambda}{20}$, $\eta = 67\%$.

2.8.4 Reciprocity

Often easier to understand antennas in terms of transmission than reception. They are equivalent. Shown by the Reciprocity theorem – R & W p127-129.

2.9 Rayleigh Distance

$$R > R_{rayleigh} = \frac{2D^2}{\lambda} \tag{34}$$

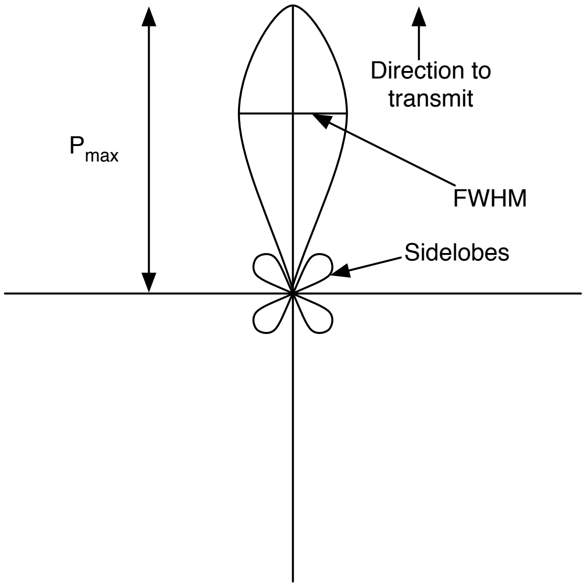
where antenna properties do not change with R. Conventionally, Rayleigh is the distance beyond which the deviations from a plane wave are less than a 16^{th} of a wavelength. Distance to source is R. Angle between source and telescope edge is θ. Radius of telescope is $D/2$.

The radius of the wave front $= \sqrt{R^2 + \left(\frac{D}{2}\right)^2}$. We require

$$\frac{\lambda}{16} = R\sqrt{1 + \left(\frac{D}{2R}\right)^2} - R \approx R + \frac{RD^2}{8R^2} - R \tag{35}$$

$$\rightarrow R = \frac{2D^2}{\lambda} \tag{36}$$

With the Lovell telescope, $D = 76m$, $\lambda = 21cm \rightarrow$ Rayleigh distance $= 55km$.



2.10 Normalized Power Pattern

The power radiated per steradian in the direction (θ, ϕ) divided by the peak power per steradian is:

$$P_n(\theta, \phi) = \frac{P(\theta, \phi)}{P_{max}} \tag{37}$$

Full Width to Half Maximum (FWHM).

2.10.1 Main beam solid angle

$$\Omega_m = \iint P_n(\theta, \phi) d\Omega \tag{38}$$

where the integral is over the main beam (integrate until the first minima in the radiation pattern are reached).

2.10.2 Antenna/beam solid angle

$$\Omega_A = \iint P_n(\theta, \phi) d\Omega \tag{39}$$

where the integral is done over 4π steradians, i.e. including the sidelobes. It represents the solid angle of an ideal antenna, where all power is concentrated into a single beam with no sidelobes.

2.10.3 Main Beam Efficiency

$$\epsilon_m = \frac{\Omega_m}{\Omega_A} \tag{40}$$

NB: not quite the same as the aperture efficiency η, but it is related.

2.10.4 Directivity (or Gain)

$$D(\theta,\phi) = \frac{P_n(\theta,\phi)}{\frac{1}{4\pi}\iint_{4\pi} P_n(\theta,\phi)d\Omega} = \frac{4\pi P_n(\theta,\phi)}{\Omega_A} \tag{41}$$

$$D_{max} = \frac{4\pi}{\Omega_A} \tag{42}$$

Gives the power radiated in the useful direction, as opposed to an isotropic emitter.

Parameters are related:

$$A_e\Omega_A = \lambda^2 \tag{43}$$

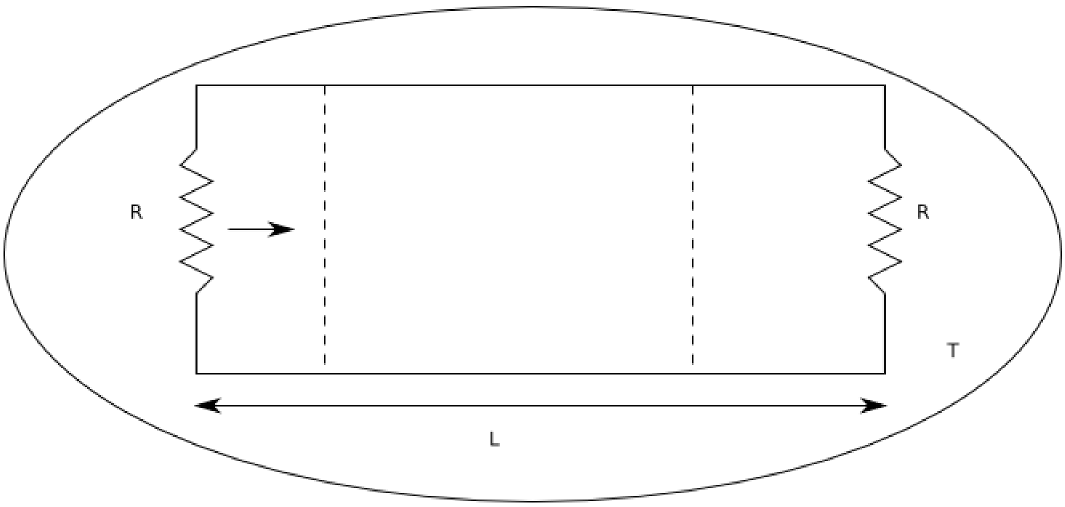
Imagine a resistor R connected to an antenna. Enclose the system in a black body cavity at temperature T. The antenna looks at the surface of the cavity, also at temperature T. A_e is the effective area and Ω_A is the antenna solid angle.

The antenna radiates power into the enclosure, which is absorbed. The black body radiates and some of the energy will be received. (Only some, as only part of the surface is viewed by the antenna.) Let the surface subtend an angle Ω_A. Assume a bandwidth $\Delta\nu$.

RJ law: $I_\nu = \frac{2kT}{\lambda^2}\Delta\nu$ in $Wm^{-2}Hz^{-1}Sr^{-1}$. Power collected by the antenna: $A_e\frac{kT}{\lambda^2}\Delta\nu\Omega_A$. Factor of 2 lost as antennas only generally record one plane of polarization so half the power. In equilibrium, the antenna radiates as much as it receives. Connected to a match load R at temperature T. From Nyquist theorem $\rightarrow$ produces noise power $= kT\Delta\nu$. In equilibrium, $kT\Delta\nu = A_e\frac{kT}{\lambda^2}\Delta\nu = \Omega_A$. Simplifying, $A_e\Omega_A = \lambda^2$.

3 Noise

3.1 Thermal noise from a resistor - Nyquist



For this resonator, the n^{th} mode has a wavelength given by

$$\frac{n\lambda}{2} = L, \;\; or \;\; n = \frac{2L\nu}{c} \tag{44}$$

The number of modes $dn/d\nu$ of bandwidth is $\frac{dn}{d\nu} = \frac{2L}{c}$. From quantum black body theory, the energy in each mode is $\frac{h\nu}{e^{-\frac{h\nu}{kT}}-1} \approx kT$ in the RJ regime. Note that there is $\frac{1}{2}kT$ for each polarization. The average noise power N from each resistor travels a time $t = \frac{L}{c}$ before reflection. It is stored as an energy ΔW.

$$\Delta W = 2Nt = 2N\frac{L}{c} = N\frac{dn}{d\nu} \tag{45}$$

But if there are dn modes in $d\nu$, $\Delta W = kTdn = N\frac{dn}{d\nu}$

$$N = kTd\nu \tag{46}$$

Also known as Johnson noise, after the person who measured it in the laboratory.

3.2 Antenna Temperature and Beam Convolution

What will the output power depend on?

- A_e
- The normalized power pattern $P_n(\theta', \phi')$
- The brightness of the sky $I_\nu(\theta, \phi)$

The power from per unit bandwidth:

$$dP = \frac{1}{2}A_e P_n(\theta', \phi')I_\nu(\theta, \phi)d\theta d\phi \tag{47}$$

$$P = \frac{1}{2}A_e \iint_{4\pi} P_n(\theta', \phi')I_\nu(\theta, \phi)d\theta d\phi \tag{48}$$

where the factor of 1/2 is due to only observing one polarization.

