Dr. M. Kramer

1. Introduction into the high-energy sky

Books:
No specific ones needed - course will be independent of a specific book.
(Longair, M.S., High Energy Astrophysics Vol. 1 & 2, 2nd Edition, CUP)

2. Essentials - a reminder

1.1 The Electromagnetic Spectrum

High energy processes manifest themselves across the whole electromagnetic spectrum.
Often in High-Energy Astrophysics, energies, frequencies and temperatures are expressed in Electron Volts, eV.
$1eV =$ the energy gained by an electron in a potential of 1 Volt $= 1.6x10^{-19}Cx1V = 1.6x10^{-19}J$
Energy of a photon with frequency ;

$$E = h\nu$$

$$E(eV) = 4.136x10^{-15}\left[\frac{\nu}{Hz}\right]$$

where $h = 6.63x10^{-34}Js$ is Planck's Constant.
For each energy, we can find a typical temperature;

$$E \sim kT\ ,\quad E(eV) = 8.62x10^{-5}\frac{T}{k}$$

	Frequency	Energy	$T(k)$
γ ray	$\geq 3x10^{19}Hz$	$\geq 100keV$	$\geq 10^{9}$
X-ray	$\geq 3x10^{16} \rightarrow 3x10^{19}Hz$	$0.1 \rightarrow 100keV$	$10^{8} \rightarrow 10^{9}$
UV	$10^{15} \rightarrow 3x10^{16}Hz$	$10 \rightarrow 100eV$	$10^{5} \rightarrow 10^{6}$
Optical	$\sim 10^{15}Hz$	$\sim 3eV$	$\sim 3x10^{4}$
IR	$3THz \rightarrow 300THz$	$0.01 \rightarrow 0.1eV$	$10^{2} \rightarrow 10^{4}$
Radio	$\leq 3THz$	$\leq 100meV$	$0.1 \rightarrow 100$

2.2 The Blackbody Spectrum

Spectrum = energy output of source as a function of frequency.
"Specific intensity" or "brightness";

$$I_\nu = \frac{Energy}{Time\ Area\ Bandwidth\ Solid\ Angle}$$

Units of $W\,m^{-2}\,Hz^{-1}\,Steradian^{-1}$.
Averaging these angles;

$$J_\nu = \frac{1}{4\pi}\oint I_\nu\, d\Omega$$

"Mean intensity", related to the "spectral energy density";

$$u_\nu = \frac{Energy}{Volume\ Bandwidth} = \frac{4\pi}{c}J_\nu$$

"(Net) flux", with;

$$S_\nu = \oint I_\nu\, d\Omega = \frac{Energy}{Time\ Area\ Bandwidth}$$

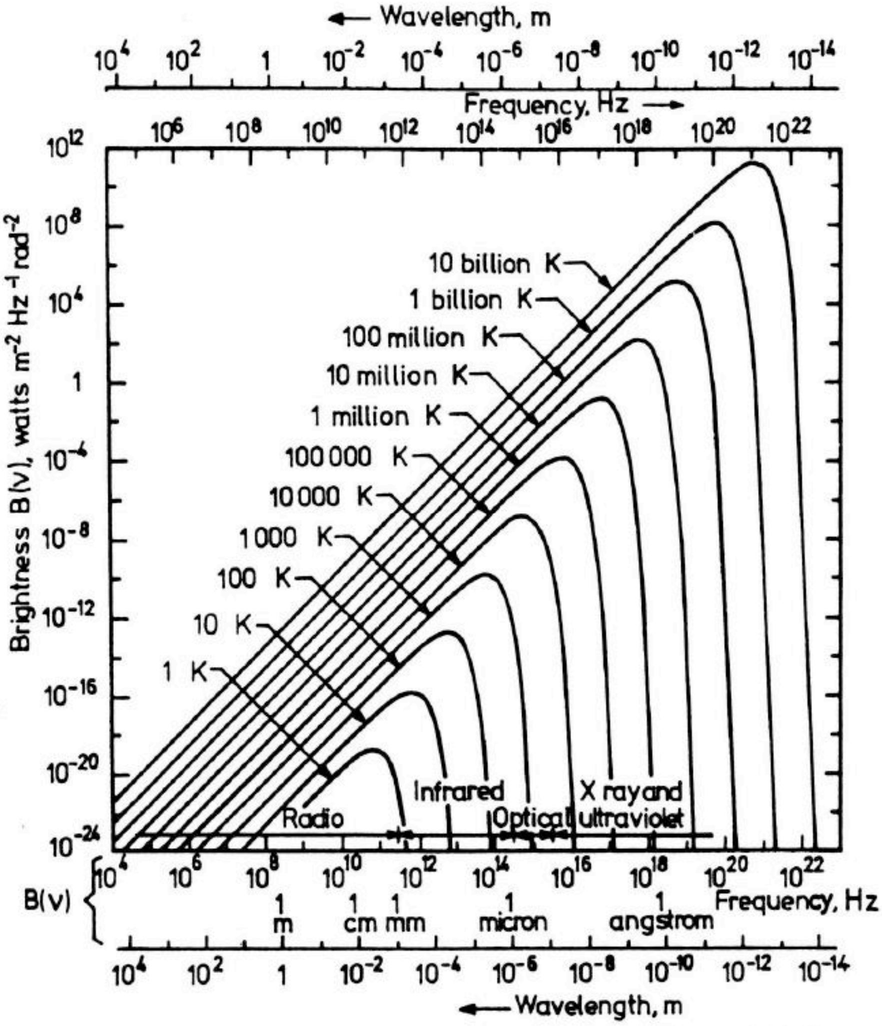
Units of $W\,m^{-2}\,Hz^{-1}$

Flux increases with $1/r^2$.

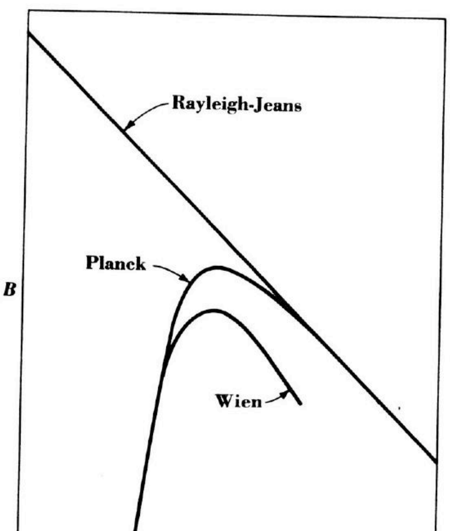
When matter and radiation are in equilibrium → black-body radiation
Given in specific intensity;

$$I_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

We will use B_ν to indicate that the radiation is from a black body.



The peak of the Planck curve indicates the frequency of the largest power output. The spectrum depends only on the temperature.



Wein's (displacement) Law:
$Tm_{max} = 0.0029 K\ m$

e.g. Sun:
$\lambda_{max} \approx 500nm$
so $T = 5800k$
Use for temperature measurements (relies on the colour of the star)
Geskerday flux:
$S_\nu = \frac{Energy}{Time\ Area\ Bandwidth}$
Integrate over all the frequencies:
$S = \int S_v dv$
For blackbody: $S = \sigma T^4$
Stefan-Boltzmann law.
$\sigma = 5.67x10^{-8} W\ m^{-2} K^{-4}$
Later we will use the Stefan-Boltzmann in form of energy density;
$u = \int u_\nu d\nu = \frac{4}{c} \sigma T^4$

At high frequencies, we can approximate the Planck curve by the "Wien Limit".
$\frac{hv}{kT} >> 1,\ B_v(T) = \frac{2kv^3}{c^2} e^{-\frac{hv}{kT}}$
→ exponential "cut-off"
At low frequencies, we approximate $B_\nu(T)$ by the "Rayleigh-Jeans Limit".
$\frac{h\nu}{kT} << 1$
$B_\nu(T) = \frac{2kT}{c^2} \nu^2$
i.e. a power law with slope +2.
Assume we observe a brightness at radio frequencies, you can then assume the source is actually a black body:
$I_\nu = B_\nu(T) = \frac{2kT}{c^2} \nu^2$
(Low frequencies)
Define the brightness temperature T_B such that $I_v = B_v(T_B)$ where we apply the Rayleigh-Jeans limit
$T_B = \frac{c^2}{2kv^2} I_v$
This is an assumption; the object might not be like a black body, hence the temperature may be wrong.
T_B is a reliable estimate for temperature if the source is "optically thick" (see below).

<u>2.3 Radiative Transfer</u>
Matter/plasma can modify the specific intensity of the radiation either by emission (increasing) or by absorption (decreasing intensity). $I_\nu \rightarrow I_\nu + dI_\nu$
Radiative Transfer Equation:
$dI_v = -\kappa_v I_v dx + \varepsilon_v dx$

where κ_v is the absorption coefficient. Unit $(length)^{-1}$. Total amount of absorption depends on the length traveled through the material (dx), and is proportional to the incoming intensity.

ε_v is the emissivity. Unit $Intensity/Length$. Describes the amount of added intensity.
We can rewrite the Radiative Transfer Equation as;
$\frac{dI_\nu}{\kappa_\nu dx} = -I_\nu + \frac{\varepsilon_\nu}{\kappa_\nu}$
We define $\varsigma_v = \frac{\varepsilon_v}{\kappa_v}$ as the "source function".
The optical depth is defined as $\tau_v = \int \kappa_v dx$ or $d\tau_v = \kappa_v dx$. This determines whether the material is optically thin or thick.

Now;

$$\frac{dI_\nu}{d\tau_\nu} = -I_\nu + \varsigma_\nu$$

We can solve this through integration; the general solution is:

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} S_\nu(\tau_\nu')e^{-\tau_\nu'}\,d\tau_\nu'$$

For a constant source function:

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

A plasma is called optically thin if $\tau_\nu << 1$ (transparent), or optically thick if $\tau_\nu >> 1$ (opaque)

Expression becomes obvious: let's assume that the plasma is not emitting, $\varsigma_\nu = 0$.

In this case, $I_\nu = I_\nu e^{-\tau_\nu}$

For $\tau_\nu << 1$, $I_\nu = I_\nu$ for optically thin.

For $\tau_\nu >> 1$, $I_\nu = 0$ for optically thick.

Be more precise:

for optically thin plasma; $\tau_\nu << 1$: $e^{-\tau_\nu} \approx 1$, $I_\nu(\tau_\nu) = I_\nu(0) + \tau_\nu S_\nu$.

For optically thick plasma, $I_\nu(\tau_\nu) = S_\nu$.

Two important cases:

1) In total Thermodynamical Equilibrium (TE), matter is in thermal equilibrium, and in equilibrium with the radiation. This is the case of a black body, and is rare in real life.

2) If the matter is in equilibrium but not with the radiation, we call this local thermal equilibrium (LTE).

In both cases, we have Kirchoff's law:

$$S_\nu = \frac{j_\nu}{\alpha_\nu} = B_\nu(T)$$

In these cases for optically thin plasma:

$$I_\nu = \tau_\nu B_\nu(T) = \tau_\nu \frac{j_\nu}{\alpha_\nu} = \alpha_\nu x \frac{j_\nu}{\alpha_\nu} = j_\nu x$$

$$I_\nu = j_\nu x$$

This is the total specific intensity of the plasma.

For optically thick plasma: in LTE or TE:

$$I_\nu = S_\nu = B_\nu(T)$$

So indeed only for optically thick plasma, brightness temperature is that of a black body.

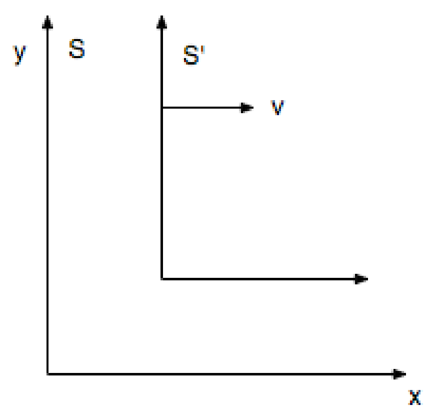
2.4 Relativity

We are dealing with high-energy particles with speeds $v \sim c$

Basic tools are summarized.

2.4.1 Lorentz-Transformation

Consider events occurring and measurements made in the rest frame S and a frame S' moving with relative speed v in the X-direction. The frames S and S' may be coincident at $t = 0$.



We define:

$$\beta = \frac{v}{c}$$

and the Lorentz factor (the "Gamma-factor")

$$c = \frac{1}{\sqrt{1 - b\frac{v}{c}l^{2}}} = \frac{1}{\sqrt{1 - b^{2}}}$$

Space-Time coordinate transforms are:

$$x = \gamma\left(x' + vt'\right) \qquad x' = \gamma\left(x - vt\right)$$

$$y = y' \qquad y' = y$$

$$z = z' \qquad z' = z$$

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right) \qquad t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Velocity transforms:

$$u_x = \frac{u_x' + v}{1 + \frac{u_x' v}{c^2}} \qquad u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}}$$

$$u_y = \frac{u_y'}{\gamma\left(1 + \frac{u_x' v}{c^2}\right)} \qquad u_y' = \frac{u_y}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

$$u_z = \frac{u_z'}{\gamma\left(1 + \frac{u_x' v}{c^2}\right)} \qquad u_z' = \frac{u_z}{\gamma\left(1 - \frac{u_x v}{c^2}\right)}$$

2.4.2 Energy and Momentum

Momentum transformation:

$$p_x = \gamma\left(p_x' + \frac{vE'}{c^2}\right) \qquad P_x' = \gamma\left(p_x - \frac{vE}{c^2}\right)$$

$$P_y = P_y' \qquad P_y' = P_y$$

$$P_z = P_z' \qquad P_z' = P_z$$

Energy transformation;

$$E = \gamma\left(E' + vp_x'\right) \qquad E' = \gamma\left(E - vp_x\right)$$

We define m_0 as the "rest mass" and find;

$m = c m_o$ "Relativistic total mass"

$p = mv = c m_o v$

$E_{total} = mc^2$

So;

$$\begin{aligned} E_{kin} &= E_{total} - E_o \\ &= mc^2 - m_o c^2 \\ &= \gamma m_o c^2 - m_o c^2 \\ &= \left(\gamma - 1\right) m_o c^2 \end{aligned}$$

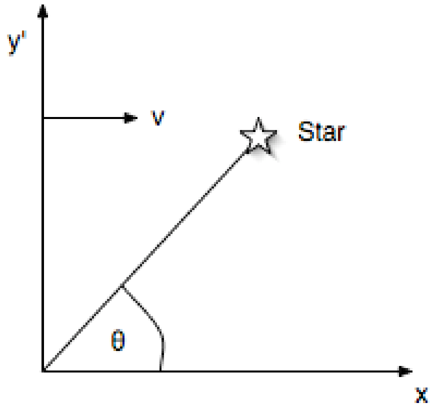
Energy and momentum are related;

$E^2 = {}^\wedge pc h^2 + {}^\wedge m_o c^2 h^2$

In each frame, rest mass must be the same.

${}^\wedge m_o c^2 h^2 = E^2 - {}^\wedge pc h^2 = E'^2 - {}^\wedge p'c h^2$

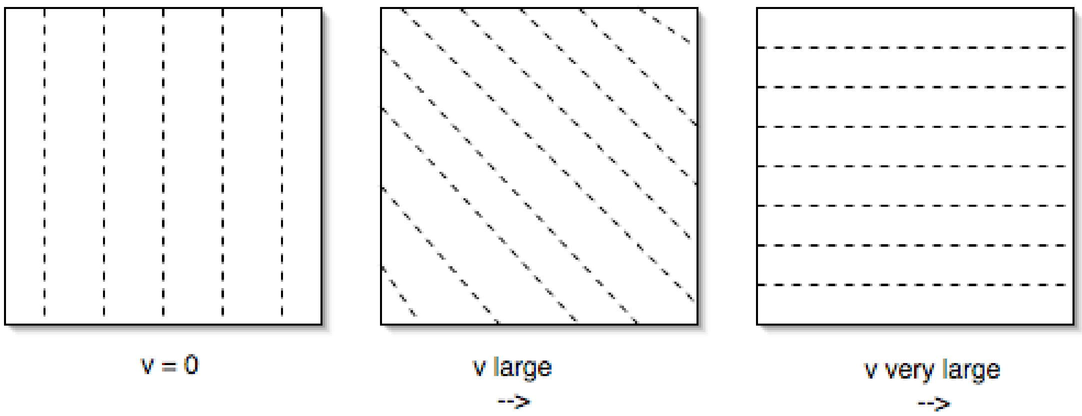
2.4.3 Propagation of Light
2.4.3.1 Doppler Effect
The frequency of photons is shifted depending on the relative motion of source and observer. If θ is the arrival angle is S,



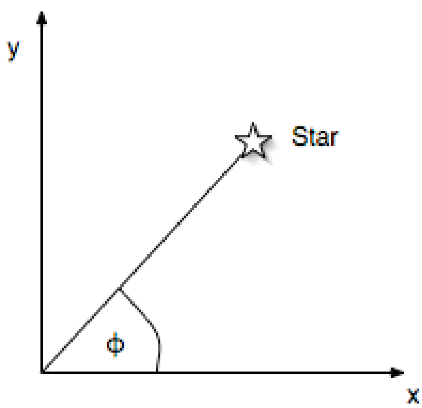
Effective velocity $v\cos\theta$.
We get;

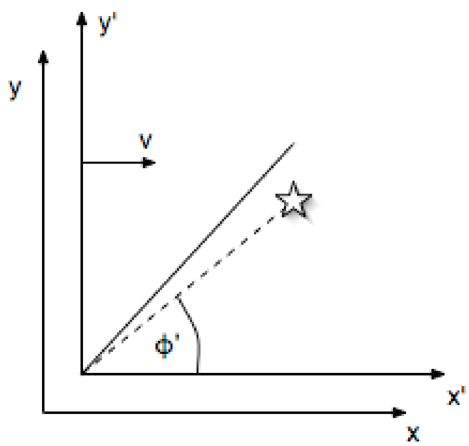
$$\frac{\nu'}{\nu} = \gamma\left(1 + \frac{v}{c}\cos\theta\right)$$

2.4.3.2 Aberration
Look at the rain outside a window in a train.



If we are in a rest frame S and see a star at angle z, this angle z will be different for an observer moving relative to the star, $z' \# z$.





Use Lorentz transformation to compute ϕ'.

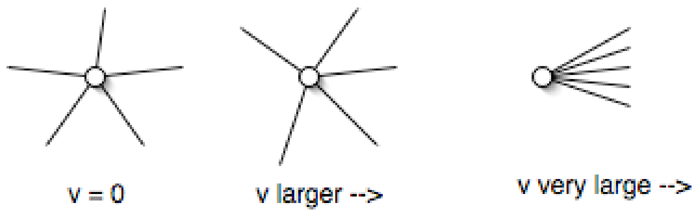
In both systems, S and S' photons travel with speed $u = c$. In S, $u_x = -c\cos\phi$, $u_y = -c\sin\phi$.

In S', $u_x' = -c\cos\phi'$, $u_y' = -c\sin\phi'$.

$$\tan z' = \frac{u_y'}{u_x'} = \frac{\sin z}{c\ b\cos z + \frac{v}{c} l}$$

2.4.3.3 Relativistic Beaming

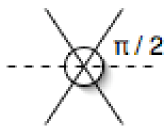
We have a source emitting isotropically radiation as seen in its' rest frame.



For large velocities, $v \to c$. $\gamma >> 1$. $\beta = \frac{v}{c} \approx 1$.

$$z' = \tan z' \sim \frac{\sin z}{c\ b\cos z + \frac{v}{c} l} \ \text{"}\ \frac{1}{c}.$$

If a particle is emitting in all directions, it can be described by a "cone" with $\phi = \frac{\pi}{2}$.



$$\phi' = \frac{\sin \pi/2}{\gamma(0+1)} = \frac{1}{\gamma}$$

→ outside observer sees radiation concentrated in a narrow cone of radius γ^{-1}.

→ brightness increases for observer. "relativistically boosted", "Doppler boosted"

$I_{abs} \sim I_o\gamma^4$

In contrast, total power is the same in all reference frames.

3. Radiation Processes

Everything we know about a source is derived by looking at the photons.

→ have to look at radiation processes such as;

- (Ionisation losses)
- Free-free Emission (Bremsstrahlung)
- Compton Scattering
- Synchrotron Emission
- Absorption process (self-absorption, pair production)
- Cherenkov Radiation
- Nuclear Interactions

One distinguishes between thermal and non-thermal processes.

- We call a process thermal or non-thermal depending on whether the temperature is the governing parameter determining the energy distribution, where $N(E)$ is the number of particles in a given energy interval.

→ if thermal, only T matters.

Examples:
Thermal:
- Black-body radiation
- Emission lines of optically thin plasma
- Thermal Bremsstrahlung

Non-thermal:
- Non-thermal Bremsstrahlung
- Inverse Compton scattering.
- Synchrotron emission.

3.1 Ionisation Losses

- Interaction and excitation of atoms and molecules by an electrostatic interaction with a high energy particle.
- Electrons are torn off by the electrostatic forces between the high-energy charged particle and the electron.

This is important for three reasons:
1) It is the main process used to detect high-energy particles in detectors.
2) Process influences the propagation of cosmic particles.
3) Provides a heating mechanism for the interstellar medium: e.g. giant molecular clouds that are heated by a transfer of the particle's kinetic energy to the electron.