We define the antenna temperature T_A by

$$kT_a = \frac{1}{2} \iint_{4\pi} P_n(\theta', \phi')I_\nu(\theta, \phi)d\theta d\phi \tag{49}$$

T_A is not the physical temperature of the antenna!

$$T_A = \frac{A_e}{2k} \iint_{4\pi} P_n(\theta', \phi')I_\nu(\theta, \phi)d\theta d\phi \tag{50}$$

Using $\Omega_A A_e = \lambda^2$,

$$T_A = \frac{\lambda^2}{2k\Omega_A} \iint_{4\pi} P_n(\theta', \phi')I_\nu(\theta, \phi)d\theta d\phi = \frac{\lambda^2}{2k} \frac{\iint_{4\pi} P_n(\theta', \phi')I_\nu(\theta, \phi)d\theta d\phi}{\iint_{4\pi} P_n(\theta', \phi')d\theta d\phi} \tag{51}$$

$$I_\nu(\theta, \phi) = \frac{2k}{\lambda^2}T_b(\theta, \phi) \tag{52}$$

Therefore,

$$T_A(\theta_0, \phi_0) = \frac{\iint_{4\pi} P_n(\theta - \theta_0, \phi - \phi_0)I_\nu(\theta, \phi)d\theta d\phi}{\iint_{4\pi} P_n(\theta, \phi)d\theta d\phi} \tag{53}$$

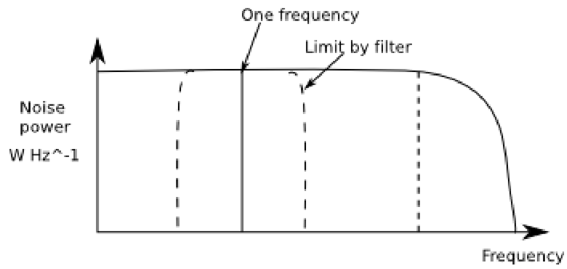
where $\theta - \theta_0$, and $\phi - \phi_0$ have been introduced as to get $T_A(\theta, \phi)$, one must scan the antenna across the sky. The numerator is the convolution of $T_b(\theta, \phi)$ with the beam power pattern.

Sky convoluted with a Beam:

$T_A = \frac{1}{\Omega_A} P_n \ast T_B$, where T_A is the antenna temperature, P_n is the beam pattern, $\ast$ represents convolution and T_p is the sky temperature.

3.3 Signal detection and noise

Noise from a resistor "Johnson noise" is "white" – the noise power is independent of frequency until $h\nu << kT$ condition breaks down, i.e. at $\nu \sim \frac{kT}{h}$. This is represented on the following diagram by the right-most dashed line.



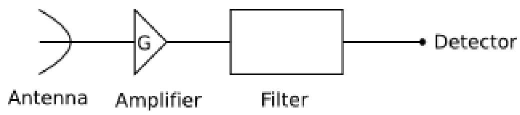
The noise from a radio receiver should be white – sometimes called Gaussian. The voltage v has a probability density function

$$P(v) = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_v} exp \left(-\frac{v^2}{2\sigma_v} \right)^2 \tag{54}$$

This has a mean $\langle v \rangle = 0$ and variance $\langle v^2 \rangle = \sigma_v^2$. The voltage as a function of time is a stationary random variable. The value at one instant is uncorrelated with that at the next instant. Plotting v vs. t produces a random scatter around a fixed voltage.

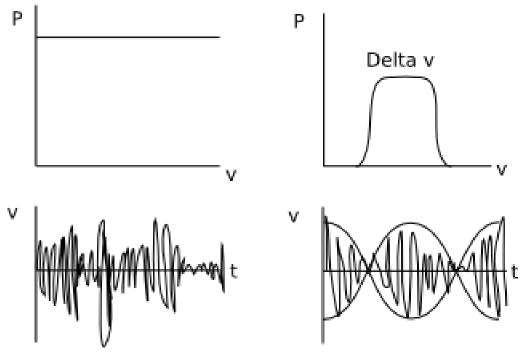
Radio astronomers limit their bands with filters with frequency range $\Delta\nu \rightarrow$ the noise is no longer white. If the band consists of a single frequency, then the noise becomes a perfect sinosoid, and is therefore predictable.

$v(t) \Leftrightarrow$ bandwidth via a fourier transform.



Amplifiers have a gain G = signal out / signal power in. It is often measured in dB. We specify the amplifier noise in terms of the "Noise Temperature", T_R, the temperature that a matched resistor at the amplifier input which would generate the observed noise power output. The power out = $Gk(T_A + T_R) = GkT_{sys}$ $[WHz^{-1}]$, where T_A is the antenna temperature, and T_{sys} is the system temperature. Both T_{sys} and $\Delta\nu$ limit the detectability of signals.

3.4 Narrow band noise



Filters convert white noise into band-limited noise (Burke & Smith Chapter 3)

Detect the signal (i.e. measure $v^2(t)$) and average it for a time δt. We will only get independent samples if $\delta t = \frac{1}{\Delta\nu}$ the coherent time.

3.5 Minimum Detectable Temperature

In an observing time τ there will be $\tau\Delta\nu$ independent samples. In any one sample, we get a random fluctuation corresponding to T_{sys}. Hence $\tau\Delta\nu$ samples give an uncertainty in T_{sys} of

$$\Delta T = \frac{T_{sys}}{\sqrt{\tau\Delta\nu}} \tag{55}$$

Note that $\Delta\nu$ is also called the bandwidth B.

3.6 Antenna Temperature from a Point Source

We showed earlier that the noise power from an antenna is

$$kT_A = \frac{1}{2}A_{eff}\iint I_\nu(\theta,\phi)P_n(\theta,\phi)d\theta d\phi \tag{56}$$

For a point source, $\iint I_\nu(\theta,\phi)P_n(\theta,\phi)d\theta d\phi = S_\nu$. Also, $P_n(\theta,\phi)$ when we point at the source, and therefore $kT_A = \frac{1}{2}A_{eff}S_\nu$. Hence, $S_\nu = \frac{2kT_A}{A_{eff}}$. $\left(\frac{2k}{A_{eff}}\right)$, measured in $Jy\ k^{-1}$, is a useful measure of the performance of a telescope. Usually want small values of $Jy\ k^{-1}$.

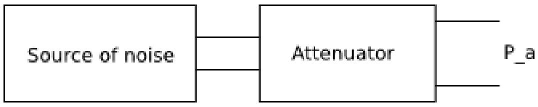
For example, $32m$ telescope, 40% efficiency. $A_{eff} = 0.4\pi 16^2 = 322m^2$. Number for $Jy\ k^{-1} \approx 9$. For a $100m$ telescope, typical efficiency, $\sim Jy\ k^{-1}$

Assume $B = 10GHz$, T_{sys}. How long an integration to detect a $5mJy$ source at 5σ? $\rightarrow$ want $1\sigma = 1mJy = \Delta S_\nu$.

$$\sqrt{B\tau} = \frac{2kT_{sys}}{\Delta S_\nu A_{eff}} \tag{57}$$

$$\rightarrow \tau = \left(\frac{2kT_{sys}}{\Delta S_\nu A_{eff}}\right)^2 \frac{1}{B} \approx 12s \tag{58}$$

3.7 Noise from an attenuator



The gain of the attenuator $G = \alpha \leqslant 1$. α represents the loss in the whole system. Attenuator also called a transformer. Have the noise source an the attenuator at the same temperature T.

Noise source produces a power kT (per unit bandwidth). Attenuator reduces noise source signal by α; αkT of the noise source signal emerges from the attenuator, but everything is matched and at T, so the total output must be kT. Therefore the attenuator produces a noise to make up the difference.

$$P_A = kT = \alpha kT + P_{att} \tag{59}$$

$$\rightarrow P_{att} = (1-\alpha)kT \tag{60}$$

This is a general expression, regardless of the temperature of the noise source – the source and attenuator need not be at the same T.

$$P_{att} = (1-\alpha)kT_{att} \tag{61}$$

where T_{att} is the actual temperature of the attenuator.

Noise temperature $= T_{att}(1-\alpha)$.

Example: Attenuation of $0.1dB$ between the antenna and the amplifier. Contribution to T_{sys}? $T_{att} = 300k$.

$\alpha = 10^{-0.01} = 0.977$

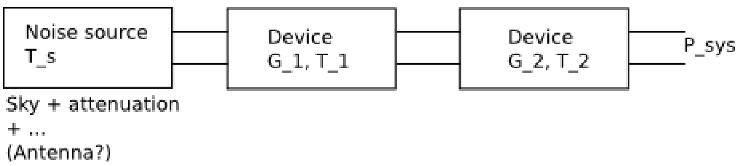
$T_{sys} = (1-\alpha)300 = 6.8k$

If $T_{att} = 20k$, then $T_{loss} = 0.5k$.

3.7.1 Optical depth and attenuation

Attenuator: $(1-\alpha)T_{att}$. Brightness of absorbing cloud: $(1-E^{-\tau})T_{cloud}$. Expressions equivalent if $\alpha \equiv e^{-\tau}$.

3.8 T_{sys}



NB: T_1, T_2 are the noise temperatures, not the actual temperatures.

Remember T_{system} is the temperature of a matched resistor at the input which will produce the observed output noise power.

Power from the noise source $= kT_s$

Power out of $1 = G_1 k(T_s + T_1)$

Power out of 2 $= kG_2(T_2+G_1(T_s+T_1))$, which by the definition of $T_{sys} = kT_{sys}G_1G_2$. Equating,

$$G_1G_2T_{sys} = G_2(T_2 + G_1(T_s + T_1)) \tag{62}$$

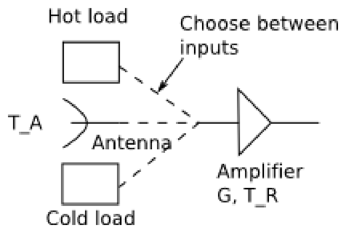
or

$$T_{sys} = T_s + T_1 + \frac{T_2}{G_1} \tag{63}$$

In general,

$$T_{sys} = T_s + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1G_2} \tag{64}$$

3.9 Measuring T_{sys}



$T_{sys} = T_A + T_R$

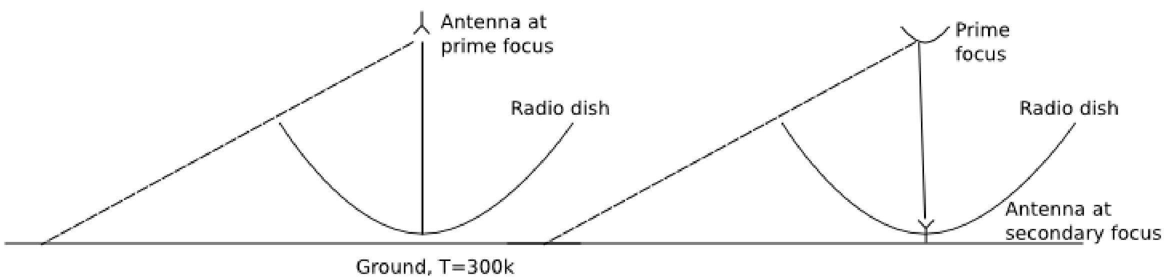
Use hot and cold loads, measure the ratio of the powers available from hot and cold loads Y.

$$Y = \frac{T_R + T_{hot}}{T_R + T_{cold}} \tag{65}$$

$$\rightarrow T_R = \frac{T_{hot} - YT_{cold}}{Y - 1} \tag{66}$$

Then measure power when antenna is connected to get T_A.