Both momentum and energy is transferred to the electron.
To be shown in examples: only a small fraction of kinetic energy is transferred unless it is an electron-electron interaction.
In general, there is hardly any deviation from the high-energy particle's path.
The maximum energy that can be transferred to an electron is $E_{\max} = 2m_e v^2$.
For derivation of the energy loss of the particle,
$$\frac{dE}{dx},$$
we assume the high-energy particle moves sufficiently fast that the electron is essentially stationary during the interaction.
For the non-relativistic case, $v << c$, then energy loss per unit length;
$$-\frac{dE}{dx} = \frac{z^2 e^4 N_e}{4\pi\varepsilon_o{}^2 m_e v^2} \ln\left(\frac{2m_e v^2}{I}\right)$$
where ze is the charge of the high-energy particle, v is its' velocity, N_e the electron density of the medium traveled, m_e the electron mass, and I the Ionisation level of the atom averaged over all electron states.
The last logarithmic term contains the maximum transferable energy $E_{\max} = 2m_e v^2$, but it is only slowly varying with energy.
Energy loss is independent of the mass of the high-energy particle.
In the relativistic case, $v \sim c$, the maximum energy that can be transferred is;
$$E_{\max} = 2\gamma^2 m_e v^2$$
and the exact formula for the energy loss is known as the "Bethe-Bloch formula".
$$-\frac{dE}{dx} = \frac{z^2 e^4 N_e}{4\pi\varepsilon_o{}^2 m_e v^2} \left[\ln\left(\frac{2\gamma^2 m_e v^2}{I}\right) - \frac{v^2}{c^2}\right]$$
This is similar to the non-relativistic case, apart from the correction and different $E_{\max}$.
This is valid for proton-atom interaction. Again, it depends only on the velocity of the high energy particle, and not the mass.
$-\frac{dE}{dx}$ is called the "stopping power".

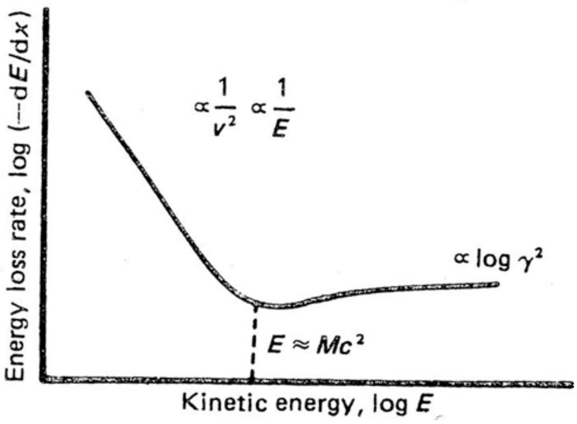
- For low energies;
$E \le mc^2$ where M is the mass of the high energy particle. The loss rate is determined by the first term: the stopping power is proportional to $\left(m_e v^2\right)^{-1}$, which is proportional to E^{-1}.

$$-\frac{dE}{dx} \propto \frac{1}{v^2} \propto \frac{1}{E}$$

- For high energies;
The In-factor becomes important.

$$-\frac{dE}{dx} \propto \ln \gamma^2$$

- By measuring the stopping power and the distance until it comes to rest, we can identify particle and its' energy.
- In cold interstellar cloud, the ionisation losses of the cosmic ray particles provide a certain degree of ionisation → we have ionized atoms at temperatures of only 10 - 50k → large number of chemical reactions and formation of complicated molecules.



3.2 Bremsstrahlung (Free-Free emission)

Free-Free as the electron is free before and after the encouter.

- When a charged particle, usually an electron, encounters an ionized atom or nucleus, it is accelerated and hence it radiates.
- As a consequence of this, wherever there is a hot ionized gas in the universe we expect such radiation.
- Kinetic energy is taken away from charged particle → it slows down → it breaks → "Bremsstrahlung" - breaking radiation.
- Since charge is unbound (free) before and after interaction → "Free-Free emission"
- Process is similar but reverse to that of ionized losses → similar computations needed.

3.2.1 Single electron

From classical electrodynamics, any accelerated charge emits dipole radiation.
The total radiation rate:

$$-\frac{dE}{dt} = \frac{q^2 |\ddot{\underline{r}}|^2}{6\pi\varepsilon_o c^3}$$

Radiation pattern is dipolar. i.e. the electric field strength varies with the polar angle as $\sin i$.
E is proportional to $\sin i$.
→ power per unit angle is proportional to $\sin^2 i$.
→ more power in direction of acceleration.
→ power is greatest perpendicular to the acceleration.
Radiation is polarized.
→ "Electric field vector oscillates in the preferred direction"
Here it is in the direction of acceleration.
For bremmstrahlung:

When the electron passed by an atom, is deviates in its' course to go around the atom slightly. The impact parameter b is the closest approach to the nucleus. The acceleration vector is constantly changing so that it is parallel to the direction of motion, i.e. pointing directly towards the center of the atom.
Let the atom have charge Ze^+, the electron e^-. Acceleration has parallel $a_{\parallel}$ and perpendicular $a_{\perp}$ components.
In order to compute the spectrum (= intensity as function of frequency), we need to integrate the Lamot formula over all angle θ and all accelerations.
Check longair Volume 1, Chapter 3 for the derivation.
The result is (classical treatment):
$\omega = 2\pi\nu$

$$I(\omega) = \frac{Z^2 e^6}{24\pi^4 \varepsilon_o^3 c^3 m_e^2} \frac{\omega^2}{\gamma^2 v^2} \left[\frac{1}{\gamma^2} I_o^2\left(\frac{\omega b}{\gamma v}\right) + I_1^2\left(\frac{\omega b}{\gamma v}\right) \right]$$

We don't need to memorize this for the exam!
$\frac{1}{\gamma^2} I_o^2\left(\frac{\omega b}{\gamma v}\right)$ is $a_{\parallel}$, $I_1^2\left(\frac{\omega b}{\gamma v}\right)$ is $a_{\perp}$.
I_o, I_1 are "modified Bessel functions" of Zeroth and first order.
The two terms in the brackets correspond to the acceleration of the electron parallel and perpendicular to the direction of motion.
The term due to $a_{\perp}$ is larger due to dipole emmision.
Considering asymptotic limit of modified Bessel function to derive high and low frequency limit:
High frequencies:
$\omega >> \frac{\gamma v}{b}$

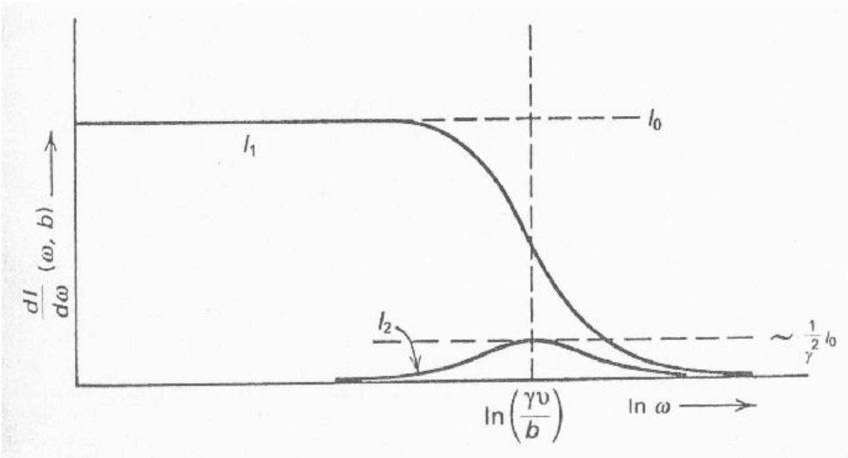
$$I(\omega) = \frac{Z^2 e^6}{48\pi^4 \varepsilon_o^3 c^3 m_e^2 \gamma v^3} \left[\frac{1}{\gamma^2} + 1 \right] \exp\left(-\frac{2\omega b}{\gamma v} \right)$$

The exponential term causes a "high frequency cutoff" i.e. beyond certain frequency emitted, the energy drops dramatically.
Say;
$\frac{\omega b}{\gamma v} \sim 1$
(See example later)
$t = \frac{b}{\gamma v}$
This is the time it takes for the closest encounter with the nucleus to take place.
The cutoff depends on the duration of the encounter. Thus there must be an upper limit to the frequencies emitted, since the electron cannot emit more that its' kinetic energy.

Low frequencies:
$\omega << \frac{\gamma v}{b}$

$$I(\omega) = \frac{Z^2 e^6}{24\pi^4 \varepsilon_o^3 c^3 m_e^2 b^2 v^2} = const$$

for given b and v.
The complete spectrum:



The cutoff is given by the kinetic energy of the electron.

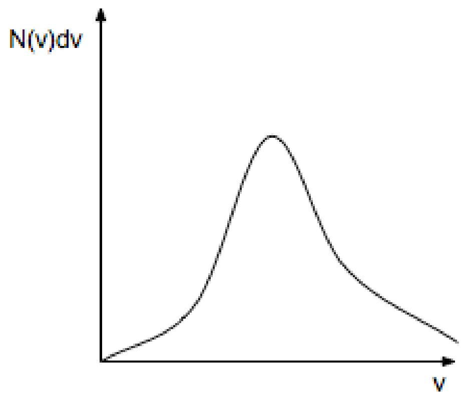
3.2.2 Thermal Bremsstrahlung

So far, we have considered only a single electron.

In hot plasma, we have many electrons with velocity distribution given my Maxwell distribution. (Non-relativistic velocities)

Only parameter that matters is the temperature → thermal radiation process.

$$N(v)dv = 4\pi N_o \left(\frac{m_e}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{m_e v^2}{2kT}}$$



Exponential tail-off.

To obtain spectra for plasma, we have to integrate the single-electron spectrum over all velocities in Maxwell distribution and all impact parameter b.

We obtain:

$$I(\omega) \propto \frac{Z^2 N_i N_e}{\sqrt{T}} e^{-\frac{\hbar\omega}{kT}} g(\omega,T)$$

$Z =$ charge of nuclei

$N_i =$ Number density of nuclei

$N_e =$ Number density of electron

$\hbar\omega = h\nu$

The last term is called the "Gaunt factor" and depends on the energy limit that is considered.

For high frequencies (X-rays!) we expect again an exponential cutoff.

For low frequencies (Radio!) we expect a constant spectrum. (Later: there will be a decrease at low frequencies as it is modified by "self-absorption" due to the plasma becoming optically thick.)

The spectrum looks the same as that for the single atom.

The cutoff occurs at the frequency (=energy) given by the thermal energy of the hot plasma. See the exponential term.