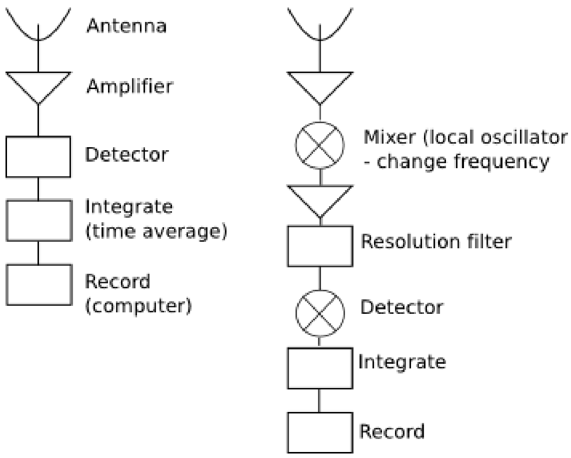
3.10 Example of noise accounting



- $3k$ from CMB
- $7k$ "spill-over" or "ground radiation" from ground at telescope ($6-7k$ for OCRA). Worse on prime focus dish than cassegrain.
- $\sim 10k$ from atmospheric emission

- $15k$ from 1st amplifier
- $1k$ from feed attenuation
- $4k$ from second stage
- etc.

3.11 Heterodyne receivers



Most receiver systems, especially for spectral line, pulsars and interferometry, have a mixer or down-converter which changes the frequency of the signal. Why?

- Signal processing is easier
- Attenuation in cables is less at lower frequencies than higher ones.
- Makes it easy to change frequency bands.

How?

The radio astronomy signal is mixed with a "local oscillator" signal. In general, a non-linear device will produce harmonics. Mixers exploit this. Square-law mixer, $I_{out} = aV_{in}^2$.

Feed two sinusoids into a square-law device.

$$V_1 \cos \omega_1 t + V_2 \cos(\omega_2 t + \phi) \tag{67}$$

$$I_{out} = a(V_1 \cos \omega_1 t + V_2 \cos(\omega_2 t + \phi))^2 \tag{68}$$

Use trig identities $\cos^2 \theta = \frac{1}{2}(1 - \cos 2\theta)$ and $2 \cos A \cos B = \cos(A - B) + \cos(A + B)$.

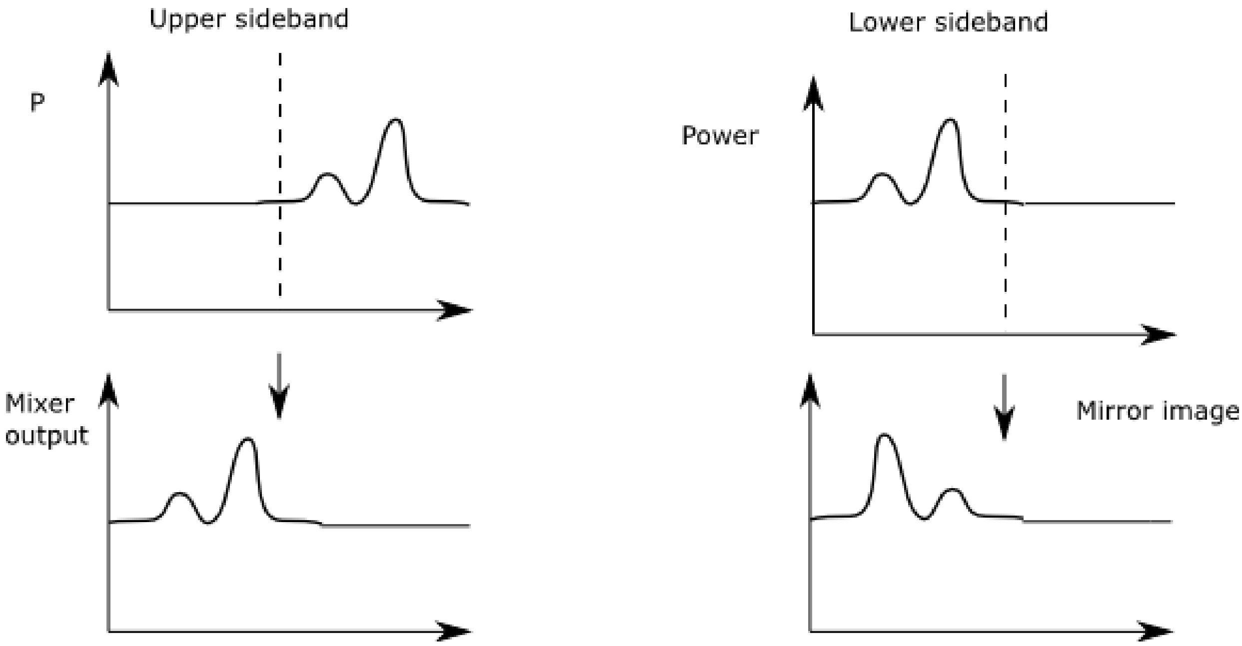
$$\rightarrow I_{out} = \frac{a}{2}(v_1^2 + v_2^2) + \frac{a}{2} v_1^2 \cos 2\omega_1 t + \frac{a}{2} v_2^2 \cos 2(\omega_2 t + \phi) + a v_1 v_2 \cos(\omega_1 t - \omega_2 t - \phi) + a v_1 v_2 \cos(\omega_1 t + \omega_2 t - \phi) \tag{69}$$

The first term is the DC term. The fourth term is the difference, and is the important one.

The down-converted signal (IF, intermediate frequency):

$$\nu_{IF} = |\nu_{RF} - \nu_{LO}| \tag{70}$$

where LO denotes Local Oscillator. For a fixed IF and LO, there are two frequency bands detected by the system. They are known as Upper Side-band and Lower Side-band.

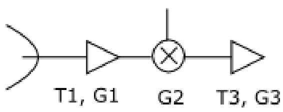


The x-axis is frequency ν. The dashed line is ν_{LO}.
RF filters can be used to exclude the unwanted side-band.
Heterodyning presumes phase information.

3.11.1 Conversion loss

The IF power out over the RF power in is significantly less than 1. This corresponds to a "conversion loss".

What is the system temperature of the following?



- $T_{sky} = 10k$
- $T_1 = 20k, G_1 = 100$
- $T_2 = 300k, G_2 = 0.1$
- $T_3 = 300k, G_3 = 100$

$$T_{sys} = T_{sky} + T_1 + \frac{T_2}{G_1} | \frac{T_3}{G_1 G_2} \tag{71}$$

$$T_{sys} = 10 + 20 + \frac{300}{100} + \frac{300}{0.1 \times 100} = 63k \tag{72}$$

Note the 30k from the second amplifier. Hence it would be a good idea to have additional amplifiers before the mixer, to minimise T_{sys}.

3.12 Square law detectors

Diodes can be used as both mixers and detectors. It depends on the terms that are kept after filtering.

Detector converts voltage to power. $P \propto V^2$

3.13 Gain stability, 1/f Noise

$$\Delta T = \frac{T_{sys}}{\sqrt{B\tau}} \tag{73}$$

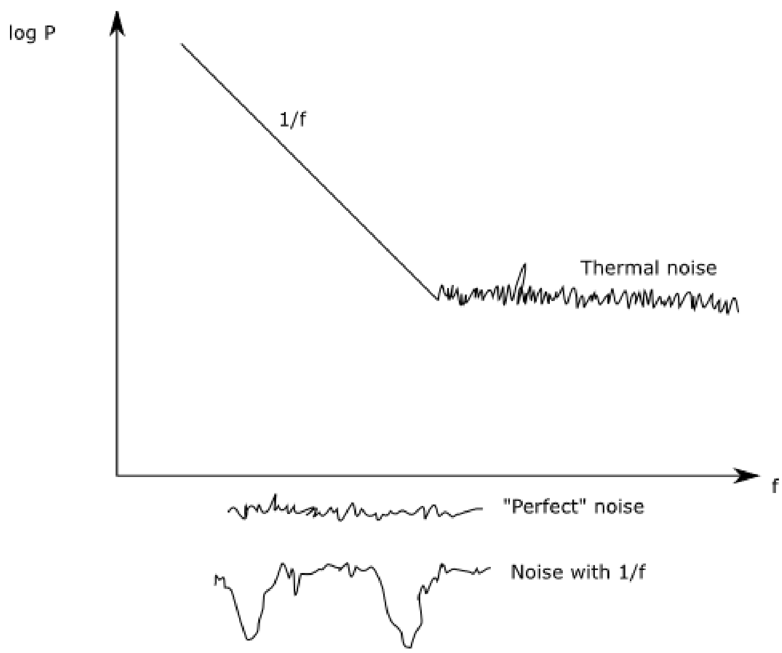
This is the random noise limit on detectability. But gain changes can prevent this limit being reached.

NB: $\frac{\Delta T}{T}$ is often very small. For $\tau = 1s$, $\Delta\nu = 10^{10}Hz \rightarrow \frac{\Delta T}{T} = 10^{-5}$.

If the gain fluctuates, one effectively gets ΔT fluctuations.

$\Delta T = \frac{\Delta G}{G} T_{sys}$ These $\frac{\Delta G}{G}$ fluctuations get larger with increasing integration time τ; they get bigger with $\frac{1}{\nu}$ where $\nu = \frac{1}{\tau}$. Hence $\frac{1}{f}$ noise.

$\frac{1}{f}$ noise seems universal.



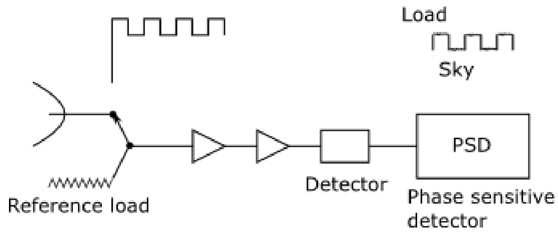
3.14 Case Study - CMB

Fluctuations are $\sim 10^{-5}k$. Want to measure them with a signal-to-noise ratio of ≈ 10. Need $\Delta T \approx 10^{-6}k$.

$T_{sys} = 20k \rightarrow \frac{\Delta T}{T} = \frac{10^{-6}}{20} = 5 \times 10^{-8}$.

3.15 Living with $\frac{1}{f}$ - Dicke Switch System

Try and continuously measure the gain of the receiver.



PSD gives $\propto (T_A - T_{load})$. Want to arrange the sky and the load temperatures so that they are as close as possible together.

Assume that the gain does not change.

State 1 – voltage $\propto kG(T_A + T_R)\Delta\nu$

State 2 – voltage $\propto kG(T_{load} + T_R)\Delta\nu$

PSD Output $kG(T_A - T_{load})\Delta\nu$

If there were no switching, ΔT fluctuations $\propto \frac{\Delta G}{G}(T_A + T_R) = \frac{\Delta G}{G}T_{sys}$. With switching, they become $\frac{\Delta G}{G}(T_A - T_{load}) \rightarrow$ an improvement $\frac{\Delta G}{G}T_{sys}\frac{G}{\Delta G(T_A - T_{load})} = \frac{T_{sys}}{T_A - T_{load}}$

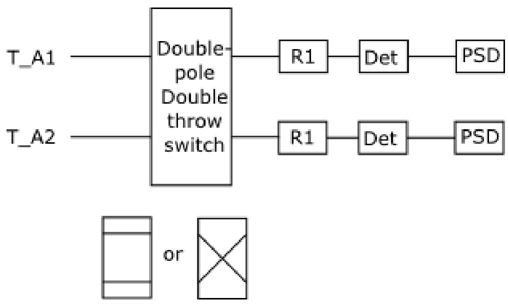
Need:

- Switch faster than gain fluctuations
- Need $T_A \approx T_{load}$

Signal-to-noise penalty. Without switching, $\frac{\Delta T}{T} = \frac{1}{\sqrt{B\tau}}$. With switching, τ becomes half the observing time. Also, each of T_A and T_{load} has a noise error, which inceases the noise on the difference by $\sqrt{2}$ on one of the quantities. Hence the random noise equation is

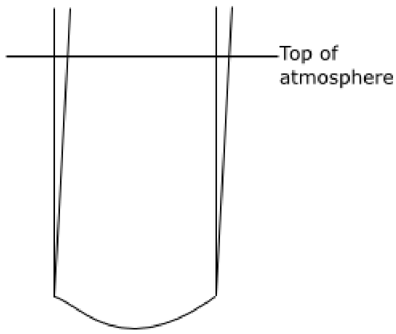
$$\Delta T_{min} = \frac{2T_{sys}}{\sqrt{B\tau}} \quad (74)$$

3.15.1 Double Dicke



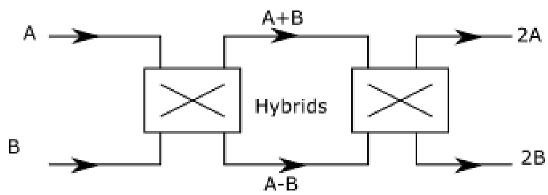
This recovers one of the lost $\sqrt{2}$'s. Two horn Double Dickie has advantages:

- $\frac{1}{f}$ cancellation
- Gains $\sqrt{2}$ over Single Dicke
- But also cancels most atmospheric emission fluctuations.



Cancellation occurs where the beams overlap. This is at heights less than the Rayleigh Distance $\frac{4d^2}{\lambda}$.

3.16 Correlation receiver



The outputs A and B have identical gain fluctuations. Hence the difference $2A - 2B$ is free of $\frac{1}{f}$. It also cancels the atmospheric fluctuations.

Why correlation receiver?

Mathematically, the action of the second hybrid and differencing is equivalent to multiplication. At the input of the hybrid, we have $\sqrt{G_1}(V_A + V_B + V_{N1})$ and $\sqrt{G_2}(V_A - V_B + V_{N2})$. Multiplying them together, we have $\sqrt{G_1 G_2}(V_A^2 - V_B^2 + V_{N2}(V_A + V_B) + V_{N2}(V_A - V_B) + V_{N1}V_{N2})$. The third, fourth and fifth terms average over time to zero, leaving the difference between the two powers.

Multiplication also gives the difference between two powers. (See Interferometers section, later.)

3.16.1 Planck LFI receivers

The aim of Planck is to measure the CMB fluctuations. It uses correlation receivers (R_x) configuration., which reduces $\frac{1}{f}$ to $f \sim 1mHz$.

4 Polarization

4.1 Astrophysics from Polarization

- Synchrotron emission which is polarized. The PA of polarization is related to the magnetic field direction.
- Faraday rotation → information about plasma between us and a distant source.
- CMB polarization fluctuations give unique cosmological information.

4.2 Notation

Specify in terms of the **E** vector of the EM wave. For a monochromatic wave,

$$E_x = E_x \cos(kz - \omega t + \phi_1) \tag{75}$$

$$E_y = E_y \cos(kz - \omega t + \phi_2) \tag{76}$$

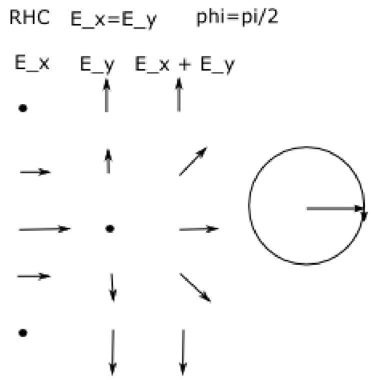
$$E_z = 0 \tag{77}$$

Only interested in phase difference $\phi = \phi_1 - \phi_2$. At $z = 0$, we can write $E_x = E_x \cos(-\omega t)$ and $E_y = E_y \cos(-\omega t - \phi)$. This means that the position vector will trace out an ellipse over time.

Special cases:

- $E_x = E_y$ and $\phi = 0$ → linear polarization.
- $E_x = E_y$ and $\phi = +-\frac{\pi}{2}$ → circular polarization

Right-hand circularly polarized (RHC) wave



4.2.1 Stokes Parameters

Define 4 parameters to specify the polarization state.

$$S_0 = I = E_x^2 + E_y^2 \tag{78}$$

$$S_1 = Q = E_x^2 - E_y^2 \tag{79}$$

$$S_2 = U = 2E_xE_y \cos\phi \tag{80}$$

$$S_3 = V = -2E_xE_y \sin\phi \tag{81}$$

I is total intensity. Q and U are related to linear polarization, and V is circular. Special cases:

- Pure RHC, $E_x = E_y$, $\phi = -\frac{\pi}{2} \to I = E_x^2 + E_y^2 = 2E_y^2$, $Q = U = 0$, $V = 2E_x^2 = I$
- Pure linear in x-direction, $E_y = 0 \to I = E_x^2$, $Q = E_x^2 = I$, $U = V = 0$
- etc...

For a monochromatic wave, the Stokes parameters are not independent. $I^2 = Q^2 + U^2 + V^2$.

Why Stokes? They have dimensions of power, so they are additive for components making up a broadband signal.

For a broadband signal, we measure:

$$I = \langle E_x^2(t) \rangle + \langle E_y^2(n) \rangle \tag{82}$$

$$I = \langle E_x^2(t) \rangle - \langle E_y^2(n) \rangle \tag{83}$$

$$U = \tag{84}$$

$$V = \tag{85}$$

Now, for broadband signals,

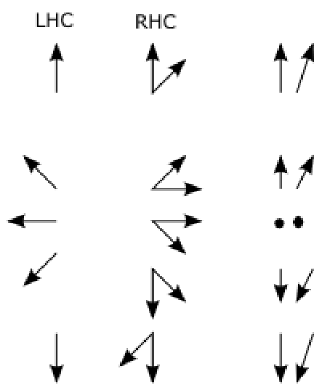
$$I^2 \geq Q^2 + U^2 + V^2 \tag{86}$$

It is possible for $\langle Q \rangle \langle U \rangle \langle V \rangle = 0$.