Thus we will have a cutoff frequency;

$h\nu_{cutoff} \sim kT$

i.e.;

$\frac{ho}{kT} \ \$ \ 1$

Example:

$h\nu_c \sim kT$

$h\nu_c = 1keV = 1000x1.6x10^{-19} \approx 10^{-16} J$

$T \approx \frac{10^{-16}}{1.38x10^{-23}} \sim 10^7 k$

Therefore we get a temperature of 10 million k measured by observing the spectrum.
The total loss rate of energy per volume;

$$\frac{-\frac{dE}{dt}}{Volume} = 1.435x10^{-40} z^2 \sqrt{T}\, N_i N_e \bar{g}$$

3.2.3 Relativistic Bremsstrahlung

In astrophysical conditions we may encounter particles with relativistic velocities that can also interact with ions.
Example: c - *ray* emission of the interstellar medium (which is filled with molecules, gas, ionized protons and free electrons, etc - see later). Here we have to consider that the electron energy distribution usually has the form of a power law:

$N(E) \propto E^{-\alpha}$

Thus we see a straight line slope in a $N(E)dE$ vs. E graph where the gradient is a.

Integrate over $N(E)dE$ to derive the spectrum.

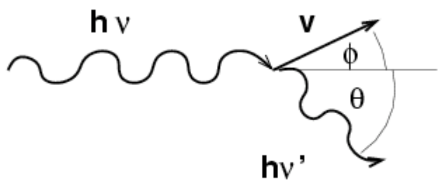
3.3 Compton Scattering

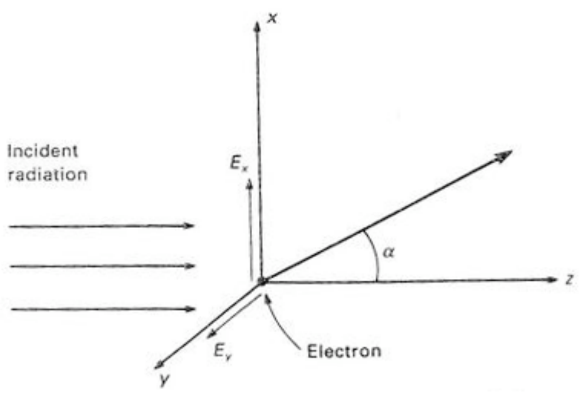
Energy loss process for high energy electrons.
In 1923, Compton discovered that the wavelength of hard (energetic) X-rays increases when they are scattered at stationary electrons.
Photon comes in with energy *h*o and hits an electron, before scattering off at angle i . After this, it has energy *h*o' which is lower than before. Thus the electron has gained energy.
X-ray emission is modified by interaction with electrons - energy is transferred.
We can consider two cases: first, where the energy of the photon is small when compared with the rest mass of the electron - $h\nu << m_e c^2$ - this is called Thomson scattering. Compton observed where the energy of the photon is comparable with the rest mass of the electron - $h\nu \sim m_e c^2$.

3.3.1 Thomson Scattering

It will become important to understand the luminosity of accreting black holes and neutron stars.
Consider the case where the energy of the photon, *h*o, is much less than the rest mass of the electron, $m_e c^2$.

$h\nu << m_e c^2$





In this case, when the photon hits the electron, the electron will interact by oscillating as it follows the electric field vector of the incoming photon. Thus it will be emitting dipole radiation, with the direction depending on the oscillating direction (Electron vibrating in the xy-direction, radiation given off in the ±z-directions.) according to the Lamor formula. Ions and protons are too heavy to interact with the incoming wave - there is no oscillation, and hence no Thomson scattering. Thus we only expect interaction with electrons.
The photon is scattered at angle α.
Since the dipole radiation has a certain intensity pattern and also polarization, we expect the same for Thomson scattering.
The intensity of the radiation depends on the direction, thus we compute it using a dipole pattern. Thus using the solid angle Ω;

$$-\left(\frac{dE}{dt}\right)d\Omega = \frac{e^4}{16\pi^2 m_e^2 \varepsilon_o^2 c^4}\left(1+\cos^2\alpha\right)\frac{S}{2}\,d\Omega$$

Where;

$-\left(\frac{dE}{dt}\right)d\Omega$ is the intensity emitted in angle Ω.

S is the "Poynting" flux which is equal to the incident energy per time and unit area upon the electron.

θ is the angle measured from the direction of incident photons.

We can define a differential cross-section for Thomson scattering;

$$d\sigma_T = \frac{e^4}{16\pi^2 m_e^2 \varepsilon_o^2 c^4}\frac{\left(1+\cos^2\theta\right)}{2}d\Omega$$

$$-\left(\frac{dE}{dt}\right)d\Omega = S\,d\sigma_T$$

$d\sigma_T$ has the unit of area.
Using the classical electron radius;

$$r_e = \frac{e^2}{4\pi\varepsilon_o m_e c^2}$$

$$d\sigma_T = \frac{r_e^2}{2}\left(1+\cos^2\theta\right)d\Omega$$

A cross section has unit area and can be considered as being the size of the electron as seen by the photons. The higher the cross section, the higher the chance for scattering, and hence the larger the scattering radiation.
The photon has a greater chance of going forward and backward than going to the side.
There is as much radiation scattered forward as there is backwards.
The scattered radiation is polarized, even if the incoming radiation wasn't.
The fraction of polarization (the intensity of the polarized light divided by the total intensity) is;

$$\frac{I_{polarised}}{I_{total}} = \Pi = \frac{1-\cos^2\theta}{1+\cos^2\theta}$$

The total Thomson cross-section is computed by integrating over all angles.

$$\sigma_T = \int_\Omega d\sigma_T = \int \frac{r_e^2}{2}\left(1+\cos^2\theta\right)d\Omega$$

$$\sigma_T = \int_0^\pi \frac{r_e^2}{2}\left(1+\cos^2\theta\right)2\pi\sin\theta\,d\theta$$

$$\boxed{\sigma_T = \frac{8\pi}{3}r_e^2 = 6.653x10^{-29}m^2}$$

Note that this is independent of frequency.

3.3.2 Compton Scattering

If photons are of similar energies, we must use proper relativistic cross sections as derived in quantum mechanics. This is similar to when electrons have low energies, but the electrons may be relativistic. The quantum relativistic cross-section is the Klein-Nishina Cross-section;

$$\sigma_{K-N} = \frac{\pi r_e^2}{\varepsilon}\left\{\left[1-\frac{2(\varepsilon+1)}{\varepsilon^2}\right]\ln(2\varepsilon+1) + \frac{1}{2} + \frac{4}{\varepsilon} - \frac{1}{2(2\varepsilon+1)^2}\right\}$$

$$\varepsilon = \frac{h\nu}{m_e c^2}$$

This is now dependant on frequency.

For low energy photons, $\varepsilon << 1$. This expression reduces to;

$$\sigma_{K-N} = \frac{8\pi}{3} r_e^2 (1 - 2\varepsilon) \approx \sigma_T (1 - 2\varepsilon) \approx \sigma_T$$

For high energies;

$$\sigma_{K-N} = \pi r_e^2 \frac{1}{\varepsilon}\left[\ln(2\varepsilon) + \frac{1}{2}\right] \propto \frac{1}{\varepsilon} \propto \frac{1}{h\nu}$$

In Compton scattering, the electron experiences a recoil that absorbs energy, and hence less energy is available for the scattered photons.
Thomson scattering is an elastic scattering of photons, while Compton scattering is an inelastic scattering of photons.
The increase in wavelength of the photon is given by the de Broglie wavelength of a particle with the rest mass as the electron.

$$E = m_e c^2 = h\nu = \frac{hc}{\lambda}$$

$$\lambda_c = \frac{h}{m_e c}$$

λ_c is the Compton wavelength of the electron.

$$\Delta\lambda = \lambda_{scattering} - \lambda_{initial} = \lambda_c(1 - \cos\theta) = 2\lambda_c \sin^2\frac{\theta}{2}$$

3.3.3 Inverse Compton Scattering

We can exchange energy between the photon and the electron in Compton scattering. If the electrons are much more energetic than the photons $(\gamma m_e c^2 >> h\nu)$, then we can reverse the Compton process.
The electrons will loose energy, and slow down, while the photons gain energy.
Since $E_{e^-} >> E_{photons}$, we have a case of Thomson scattering.
Assuming that the photons are monochromatic at a single frequency ν_o, the intensity per unit angle and unit bandwidth is given by;

$$I(\nu) = \frac{3\sigma_T c}{16\gamma^4} \frac{N(\nu_o)}{\nu_o^2} \nu f(\nu) d\nu$$

$N(\nu_o)$ is the photon number density at frequency ν_o.
$f(\nu)$ is rather complicated (Equation 4.32 in Longair), but is essentially constant at low frequencies.
For low frequencies;

$I(\nu) \propto \nu$

At high frequencies, there much be a "cut-off" as the maximum energy that can be acquired by the photon is in a head-on collision in which the photon is sent back along its' original path.

$E_{\max} \approx 4\gamma^2 h\nu_o$

We have a cut-off at $\nu_c = 4\gamma^2 \nu_o$.
inverse Compton scattering can boost photon energy by a factor $4\gamma^2$.

3.4 Synchrotron Radiation

It is produced by relativistic electrons spiraling in a B field.

$$\frac{d\underline{p}}{dt} = \frac{d(\gamma m_e \underline{v})}{dt} = \gamma m_e \frac{d\underline{v}}{dt} = q(\underline{E} + \underline{v} \times \underline{B}) = -e\underline{v} \times \underline{B}$$

This is in the lab frame. - magnetic field only.
We only have to worry about the component of v which is perpendicular to B, because cross product disappears for the parallel component of v. Hence there is no change in $v_{||}$.
Without loss of generality, we can choose a coordinate system such that B is along the z-direction.
The electron will be spiraling around the z-axis. The equation of motion will involve the x and y components of the velocity. It is given by;

$$v_{\perp} = (v_x, v_y) = (\dot{x}, \dot{y})$$

$$\ddot{x} = \dot{v}_x = -\frac{eB}{\gamma m_e} v_y$$

$$\ddot{y} = \dot{v}_y = -\frac{eB}{\gamma m_e} v_x$$

Assume that the electron is moving in a circle. Guess that;

$r_x = R\cos\omega_g t$

$r_y = R\sin\omega_g t$

$r_z = v_z t + r_{z_o}$

This is of course circular motion. Plug these back into the above equations.

$\dot{r}_x = -R\omega_g \sin\omega_g t$

$\dot{r}_y = R\omega_g \cos\omega_g t$

$\ddot{r}_x = -R\omega_g^{\,2} \cos\omega_g t$

$\ddot{r}_y = -R\omega_g^{\,2} \sin\omega_g t$

$$\dot{v}_g = -R\omega_g^{\,2}\cos\omega_g t = -\frac{eB}{\gamma m_e} R\omega_g \cos\omega_g t$$

$$\omega_g = \frac{eB}{\gamma m_e}$$

This is the relativistically correct gyrofrequency.
Putting in some typical numbers;

$$v_g = \frac{\omega_g}{2\pi} = \frac{28}{\gamma} GHz.t^{-1}$$

3.4.2 Total power radiated

Take the lab frame S, and call the electron frame at any instant S'.
Compute;

$$\frac{dE'}{dt} = \frac{q^2 |\dot{v}'|^2}{6\pi\varepsilon_o c^3}$$

Aside: the Larmor formula is;

$\frac{\ddot{p}^2}{6\pi\varepsilon_o c^3}$ is the rate of energy loss for a charged particle with a dipole of $\underline{p} = e\underline{r}$.