Define the degree (or percentage) of linear polarization $= \frac{\sqrt{Q^2+U^2}}{I}$. The position angle of the polarization $\chi = \frac{1}{2} tan^{-1} \left(\frac{U}{Q} \right)$. The degree of circular polarization $= \frac{|V|}{I}$.

4.3 Faraday Rotation

The refractive index for RHC and LHC are different in magnetized plasmas.



Rotation of the plane is $\frac{\Delta\theta}{2}$ where $\Delta\theta \propto \lambda^2 N B_{||} dz$, where N is the electron density, $B_{||}$ is the magnetic field parallel to the direction of propagation, and z is the direction of propagation.

$$\theta = k\lambda^2 \int_0^L B_{||}(z)N(z)dz \tag{87}$$

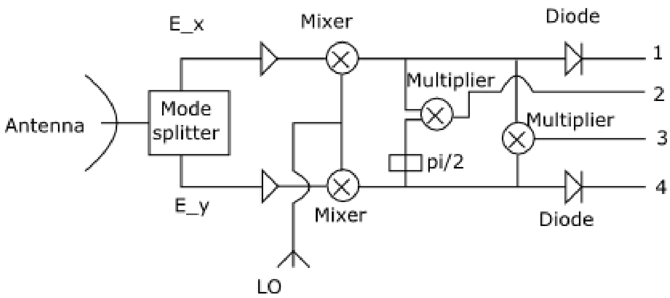
Rotation measure (RM) defined in radians m^{-2}.

$$RM = 8.1 \times 10^5 \int_0^L B_{II} N dz \tag{88}$$

where RM is radians per m^2, L and dz is in parsecs, $B_{||}$ in gauss and N in cm^{-3}

RM is the rotation in radians at $\lambda = 1m$.

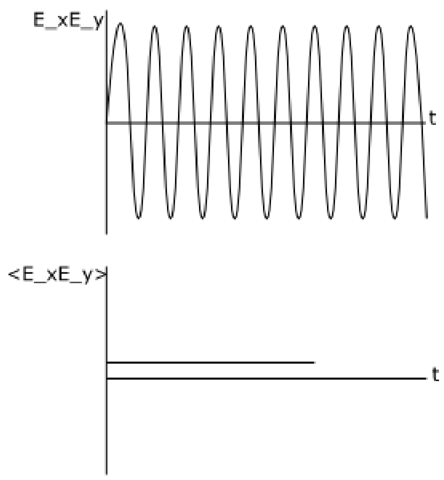
4.4 Polarization receivers



Measure E_x and E_y and multiply.

$E_x E_y = E_x E_y \cos(-\omega t - \phi_1)\cos(-\omega t + \phi_1 - \phi) = \frac{1}{2} E_x E_y \cos(-2\omega t + 2\phi_1 - \phi) + \frac{1}{2} E_x E_y \cos(-\phi)$

The first of these terms ($\frac{1}{2} E_x E_y \cos(-2\omega t + 2\phi_1 - \phi)$) will average to zero, leaving us with the second term, which is $\frac{1}{4}U$.



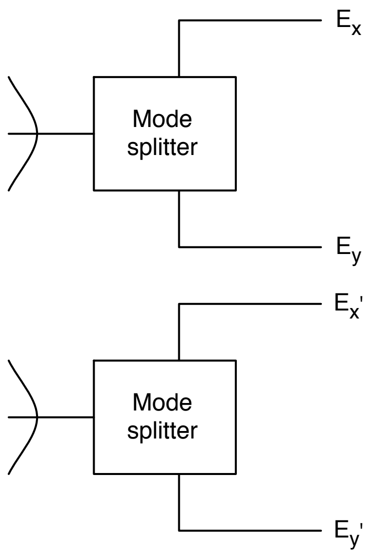
$$\langle \frac{1}{2} E_x E_y \cos(-\phi) \rangle = \frac{U}{4} \tag{89}$$

- Output 1 $\propto \langle E_x^2 \rangle$
- Output 2 $\propto \langle E_x E_y \cos\phi \rangle$
- Output 3 $\propto -\langle E_x E_y \sin\phi \rangle$
- Output 4 $\propto \langle E_y^2 \rangle$
- $I \propto 1 + 4$
- $Q \propto 1 - 4$
- $U \propto 2$
- $V \propto 3$

Need to calibrate the gains of each channel accurately.

4.5 Polarization with an Interferometer

Assume that two telescopes are looking at the same source, and that the wavefront they are measuring is coherent.



$$E_x E_{x'} = \langle E_x^2 \rangle \tag{90}$$

$$E_y E_{y'} = \langle E_y^2 \rangle \tag{91}$$

$$E_x E_{y'} = \langle E_x E_{y'} \cos(\phi) \rangle \tag{92}$$

etc.

5 Spectral Line Measurement

Radio spectrum is rich in molecular line emission, especially at high frequencies. Line widths range from 300kms^{-1} (AGN) to 0.03kms^{-1} from galactic masers. Hence want of the order of $\sim 10^5$ frequency channels.

5.1 Detectability of Spectral Lines

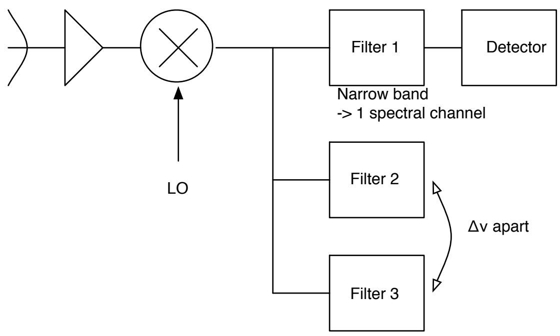
Consider an unresolved line ($\Delta\nu_{lim} << \Delta\nu_{resolution}$). The flux in line $\int S_\nu d\nu$ (in Wm^{-2}). The flux is fixed, but the flux density depends on the bandwidth (the number of Hz over which we measure).

$$S(\nu) \propto \frac{1}{\Delta\nu_{resolution}} \tag{93}$$

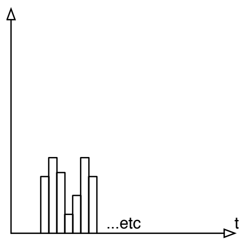
Recall that $\Delta S_{rms} \propto \frac{1}{\sqrt{\Delta\nu\tau}}$. Therefore, $\frac{\Delta S_{rms}}{S} \propto \frac{\Delta\nu_{resolution}}{\sqrt{\Delta\nu_{resolution}}} \propto \sqrt{\Delta\nu_{resolution}}$.

Therefore the signal-to-noise increases as we decrease the size of our frequency resolution elements. Maximize detectability by matching resolution to width of line.

5.2 Spectral Line receivers



Measure a single channel. The filter has a bandwidth $\Delta\nu$. Step the local oscillator in steps of $\Delta\nu$ to change the frequency that is being looked at.



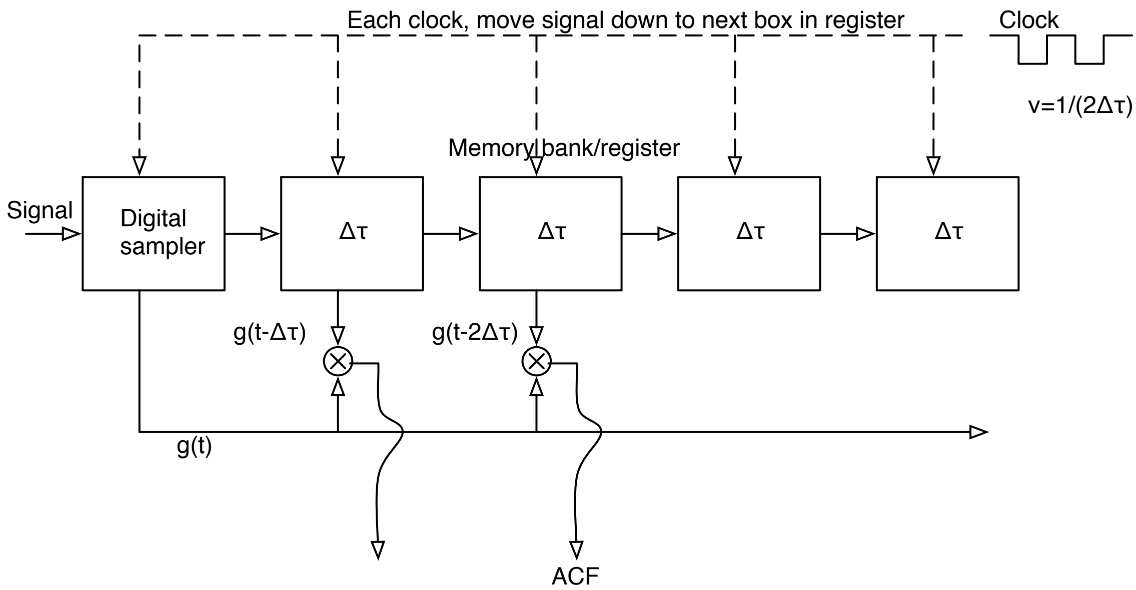
Add additional filters and detectors to detect multiple spectral lines simultaneously using a single radio telescope. Called a "filter bank". Gets very complex when you're wanting to observe many frequencies.