(In S', v'=0 but $\dot{v}' \neq 0$)
In S', a force is experienced.

$F = m_e \dot{v}' = -e\left(\underline{E}' + \underline{v}' x \underline{B}'\right) = -eE'$

i.e. while in the lab frame there is only a magnetic field, in the electron frame there is only an electric field.
Lorentz transformations for electric and magnetic fields;

$E_x' = E_x$

$E_y' = \gamma\left(E_y - vB_z\right)$

$E_z' = \gamma\left(E_z - vB_y\right)$

So;

$E_x' = 0$

$E_y' = -\gamma vB$

$E_z' = \gamma\left(E_z - vB_y\right) = 0$

Therefore;

$\dot{v}_x' = 0$

$\dot{v}_y' = -\frac{\gamma evB}{m_e}$

$\dot{v}_z' = 0$

Insert this into the formula for the rate of energy loss.

$$\left(\frac{dE}{dt}\right)' = \frac{\gamma^2 e^4 v_\perp^2 B^2}{6\pi\varepsilon_o c^3 m_e^2} = -\frac{dE}{dt}$$

as this is constant in both frames.
Rate of radiation is proportional to $\gamma^2 B^2$
Normally, $v_\perp = v\sin\theta$ where θ is the pitch angle. This is the angle between $\underline{v}$ and $\underline{B}$.

Hence;

$$-\frac{dE}{dt} = 2\left(\frac{e^4}{6\pi\varepsilon_o^2 c^4 m_e^2}\right)\left(\frac{v}{c}\right)^2 c\,\frac{B^2}{2\mu_o}\gamma^2\sin^2\theta$$

Remember that;

$$\left(\frac{e^4}{6\pi\varepsilon_o^{\ 2}c^4 m_e^{\ 2}}\right) = \sigma_T$$

is the Thomson cross-section, and note that

$$U_{mag} = \frac{B^2}{2\mu_o}$$

is the energy density of the magnetic field per unit volume.

Thus;

$$-\frac{dE}{dt} = 2\sigma_T c U_{mag}\left(\frac{v}{c}\right)^2\gamma^2\sin^2\theta$$

Inserting some numbers:

$$1.587x10^{-14}B^2\gamma^2\left(\frac{v^2}{c^2}\right)\sin^2\theta\ Watts$$

Suppose that there are a range of electrons with different pitch angles. Assume an isotropic pitch angle distribution.

$$p(\theta) = \frac{1}{2}\sin\theta\,d\theta$$

Integrate over 0 to π.

→ $<\sin^2\theta>$

$$-\frac{dE}{dt} = \frac{4}{3}\sigma_T c U_{mag}\left(\frac{v}{c}\right)^2\gamma^2$$

$$1.058x10^{-14}B^2\gamma^2\left(\frac{v^2}{c^2}\right) Watts$$

This is the equation when you have an unknown pitch angle.

3.4.3 Cyclotron Emission

This is similar to synchrotron emission, except the energies are lower.

3.4.3.1 Non-relativistic Motion

Here, $v << c$ so $\gamma = 1$.

Using the perpendicular component of the velocity through $v_\perp = v\sin\theta$.

$$-\frac{dE}{dt} = 2\sigma_T c U_{mag}\left(\frac{v}{c}\right)^2\sin^2\theta = \frac{2\sigma_T}{c}U_{mag}v_\perp^2$$

This emits only at one specific frequency; there is no spectrum of emission. (one fringe).

The frequency is the cyclotron frequency, or the non-relativistic gyro frequency.

$$\nu_{cycl} = \nu_g(\gamma = 1) = \frac{eB}{2\pi m_e} = 28B_{(T)}GHz$$

The radiation is polarized, similar to bremsstrahlung and Thomson Scattering. It is linearly polarized if the magnetic field is perpendicular to the line of sight. If the B field is parallel to the line of site, it is circularly polarized. At all other angles, it is elliptically polarized (i.e. a mixture of linear and circular polarization).

Often, it is seen as an absorption line rather than an emission line. By observing the frequency of the emission given off, we can estimate the magnetic field in the region it was emitted.

Example:

Observing a gyro-radiation feature allows us to determine the B field. In Hercules X-1, the absorption line is at $34KeV$.

$$h\nu = 34KeV$$

$$\nu = 8.2x10^9\,GHz = 28B\,GHz$$

$$B = \frac{8.2x10^9}{28} = 3x10^8\,T$$

→ highly magnetized neutron star.

3.4.3.2 Mildly-relativistic motion

Still the velocity is much smaller than c, but we need to take relativity effects into account. Relativistic beaming is caused by the fact that something is moving at a large speed with respect to the observer. → everything is beamed forward.
When an electron moves faster, the emission is beamed forward. Therefore we see the radiation for a shorter time of orbit around the field lines. The radiation is also blueshifted
The net effect is that the electron is seen for a shorter time dt at a higher frequency. The pulse then changes into harmonics up to the maximum frequency, given by;

$\omega_{max} \sim \frac{1}{dt}$.

We can then write the spectrum as a sum of all the harmonics.

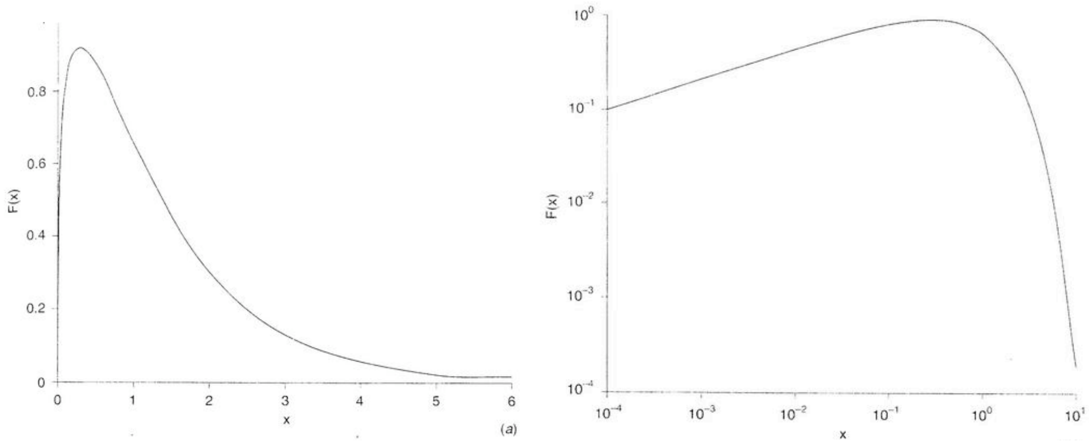
$I(\omega) = \sum_{i=1}^{max} I_i(\omega_{cycle}, \Delta\omega_i)$

The lines become broader due to the last part of this, $\Delta\omega_i$.
This effect is seen in real life.

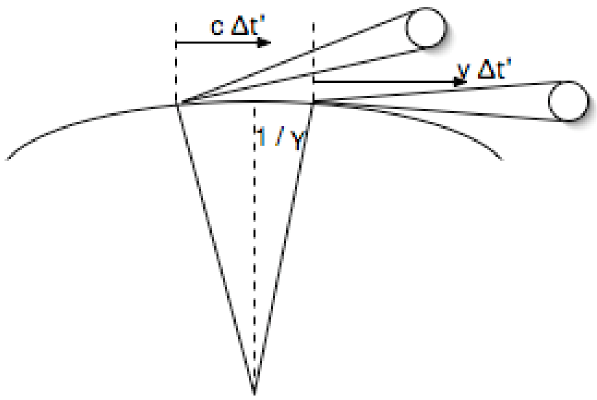
3.4.4 Single Relativistic Electron

Now $\gamma >> 1$. Electron moving in a magnetic field. Use the cases from above to derive the spectum.

The beaming is now much more extreme than in the simpler case. We also have blueshifting of the beams of width of approximately γ^{-1}. Harmonics for the mildly relativistic case blend together, leading to a single continuous spectra.



The electron is traveling around a circular path. Look at two cases; a beam coming directly to the observer, and one slightly off.



Due to the beaming, a distant observer will see emission only for a fraction of approximately

$\frac{2}{\gamma}$

Hence;

$$\Delta t' = \frac{1}{\nu_g}\frac{2}{\gamma} = \frac{2}{\nu_g \gamma}$$

ν_g is the relativistic gyro frequency.

The first photon to reach the observer will have traveled $x = c\Delta t'$, while the second will travel $x = v\Delta t'$. The difference is $(c - v)\Delta t'$.

The difference between the first and second photon arrival times becomes simply;

$$\Delta t = \frac{\Delta x}{c} = \Delta t'\left(1 - \frac{v}{c}\right) = \frac{2}{\nu_g \gamma}\left(1 - \frac{v}{c}\right)$$

For large γ;

$$1 - \frac{v}{c} \sim \frac{1}{2\gamma^2}$$

So;

$$\Delta t \sim \frac{2}{\nu_g \gamma}\frac{1}{2\gamma^2}$$

So the highest frequency observed will be;

$$\nu_c \approx \frac{1}{\Delta t} = \gamma^3 \nu_g = \gamma^2 \nu_{cyclatron}$$

The exact solution;

$$\nu_c = \frac{3}{2}\gamma^2 \frac{eB}{2\pi m_e}\sin\theta$$

Note that;

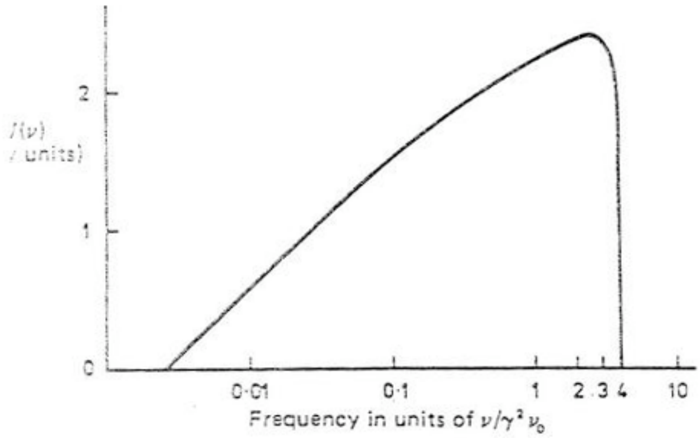
$$\nu_{cycle} = \frac{eB}{2\pi m_e}$$

and $\nu_{cycle} = \gamma \nu_g$.

This is approximately $4.2x10^{10}\gamma^2 B\sin\theta$.

Above ν_c we have high frequency "cut-off". One can show that for $v << v_c$,

$$-\frac{dE}{dt} \propto v^{1/3}.$$



Spectrum peaks at $0.29 v_c$.

What is the total energy output?

Integrate the spectrum $I(\nu)$ over all frequencies.

$$P = \frac{1}{6\pi\varepsilon_o}\frac{e^4 B^2 \sin^2\theta}{m_e^2 c}\gamma^2$$

We can write this in terms of the total electron energy $E = \gamma m_e c^2$.

$$P = \frac{1}{6\pi\varepsilon_o c}\frac{e^4 B^2 \sin^2\theta}{m_e^4 c^5}E^2$$

- Strong dependence on mass (proportional to m_e^{-4}) shows why protons are usually not important since they radiate

$$\left(\frac{m_p}{m_e}\right)^4 = 1836^4 = 10^{13}$$

times less energy than electrons - hence we only need to consider electrons normally (protons become important at very large magnetic fields, though)

We expect synchrotron emission to be polarized such as cyclotron emission → elliptically polarized for a single electron.

Example;
Consider an ultra-relativistic electron, $\gamma = 10^4$ (cosmic ray electrons) - hence $v \approx c$ - spiraling in a magnetic field of a galaxy, $B \sim 6\mu G = 6x10^{-10} Tesla$.
NB: 1 Gauss = 10^{-4} Tesla.
Frequency of circular motion;
$$\nu_g = \frac{eB}{2\pi m_e \gamma} = \frac{28}{\gamma} Bx10^9 Hz = \frac{28}{10^{11}} x6x10^{-10} x10^9 = 0.002Hz$$
Radius of circular motion;
$$R_g = \frac{v}{\omega_g} = \frac{v}{2\pi \nu_g} = \frac{c}{2\pi \nu_g} = 2.4x10^{10} m$$
This is about 1/5 of the Sun-Earth distance. Very large for us, but on the cosmic scale it is fairly small.
We expect the peaks of synchrotron emission at;
$$\nu_c = 4.2x10^{10} \gamma^2 B \sin\theta = 4.2x10^{10} x10^8 x6x10^{-10} = 2.5x10^9 Hz = 2.5GHz$$
(ignoring θ)

3.4.5 Multiple Electron: Electron density distribution
We now have a number of electrons per energy interval distributed in some way.
$$N(E)dE$$
We have to integrate this to derive the spectrum.
Most high energy electron distributions are not thermal but the distribution is a power law;
$$N(E)dE = \kappa E^{-p} dE$$
where p is the power law index.
One can show that the spectrum also becomes a power law;
$$-\frac{dE}{dt} \propto v^{-\alpha} = v^{-\frac{p-1}{2}}$$
So the energy spectrum of the radiation is also a power law with index;
$$\alpha = \frac{p-1}{2}$$
This is quite "cute".
From measuring the spectrum, i.e. α, we can derive p and hence the energy distribution.
For a wide variety of non-thermal cosmic sources we observe;
$$0.2 \leq \alpha \leq 1.2$$
For extragalactic sometimes $\alpha \sim 2$.
For the galactic cosmic electrons (we can measure them at the top of the atmosphere at Earth);
$$p = 2.6$$
$$\alpha = \frac{2.6-1}{2} = 0.8$$
A word of caution: this spectral index is for optically thin sources. For optically thick emission, we have a process called self-absorption where the electron is immediately absorbed by the material that emitted it.
For the remainder of the lecture, synchrotron emission is from an ultra-relativistic electron.

3.4.6 Polarization properties of synchrotron emission
The single electron was elliptically polarized. For many electrons we have superposition of polarization. When you add all of these up, the circular polarization will cancel and all that will be left is linear polarization. the E field of the EM radiation is perpendicular to the B field.
Measuring synchrotron emission as intensity gives you the B field strength, and measuring the polarization gives the direction of the B field.

Given cancellation of some polarization emission is not 100% polarized, and the degree of polarization depends on p where p is the energy density.
$$\Pi = \frac{p+1}{p-1}$$
For $p = 2.6$ for Galactic cosmic ray electrons;
$\Pi = 0.72$ or 72%.
This is the maximum value as you can always "destroy" polarization e.g. by superposition of different sources.

Sidenote of synchrotron emission;
For low speed, on intensity vs. time graph, it is roughly sinusoidal and on a period vs. frequency these is a single spike.
Speeding up: the peaks become more pointed, with some time in between each of them, with more harmonics.
Even faster: the peaks will become pulse like, with more harmonics appearing.

3.5 Absorption Processes
3.5.1 Photoelectrical Absorption
An incoming photon interacts with an atom, ejecting an electron from an inner shell. The ejected electron receives a kinetic energy

$$E_{kin} = h\nu - E_B > 0$$

where E_B is the binding energy of an ejected electron.
Corresponding cross-section;

$$\sigma_{ph} \propto \sigma_T Z^5 \left(h\nu\right)^{-3}$$

For higher energies, Compton scattering is dominant. At even higher frequencies, pair-production becomes important.

3.5.2 Pair Production
If

$$E = h\nu \geq 2m_e c^2$$

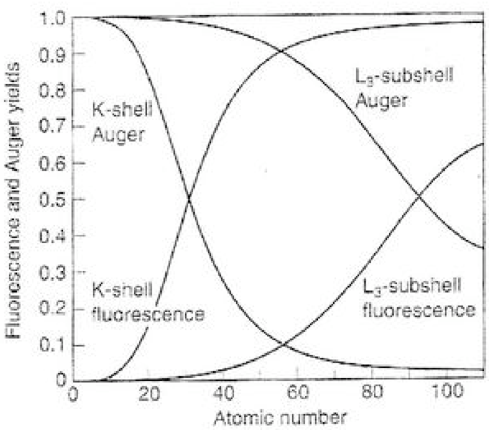
it is possible to create an electron-positron pair with

$$E_{kin}\left(e^-\right) = E_{kin}\left(e^+\right) = \left(\gamma - 1\right) m_e c^2$$

The inverse process is called annihilation;

$$e^+ + e^- \rightarrow \nu + \nu$$

3.5.3 Auger effect and Fluorescence emission
In Auger effect a hole in an inner electron shell is filled with an electron from an outer shell. The gained energy is emitted as electron.
The Auger effect is competing with Fluorescence emission.
In fluorescence emission an excited atom drops back into the ground state and emits a photon of corresponding energy. The result; a line spectrum characteristic for atom.



<u>*3.5.4 Self-absorption*</u>
Cosmic sources can be opaque to their own radiation → they can absorb their own radiation.
Optically thick plasma may show self-absorption. Optically thin plasma is transparent.

<u>Optically thin:</u>

$$I_o = f_o x$$

where f_o is the emissivily, and x the length of the object emitting the radiation.

→ observed radiation is that of the actual emission process, and is equal to the number of photons that can escape to the observer.
The observed spectrum is that of the emission process.
For instance; for an optically thin plasma radiating thermal bremsstrahlung we expect the intensity to be constant.
For synchrotron emission, we expect a power law;

$$I_\nu \propto \nu^{-\frac{p-1}{2}}$$

<u>Optically thick plasma;</u>

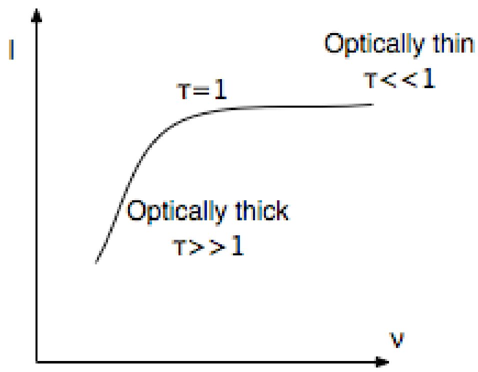
<u>3.5.4.1 Thermal Bremsstrahlung self-absorption</u>

An electron passing an ion can also absorb the photon, not only emit one. This is the reverse of bremsstrahlung. It is sometimes also called free-free absorption, as the electron is free both before and after the absorption process.
Radiative transfer equation suggests that for optically thick plasma we see the spectrum and intensity of a black body of the same temperature.
Low frequencies → Rayleigh-Jeans limit.

$$I_\nu = B_\nu(T) = \frac{2kT}{c^2}\nu^2$$

We expect $I_\nu \propto \nu^2$.

Optically thin: I_ν is constant. $\tau << 1$. Optically thick: power law.



Typical examples: HII region (hot ionized regions of hydrogen)
At transition frequency: $\tau = l\,\alpha_\nu \approx 1$
→ number density information.

<u>3.5.4.2 Synchrotron self-absorption</u>

We expect an energy distribution as a power law as before.

$$N(E)dE = kE^{-p}dE$$

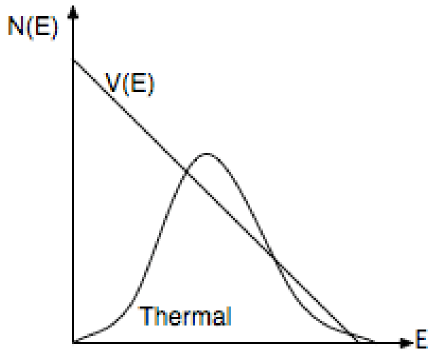
So we expect more electrons at lower energies than higher energies.
For optically thin plasma, we expect;

$$I_\nu \propto \nu^{-\frac{p-1}{2}}$$

For optically thick plasma we may expect;

$$I_\nu = \frac{2kT}{c^2}\nu^2$$

For this, there are two constraints;
1)



In contrast to thermal bremsstrahlung, the number of available electrons per energy interval dE is given by the power law, rather than the Maxwell distribution. Hence we have different numbers per interval.

2)

No radiation process (of incoherent emission) is more effective than black body.

We need to take both (1) and (2) into account simultaneously.

Synchrotron emission spectrum can be viewed as the superposition of single electron spectra with each electron radiating at a characteristic frequency

$$o_c = \frac{3}{2} c^2 \frac{eB}{2r\, m_e} \sin i$$

which is essentially proportional to c^2.

For a given magnetic field, emission seen at the frequency o can be viewed as the radiation of just those electrons that have the corresponding gamma-factor.

$$\gamma \propto \nu^{1/2}.$$

For each energy interval dE, the radiation can only have an intensity that is given by the Plank formula (or the Rayleigh-Jeans limit) for blackbody emission at that energy.

$$E \sim kT = c m_e c^2$$

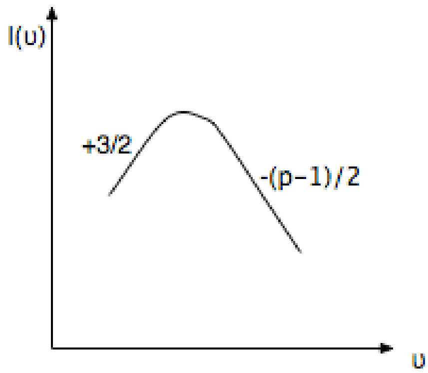
Solving for T:

$$T \sim c \frac{m_e c^2}{k} \text{ a } o^{1/2}$$

Using the equivalent black body intensity;

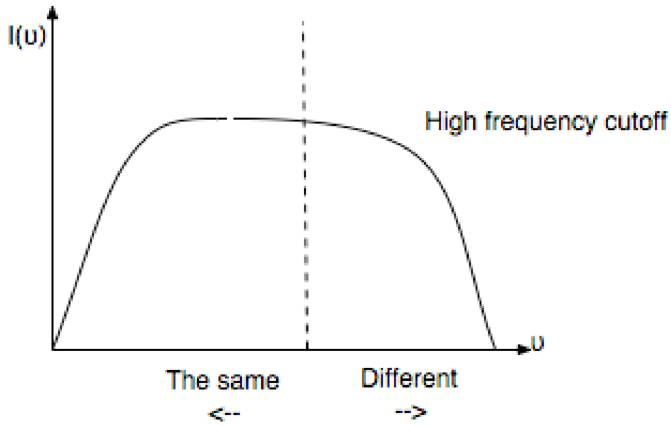
$$I_o = \frac{2kT}{c^2} o^2 \text{ a } o^{5/2}$$

This is independent of the energy distribution.