5.3 Digital Autocorrelation Spectrometer (ACS)

The Fourier Transform (FT) of the convolution of two functions g_1, g_2 is equal to the product of their individual FTs G_1 and G_2.
FT of (g_1 convolved with g_1) $= G_1 G_2$.

5.3.1 Autocorrelation

Convolve a function with itself.
g convolved $g = \int_{\inf}^{\inf} g(t)g*(t+\tau)d\tau = ACF(\tau)$
The FT of $g(t)$ convolved $g(t) = GG* = |G(\nu)|^2 = FT$ of $ACF(\tau)$.
$|G(\nu)|^2$ is the power spectrum.
The FT of the ACF is the power spectrum.
i.e. $\int_{-\inf}^{\inf} g(t)g*(t+\tau)d\tau = \int_{-\inf}^{\inf} |G(\nu)|^2 e^{i2\pi\nu t} d\nu$
Weimer-Kimchin Theorem.

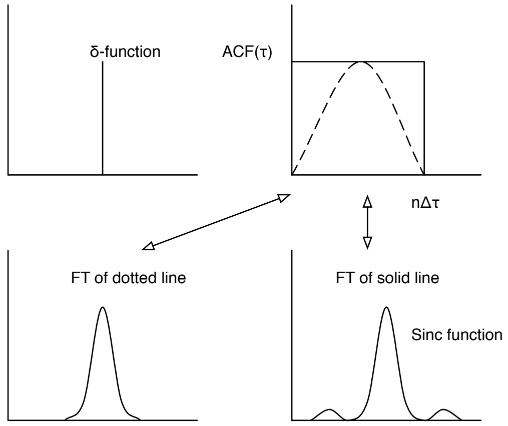


5.3.2 ACS using shift register

Shift register moves the sample down the chain each time it receives a timing pulse. The register stores a delayed version of the signal. Multiplier outputs give a sample of the ACF.

5.3.3 Window Function

Pure monochromatic wave.



How close the spike is to a delta function depends on the number of delays that we measure. But get sidelobes (sinc function). This becomes a problem if we have two lines close together, then the sinc functions overlap and things start to get confusing. By choosing the window f^n (weight the ACF [dashed line]) we suppress the sidelobes, and degrade the resolution. This is normally acceptable. Normally use a $\cos^2$ weighting function.

Advantages:

- Digital, hence stable and compact
- Good dynamic range (no saturation)
- Noise falls as (integration time)$^{-1/2}$
- Bandwidth can be changed
- Very high resolution is possible

Nowadays this tends to be done in software rather than hardware.

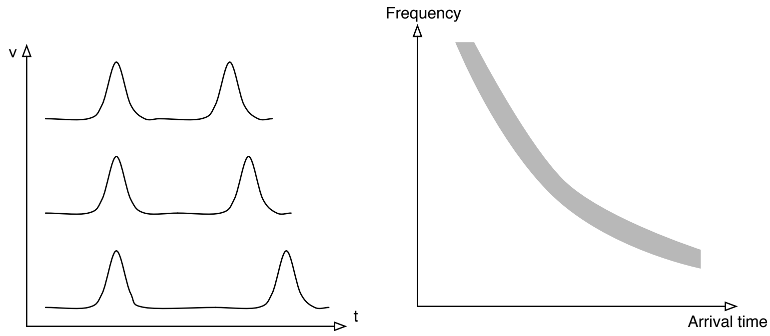
5.4 Shannon Sampling Theorem (Nyquist sampling)

If we have a bandwidth B_1 we need only sample at a frequency $\nu = 2B$ to obtain full information about the power spectrum.

e.g. 10MHz bandwidth needs a 20MHz sampling. To sample lines of 10kHz in our spectrum, how many lags do we require in the ACS? Need a resolution of 5kHz, hence need 2000 lags.

6 Pulsars and Pulsar Receivers

Need time resolution, and need to deal with dispersion. The ISM delays low frequencies with respect to high, giving rise to dispersion.



Plasmas have a plasma frequency of $\nu_p = \frac{e}{2\pi}\sqrt{\frac{n_e}{\epsilon_0 m_e}}$ At frequencies above ν_p, we get propagation with dispersion. For frequencies below, we get no propagation, i.e. no signal.

The group velocity is $v_g = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$.

To maximize pulse detectability, we need to match the sampling time to the intrinsic pulse width.

Relationship between the flux density S and the frequency ν is typically something like $S \propto \nu^{-2}$.

6.1 Example

What is the propagation delay for a pulsar at 1kpc at 100MHz? (Assume $n_e = 0.03cm^{-3}$.)

Since $\frac{\omega}{\omega_p} >> 1$, $v_g \approx c(1 - \frac{1}{2}\frac{\omega_p^2}{\omega^2})$.

Travel time $= \int \frac{1}{v_g} ds \approx \frac{d}{c} + \frac{1}{2c\omega^2}\int \omega_p^2 ds$, where the first part is light travel and the second part is the propagation delay. Therefore, the delay $= \frac{1}{2c\times4\pi^2(10^8)^2}\int_0^{1kpc} \frac{e^2}{\epsilon_0 m_e}\sqrt{3\times10^4} ds =$ $12.5s$.

What is the delay for 101MHz? 12.25s. Note that pulse periods range from about 2ms to several seconds. More importantly, the pulse widths and features within the pulse have a time scale very much less than a quarter of a second. So this is significant.

6.2 Dispersion Measure

Define Dispersion Measure (DM).

$$\mathrm{DM} = \int \frac{\mathrm{n_e}}{\mathrm{cm^{-3}}} \mathrm{d}\frac{\mathrm{l}}{\mathrm{pc}} \tag{94}$$

A pulsar with a given DM has a propagation delay given by

$$0.83 \times 10^4 \times \mathrm{DM} \times \frac{\Delta\nu}{\mathrm{MHz}} \times \frac{1}{\nu/\mathrm{MHz}}^{3} \mathrm{s} \tag{95}$$

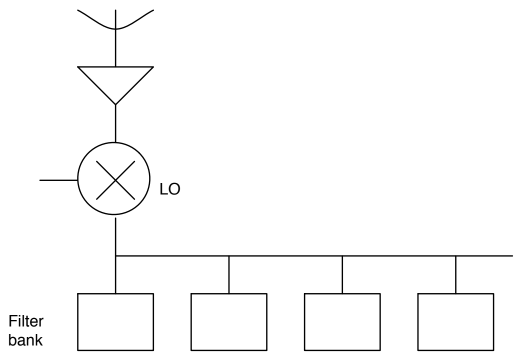
6.3 Faraday rotation

Pulsar emission is highly polarized. Hence can measure the Faraday rotation. Rotation Measure (RM) $= 0.81 \int \frac{n_e}{\mathrm{cm^{-3}}} B_{||} \frac{dl}{\mathrm{pc}}$

$$\frac{RM}{DM} = \frac{0.81 \int n_e B_{||} dl}{\int n_e dl} = 0.81 \langle B_{||} \rangle \quad (96)$$

The $\langle \ \rangle$ denote $B_{||}$ weighted by n_e.

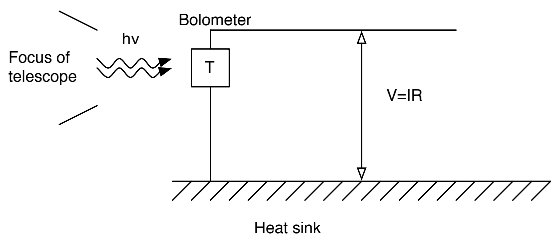
6.4 Pulsar receivers



Nowadays digitize the full band as it comes out of the mixer, and do the dispersion correction in software. Known as Coherent De-dispersion. Essentially a software receiver. Multiply the digital signal by the inverse of the dispersion transfer function.

7 Bolometers

These are mm and sub-mm devices, which are only suitable for continuum (i.e. broadband) applications. They are incoherent receivers, i.e. no amplitude or phase. They are temperature sensitive resistors.



A dc bias is applied. Photons are absorbed → the temperature rises ⇒ V changes and is amplified. The heatsink is present to cool the bolometer down as quickly as possible once it's received some energy.

Broadband – good for sensitivity, but not good for spectra. Pick up everything. Biggest disadvantage is the expense of the cryogenics, as they need to be cooled to mK. But can build large arrays - currently up to about 100 elements. They are the detector of choice above around 100GHz.

7.1 Noise Equivalent Power (Flux density)

The NEP is the power that must fall on the detector to raise its output by an amount equal to the RMS noise.

If the bolometer noise is dominated by sky background of emissivity ϵ, the incident power is $P_{BG} = \epsilon 2kT_{BG}\Delta\nu$, where the factor of 2 comes from the fact that the receiver is sensitive to both polarizations. Bolometers can be made to detect only one polarization, though.

The fluctuations give an $NEP = \epsilon 2kT_{BG}\sqrt{\Delta\nu}$. (Rohlfs and Wilson pp72, 73).

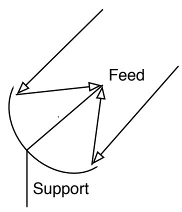
If we have a source producing T_s in the telescope beam,

$$\frac{S}{N} = \frac{2kT_s\Delta\nu\sqrt{\tau}}{NEP} \tag{97}$$

e.g. SCUBA. NEFD = $60mJyHz^{-1/2}$. For $\Delta\nu = 60GHz$, what is the integration time to detect $1mJy$ source with SNR of 5? $\frac{S}{N} = 5 = \frac{\sqrt{t}S}{NEFD} \Rightarrow t \approx 25$ hours.

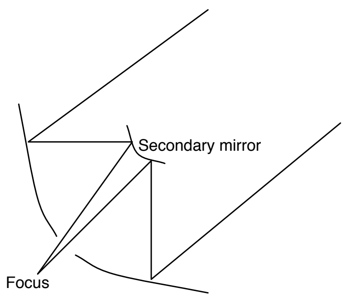
8 Types of Telescope

Paraboloid: dish + feed system. Nearly all are alt/az.



Prime focus - simple, just one reflecting surface. Difficult to feed, as the feed is not very far from the dish surface, so power needs to be collected from a wide spread of angles. Also means that the feed will see some power from over the edge of the dish - the ground, which can produce a large amount of noise. Lowish maximum efficiency, less than around 60%.

Cassegrain design:



Advantages:

- Focus is accessible
- Feed looks at the sky
- Less spillover – ground radiation
- Can have up to 80% efficiency

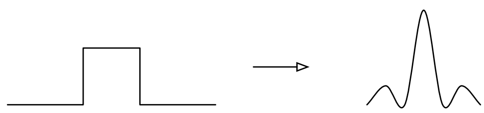
- Can have a wide field operation

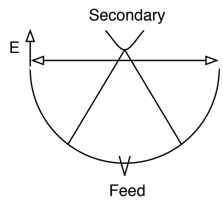
A disadvantage of the Cassegrain is that the feed needs to be quite large – problem at low frequencies. The feed needs to look at the whole of the secondary focus, so the diameter needs to be greater or equal to about 6λ (from λ/D).

8.1 Aperture Distributions

Optical	$\leftrightarrow$	Radio
Single slit	$\leftrightarrow$	single dish
Young's double slits	$\leftrightarrow$	interferometer
Diffraction gratings	$\leftrightarrow$	Array

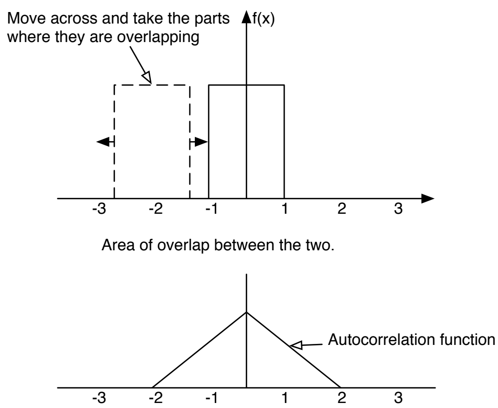
The diffraction pattern is the fourier transform of the aperture function.





For a radio telescope, the aperture distribution is the [electric] field (both amplitude and phase) across the front of the dish.

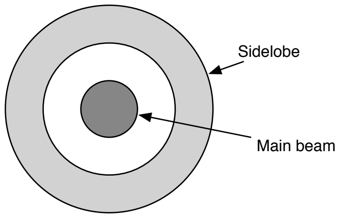
Apply Wiener-Khimelim theorem. If we take the autocorrelation of the aperture distribution g correlated with g, the W-K theorem tells us that the fourier transform of this is the power spectrum of g. **The power gain is the FT of the autocorrelation of the aperture distribution.**



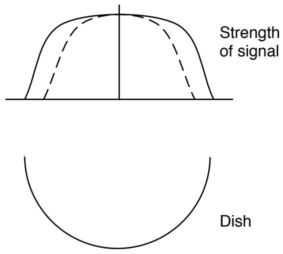
8.1.1 Single dishes

Resolution is $\approx 1.22\frac{\lambda}{D}$.

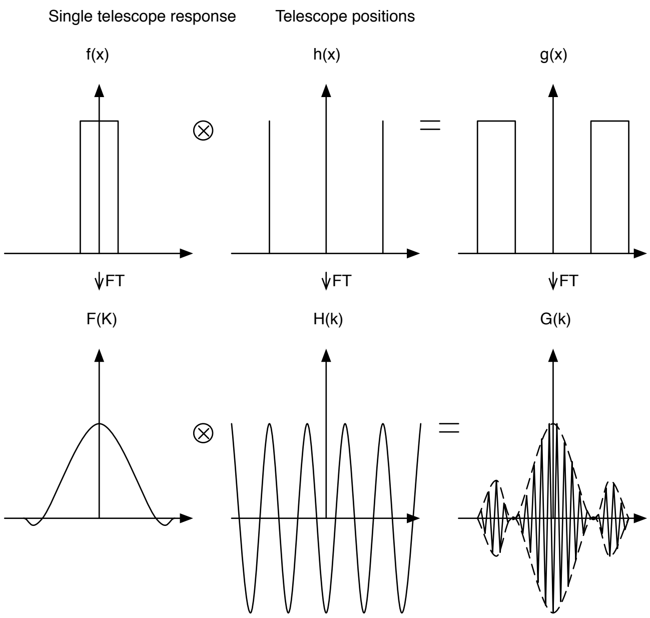
First side lobe is $\sim$ 20dB down from the peak. However, it covers more area.



If one "grades" the aperture distribution, one can reduce sidelobes.

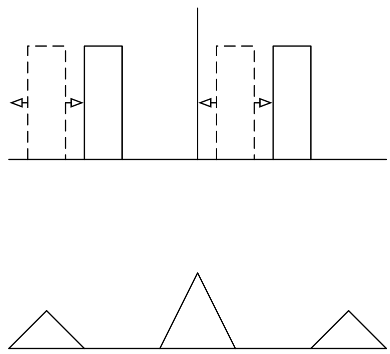


8.1.2 Interferometers



F(K) is the FT of f(x) etc.

The fringe pattern is limited by the size of the individual dishes, while fringe separation is determined by the separation of the elements.



$g(x) = \int_{\infty}^{\infty} f(x)h(x - x')dx'$. Interested in the outside two; the inside one is not of such interest.

8.2 Antenna Smoothing

The antenna temperature T_A is the convolution of the sky brightness $T_b(\theta, \phi)$ with the antenna pattern.

$$T_A(\theta, \phi) = \frac{A_e}{\lambda^2} T_b(\theta, \phi) * P_n(\theta, \phi) \tag{98}$$

where * denotes convolution. Using the convolution theorem, the F.T¿ of $T_A(\theta, \phi)$:

$$t(u, v) = \frac{A_e}{\lambda^2} t_b(u, v) \times c(u, v) \tag{99}$$

where $t_b(u, v)$ is the fourier transform of T_b, and $c(u, v)$ is the FT of P_n. $c(u, v)$ is the transfer function.

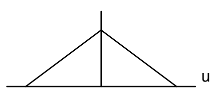
$$c(u, v) = g(u, v) \otimes g(u, v) \tag{100}$$

is the autocorrelation of the aperture distribution.

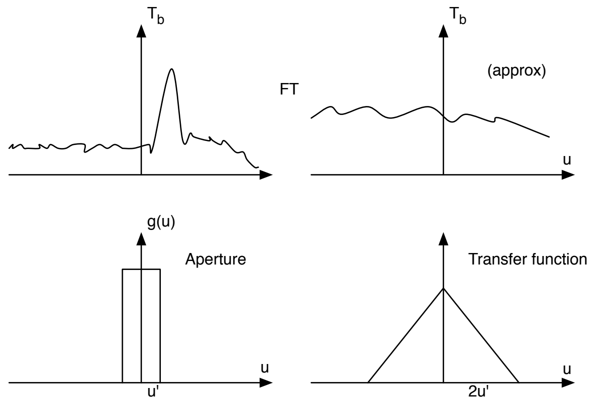
The coordinates u, v are coordinates in the plane of the aperture measured in the wavelength λ; $u = \frac{x}{\lambda}$, $v = \frac{y}{\lambda}$.

$c(u, v)$ vanishes outside the aperture distance. Hence the telescope acts like a low pass filter - it accepts only low frequencies or long wavelength Fourier components.

$c(u) = g(u) \otimes g(u)$. Single dish: This is the filter function. It goes to zero for greater



or less than certain values of u and v, hence it has an absolute cutoff in terms of the fourier components that it accepts.



Implies that values of Fourier components in the sky $\geq |2u'|$ are rejected.

8.3 Deconvolution

You know the transfer function - it has weighted down high spatial frequencies. Want to reconstruct a map with a nicer transfer function.

Example: A gaussian source observed with a Gaussian beam. The convolution of two gaussians is another. θ_{mes} is measured. Θ_s is the source size, and Θ_b is the beam size. $theta_{mes} = \sqrt{\Theta_b^2 + \Theta_s^2}$.

If $\Theta_b = 10'$, $\theta_{mes} = 12' \Rightarrow \Theta_s = 6.6'$.

NB: to be quantitative, have used a Gaussian prior - i.e. assumed that the source is gaussian.