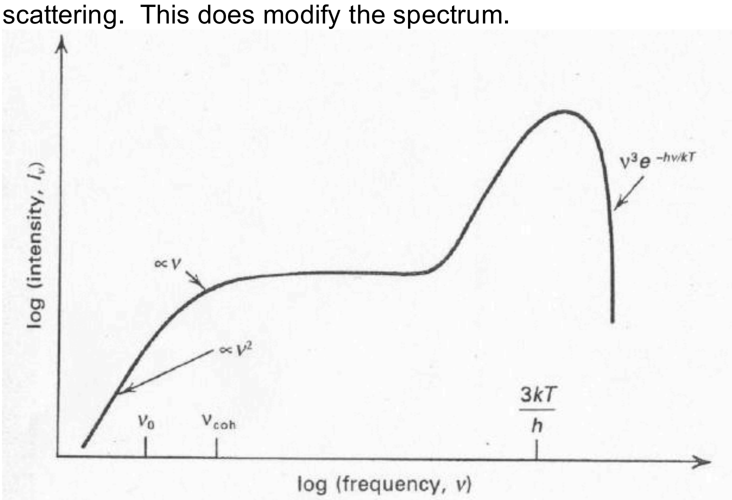


3.6 Final remarks about radiation

We have to look at the relative importance of competing radiation processes. e.g. "Comptonization" that happens when there is a hot, ionized region that produces thermal bremsstrahlung.



Photons may be Thomson scattered but that doesn't change the spectrum. However at high energies the photon energy is similar to the electron rest mass. → Compton scattering becomes important. The photons may be boosted to high energies via inverse Compton scattering. This does modify the spectrum.



The bump is similar to the Wien part of black body spectrum.

$\propto \nu^3 e^{-\frac{h\nu}{kT}}$

Peak of the spectrum is at roughly

$\frac{3kT}{h}$

Second example: competition between synchrotron emission and inverse Compton scattering.

Electron in a magnetic field can loose energy due to Synchrotron emission and inverse Compton scattering with the background photons of the cosmic microwave background.

For Synchrotron emission;

$$-\left(\frac{dE}{dt}\right)_{sync} = \frac{4}{3}\sigma_T c U_{mag}\left(\frac{v}{c}\right)^2 \gamma^2$$

For large $v \approx c$;

$$\boxed{-\left(\frac{dE}{dt}\right)_{sync} = \frac{4}{3}\sigma_T c U_{mag}\gamma^2}$$

For inverse Compton scattering;

$$\boxed{-\left(\frac{dE}{dt}\right) = \frac{4}{3}\sigma_T c U_{rad}\gamma^2}$$

U_{mag} was the energy density of the B field, while U_{rad} is the energy density of the photon field.

Remember:

$$U_{mag} = \frac{4}{c}\sigma T^4$$

where σ is the Stefan-Boltzman constant.

So;

$$\frac{\left(\frac{dE}{dt}\right)_{IC}}{\left(\frac{dE}{dt}\right)_{sync}} = \frac{U_{rad}}{U_{mag}}$$

For instance;
For an electron in a Galactic B-field of $B = 3\,\mu G = 3x10^{-10}\,T$.
Energy density;

$$U_{mag} = \frac{B^2}{2\mu_o} = 3.6x10^{-14}\,Jm^{-3}$$

cf. energy density of the CMB ($T = 3.73k$)

$$U_{rad} = \frac{4}{c}\sigma\,(2.73)^4 = 4.2x10^{-14}\,Jm^{-3}$$

These numbers are very similar. So;

$$\frac{\left(\frac{dE}{dt}\right)_{IC}}{\left(\frac{dE}{dt}\right)_{sync}} = 1.17$$

Both processes are equally important.
→ consider relative importance.

Resolution of a telescope; $\theta = 1.22\frac{\lambda}{D}$

5. The Interstellar Medium

See also the lecture course on ISM, and stars and stellar structure.

5.1 Composition of the ISM

- Dust
- Gas

5.1.1 Interstellar Dust

Composition is unknown, mostly graphite (perhaps iron-based particles). Wide range of sizes, typically around 100nm. The dust absorbs EM radiation with comparable or smaller wavelengths.

$\tau \propto \lambda^{-1}$

(τ is optical depth)

→ mostly UV and optical light is absorbed. "extinction". Also called "reddening" as short wavelength blue light is more readily absorbed than red light.
Absorbed energy is re-emitted as infrared radiation. → strong IR sources.
Elongated dust grains that are aligned e.g. by an external magnetic field cause polarization of optical light.

5.1.2 Interstellar Gas

There are four main gas types.

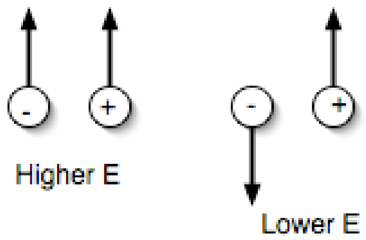
Type	Constituent	Detection	V (%)	M (%)	$N\left(m^{-3}\right)$	T (k)
Molecular ISM	H_2, CO	Molecular lines, dust emission	~ 0.5	~ 40	$\geq 10^9$	$10 \rightarrow 30$
Cold ISM	H, C, O, some ions	21cm line	~ 5	~ 40	$10^6 \rightarrow 10^8$	80
Warm ISM	H, H^+, e^-	21cm, H^α pulsars	40	20	$10^5 \rightarrow 10^6$	8000
Hot ISM	H^+, e^-, highly ionized e.g. O^{8+}	UV, soft X-ray	50	0.1	$\sim 10^3$	10^6

Remember; $p = Nk_BT$

For same p, $T \propto N^{-1}$.

5.1.2.1 Molecular ISM
- Low temperatures are needed.
- Dust cloud around molecules to protect them from UV photons. "Molecular clouds".
- Size of clouds; tens of parsecs. $1pc \approx 3x10^{18}m$
- Total mass up to $10^6 M_\odot$.
- Sites of star formation (e.g. Orion cloud)
- Most abundant molecule is H_2. Difficult to observe. Mostly observed in IR.
- First detected in 1963; OH in CasA. 1968; NH_3.
- By now many more detected, quite complex molecules too.

5.1.2.2 Cold medium
- Cold medium consists of diffuse clouds, HI clouds (atomic hydrogen; HII is ionized hydrogen), and cold neutral medium.



$E_{\uparrow\uparrow} \geq E_{\downarrow\uparrow}$

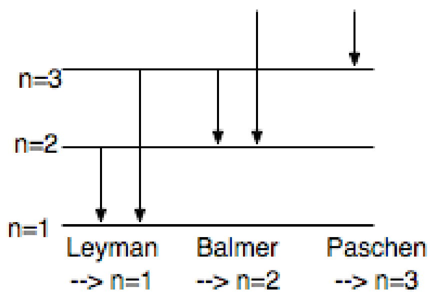
When the electron flips (this is forbidden, but QM says there is a probability of $2.86x10^{-15}s^{-1}$ → it takes $3.5x10^{14}$ seconds = 11.1 million years for an electron to flip - but there are so many hydrogen atoms that this "regularly' happens), the energy difference corresponds to a photon of energy

$$\boxed{\Delta E = h\nu = 1420.4058MHz = 21.1cm}$$

This is the "21 cm line".
It also lets us know how fast it is moving using Doppler shift of this line → gives motion in the galaxy.

5.1.3.3 Warm medium
Neutral component is diffuse, no clouds.
- Important for absorption of soft X-rays
- Strong absorber for soft X-rays, making the spectrum "harder"
- Since gas densities can be inferred from the 21cm line, distance estimate for X-ray source.

Ionized component (WIM)
- Diffuse component and HII regions
- HII regions are located around star-forming regions (need UV photon of $13.6eV$ for ionization → UV photons are mainly found around larger stars)
- Emission due to recombination lines.



Lyman - UV
Balmer - Optical.

Paschen - IR
$n = 3$ → $n = 2$ is the H^{α} line.
$n = 4$ → $n = 2$ is the H^{β} line.
Diffuse component; free electrons.
- Diffuse component density $n_e = 0.02 \rightarrow 0.03\,Cm^{-3}$, with a higher concentration towards the spiral arms of the galaxy.
- Have a higher concentration near stars that are giving off UV
- The spiral arm structure inferred from
    - $H\alpha$ observation of HII-region
    - Motion from HI 21cm line
    - n_e from pulsars

Pulsars; pulses are delayed by 25M at lower frequencies ("dispersion") - delay is proportional to DM (dispersion measure). $DM = \int n_e d\ell$ along line of sight.

$DM = \int n_e d\ell$ = column density of free electrons.

Also; emission measure $EM = \int n_e^{\,2} dl$.
In an ionized medium the refractive index n depends on frequency. For plasmas;

$$n = \sqrt{1 - \left(\frac{\nu_p}{\nu}\right)^2} \leq 1$$

ν_p is the plasma frequency.

$v_{group} = nc \leq 1$

→ they travel at a velocity that depends on frequency. Propagation velocities of the pulse (v_{group}) is smaller than c.

$$\nu_p = \sqrt{\frac{e^2 n_e}{4\pi^2 \varepsilon_o m_e}} = 8.98 n_e^{1/2} Hz$$

If $\nu_p > \nu$, $\frac{\nu_p}{\nu} > 1$ → n is imaginary.
→ wave cannot propagate in the plasma.
(see examples sheet about X-ray mirrors).
For USM, $\nu_p < \nu$ → pulses propagate but with v_{group} depending on ν.
If $\nu >> 1$, $n \rightarrow 1$
$v_{group} \rightarrow c$
Delay between low and high frequency pulse;

$$\Delta t = \frac{e^2}{8\pi^2 \varepsilon_o m_e}.DM.\left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right)$$

It is convenient to express the dispersion measure in units of pc/cm^3 and the frequencies in Mhz.

$$\Delta t = 4.14x10^3 \times DM\left[\frac{pc}{cm^3}\right] \times \left(\frac{1}{\left(\nu_1\left[MHz\right]\right)^2} - \frac{1}{\left(\nu_2\left[MHz\right]\right)^2}\right)$$

Measuring the DM for the observed pulsars, we can infer the electron density distribution in the Galaxy.