8.4 Interferometers

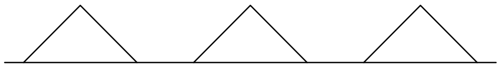
Advantages:

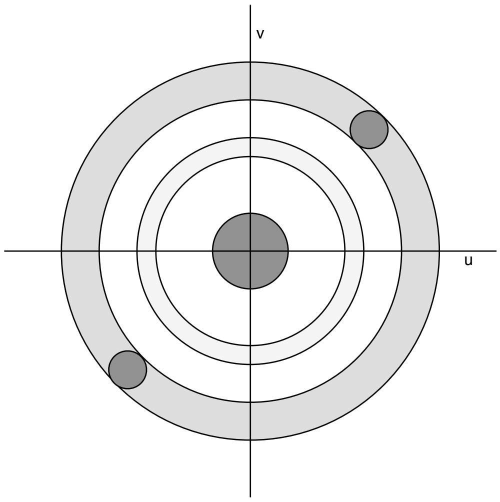
- Angular resolution $\sim \frac{\lambda}{D}$ where D is the maximum baseline (¡1mas)
- More stable to $\frac{1}{f}$ noise - they only measure the correlated noise from both elements.
- Less susceptible to radio frequency interference (FRI), as it is not usually correlated.
- Less affected by atmospheric emission
- Less confused
- Good for astrometry – can measure positions to the resolution $\frac{\lambda}{D} \times 10^{-2}$.

Disadvantages:

- The transfer function is complex $\Rightarrow$ must deconvolve the resulting map. (CLEAN algorithm – well known)
- Poor surface brightness sensitivity

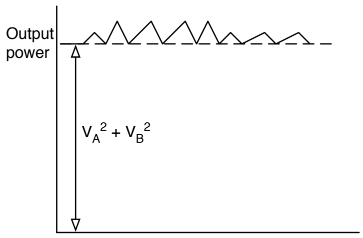
For an interferometer, the transfer function looks something like:





8.4.1 Adding interferometer

Take signals, delay, add and detect. Setting V_A as the signal from telescope A, etc, the output $\propto V_A e^{i(\omega t+\phi/2)} + V_B e^{i(\omega t-\phi/2)} \propto V_A^2 + V_B^2 + 2V_A V_B \cos\phi$. The first and second terms are the sum of the total power; the third term is the fringes, and is the interesting part.



Any gain fluctuations in either A or B will modulate the DC component up or down, increasing the difficulty of measuring the fringe pattern.

8.4.2 Correlating (Multiplying) Interferometer

Take output from telescopes pointing in different directions. Sample both signals, $A(T_n).B(t_n)$. If $A(t_n)$ is positive, $B(t_n)$ is just as likely to be positive as negative.

Therefore multiplying $A(t_n)$ with $B(t_n)$ is just as likely to give a positive as a negative result. Hence the product will average to zero.

Now point telescopes at the same source. Most output still uncorrelated noise (Ω_x+ atmosphere). But now there will be a slight bias; whenever $A(t_n)$ is positive there is a greater chance that $B(t_n)$ will be positive too.

Multiplying $A(t_n)$ with $B(t_n)$ will result in a non-zero average because they are more likely to be of the same sign. (1-bit correlation works.)

Qualification: if the correlated parts of A and B are $\pi/2$ out of phase, this will not work. The solution is to multiply twice, once with no phase shit and once with a $\pi/2$ phase shift. (often called cosine and sine outputs.)

Also, averaging to detect the correlated signal works only if ϕ s constant.

Output from telescopes V_{RA}, V_{RB} are incoherent. V_A, V_B are coherent. Multiply:

$$\text{Output} = G_A G_B^* (V_A e^{i(\omega t+\phi/2)} + V_{RA})(V_B e^{-i(\omega t-\phi/2)} + V_{RB}) \tag{101}$$

$$= G_A G_B^* (V_A e^{i(\omega t+\phi/2)} V_B^* e^{-i(\omega t-\phi/2)} + V_{RA} V_B^* e^{\cdots} + V_{RB} V_A e^{\cdots} + V_{RA} V_{RB}) \tag{102}$$

The last 3 terms will average to zero. The integrated output $= G_A G_B^* V_A V_B e^{i\phi}$.

Real part $\propto \cos\phi$, imaginary $\propto \sin\phi$. As ϕ is changing, resultant output is sinusoidal. The amplitude of this (the fringe amplitude) is the strength of the correlated part of the signal.

Real correlators deal with real signals, hence complex multiplication must be done in two steps.

1. Multiply. $\propto \cos\phi$
2. Phase shift by $\pi/2$ and multiply. $\propto \sin\phi$.

Adding gives real + imaginary. Hence we measure $V_A V_B^* e^{i\phi} = V_A V_B^* e^{i\omega\tau}$ where $\tau = \frac{\mathbf{D}.\hat{\mathbf{n}}}{c}$.

8.4.3 Extended sources

The output will be the sum of the correlated signals over the extent of the source.

$$\text{Output} \propto \int\int T_b(x,y) P(x,y) e^{i\omega \frac{\mathbf{D}.\hat{\mathbf{n}}}{c}} dxdy \tag{103}$$

where T_b is the sky brightness, P is the antenna response $e^{i\omega\frac{\mathbf{D}.\hat{\mathbf{n}}}{c}} \equiv e^{i\omega(ux+vy)}$.

Therefore the fringe strength (visibility)

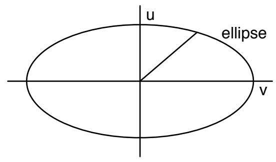
$$V(u,v) \propto \int\int T_b(x,y) P(x,y) e^{i2\pi(ux+vy)} dxdy \tag{104}$$

The complex fringe visibility is the 2D Fourier Transform of the sky brightness distribution. Can often think of $P(x,y)$ as a constant.

To get T_b, we Fourier Transform the visibility V.

The key is to sample $V(u,v)$ as well as possible $\rightarrow$ Aperture Synthesis. Either:

- Lots of interferometers
- Observe the sky for hours while the Earth rotates in respect to the sky.



8.5 Bandwidth and Delay

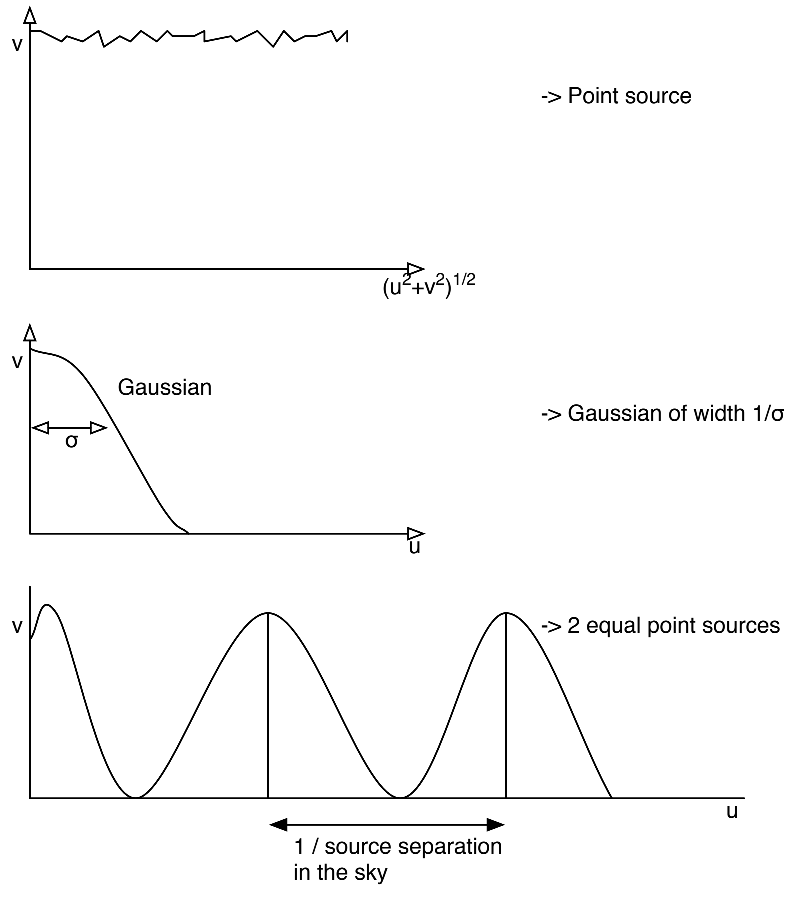
Signals of non-zero bandwidth B have a coherence length $\approx \frac{1}{Bc}$ (c is the speed of light), or a coherence time of $\sim \frac{1}{B}$. If the delay is out by greater than $\sim \frac{1}{B}$, then coherence is lost.

The delay changes with θ (direction), so coherence may be lost for extended sources. The Effective "delay beamwidth"

$$\theta_b \approx \theta_{synth} \frac{\nu}{B} \tag{105}$$

Limit the field of view. Can solve the delay smearing problem by correlating many small sub-bands.

8.6 Visibilities and Source Structure



8.7 Deconvolution

This is necessary because the transfer function (or synthesised beam) of an inteferometer array is complex

Terminology:

- Dirty map: map produced by FT of gridded visibility data
- Dirty beam: Is the dirty map of a point source. FT of UV coverage.
- CLEAN: deconvolution algorithm.

CLEAN exploits the fact that most of the radio sky is empty. Assumes that the brightness distribution can be represented by the sum of point sources. Implementation:

- Make dirty map
- Look for brightest point
- Subtract a scaled dirty beam from that point, and record where from and how much.
- Look for the next brightest source
- Subtract, record
- etc.
- Stop once the residual map is something that looks like noise.
- Take the array of "clean components" and you convolve these with a "Clean beam" - usually a gaussian of width equal to the main lobe of the dirty beam
- Add in the residuals

Advantages:

- Simple to use
- Seems to work
- People trust it

Disadvantages:

- No rigorous theory to underpin the method - empirical.
- Errors are not easy to quantify
- Doesn't work well on extended sources