5.1.3.4 Hot Medium
- Observable at X-ray and UV absorption lines.
- Solar system is located in a local structure ~ 500 parsecs across filled with hot gas. Slightly more extended above and below plane. ("Local Bubble")

Heating mechanisms?
- Supernova explosions
- UV radiation of young stars
- In particular for neutral medium and molecular gas, ionization losses of cosmic ray particles.

Cosmic rays;
- Primary cosmic rays; photons (84%) and α particles (He nuclei) (14%). Of the remaining 2%, 50% are electrons and 50% are heavier particles.

Energy spectrum has an overall form of a power law.

$N = e^{-\alpha}$

→ galactic synchrotron emission

$\alpha \approx 2.6 \to 3.1$

- Origin is still unknown.

→ traveling at around the speed of light. How are they accelerated to this velocity?

e.g. supernova, pulsars, magnetars, AGN.

5.2 The Magnetised ISM

As we have seen from the observed Galactic synchrotron emission, the Milkey Way has a magnetic field, magnetizing the ISM. Are there ways to measure its strength?

One way we discuss here is that of measuring Faraday Rotation.

For the ionized, magnetized medium the plasma frequency

$$v_p = 8.98 \times n_e^{1/2} Hz$$

and the gyrofrequency

$$v_g = 2.8 \times 10^{10} B.Hz$$

are smaller than radio frequencies.

Under these conditions, a left-handed circularly polarized (LHC) radio wave propagates with a different phase velocity than a right-handed-polarized (RHC) wave. Since a linearly polarized wave can be separated into a LHC and RHC wave, at the end of a propagation of a length l, the phases are different. The result is a rotation of the position angle of the linearly polarized wave of frequency v by an angle

$$\theta = \frac{\pi}{cv^2} \int_0^1 v_p{}^2 v_g \cos\theta dl$$

which can be rewritten as

$$\theta = 8.12x10^3 \lambda^2 \int_0^1 n_e B_{||} dl$$

where λ is the wavelength and $B_{||}$ is the component of the magnetic field along the line-of-site.

The quantity

$$RM = 8.12x10\ 3\int_0^1 n_e B_{||} dl$$

is called the Rotation Measure, measured in radians per m^2.

The average magnetic field, weighted by the electron density along the line-of-sight, is computed from

$$\langle B_{||} \rangle \propto \frac{\int n_e B_{||} dl}{\int n_e dl} = \frac{RM}{DM}$$

$RM > 0$: the average magnetic field is directed towards us.

$RM < 0$: the average magnetic field is directed away from us.

<u>6. Supernovae and their remnants</u>

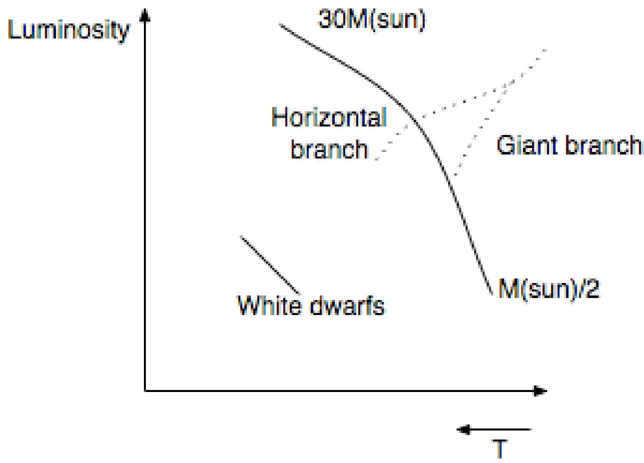
→ Bosons and Fermions

→ stars and stellar evolution

- A supernova is the explosion of massive stars.

- Important; birth events of white dwarfs, neutron stars and black holes.
→ X-rays from accretion around these objects
- Explosions are powerful sources of heating for ISM. (they liberate $10^{46} J$ into their surroundings, and ionize atoms. Shock waves radiating from the central point.)
- Intensive radio and X-ray sources
- Synchrotron emission of relativistic electrons
- Origin of most heavy elements in nature.
- They are probably the origin of high-energy cosmic rays.

The evolution of a star can be described on a HR diagram.



- During "normal" life on the MS, radiation produced by nuclear fusion balances gravity.

$L \propto M^4$

$t_{life} \propto M^{-3}$

We start with a reservoir of gas. This condenses until nuclear reactions are triggered to form stars, and spherical bodies are formed.

Stars that form with around 1/20 of the mass of the sun are too faint to see, and have insufficient mass to start nuclear reactions.

Ones above between 1/20 to 1/2 the solar mass have nuclear reaction. They burn hydrogen into helium, and are main sequence stars. This will last for a long time in many cases, and eventually the inner part is just helium and the outer part will expand to form a red giant. The red giants start to expel material into outer space, that gets passed back to the reservouir of gas around the stars. At the end, all that is left is the helium core that makes up a white dwarf. This has is fairly dense, and does not have any nuclear reactions. It has a high temperature to start with, but cools off over time.

Stars greater than around 8 masses of the sun will follow the same sequence, but will be quicker, and after they become red giants they explode in supernovae, again sending out gas back into the surroundings. This forms a neutron star, and fades away.

If the star is much more massive, somewhere above 15 solar masses, it will pass through the MS phase much quicker, possibly even just passing straight into the red giant stage, and the supernova will arrive much more quicker. The neutrons in the centre of the star are insufficient to hold up the mass, and a black hole is formed.

Main Sequence lifetimes are;

$$t = 9x10^9 \, years \left(\frac{M}{M_\odot} \right)^{-3}$$

Luminosity

$$L \sim 4x10^{26} W \left(\frac{M}{M_\odot} \right)^4$$

6.1 Main sequence evolution

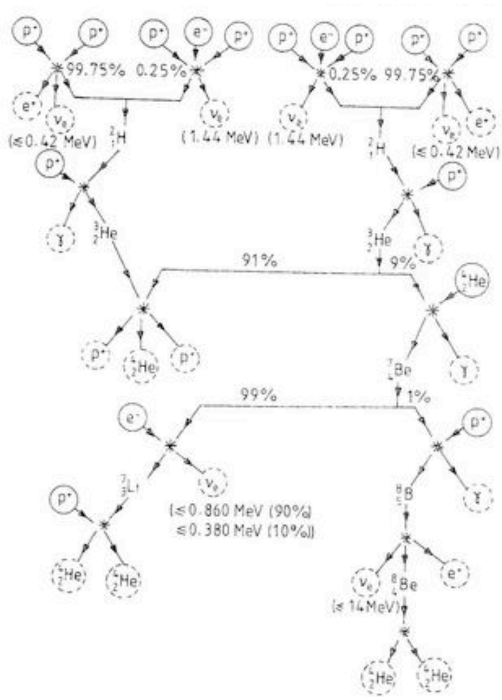
$M < 1.5M_\odot$, hydrogen is changed to helium in the "pp chain".

$$p + p \rightarrow {}^2_1H + e^+ + \nu_e$$

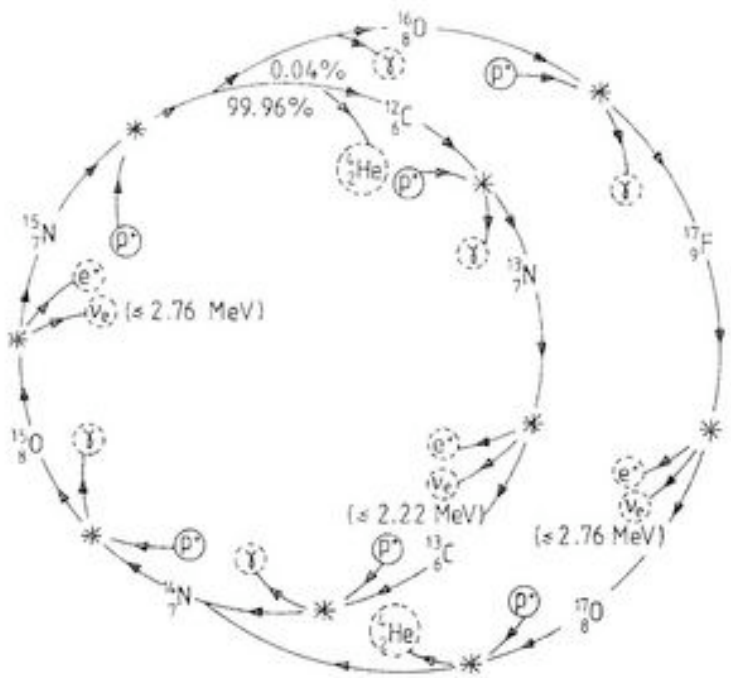
$${}^2_1H + p \rightarrow {}^3_2He + \gamma$$

$$2\,{}^3_2He \rightarrow {}^4_2He + 2p$$

… and back to start with the protons.



A second cycle is the CNO cycle. This occurs in stars that are greater than $1.5M_\odot$.



A typical MS star ends up consisting of a helium core, with a hydrogen outer. As there is not enough heat to fuse further, the star will collapse. This will then provide additional heat, which can trigger off higher fusion (helium → carbon) depending on the mass, while the shell fuses hydrogen into helium.

$M < 8M_{\odot}$, the outer layers will expand. The helium core will not be hot enough to ignite, so you end up going into the red giant phase.

6.2 White Dwarfs

Typical white dwarf properties;

Property	Earth	Sirius B	Sun
Mass (M_{sun})	$3x10^{-6}$	0.94	1
radius (R_{sun})	0.009	0.008	1.00
Luminosity (L_{sun})	0.0	0.0028	1
Surface temp (k)	287	27000	5770
Mean density $\left(g/cm^3 \right)$	5.5	$2.8x10^6$	1.41
Central temp (k)	4200	$2.2x10^7$	$1.6x10^7$
Central density $\left(g/cm^3 \right)$	9.6	$3.3x10^7$	160

For a helium white dwarf, $M < 1.46M_{\odot}$ and density $6x10^9\,kgm^{-3}$.
Under such temperatures, the matter becomes degenerate. The gas of electrons (fermions) provides the support for the star. Thus, we have the Pauli-principle, which leads to a pressure called the "Fermi pressure" (related to Fermi energy).
Origin of this pressure is from Heisenberg's uncertainty principle

$$\Delta x \Delta p \leq \frac{\hbar}{2}$$

If we pack all electrons, we reduce Δx, hence we get a momentum

$$\Delta p \geq \frac{\hbar}{2}\frac{1}{\Delta x}$$

which will result in a pressure.

$$\rho = \frac{M}{V}$$

$$n = \frac{N}{V} \propto \frac{N}{(\Delta x)^3}$$

$$E = \frac{p^2}{2m} = \frac{\hbar^2}{2m_e (\Delta x)^2} = \frac{\hbar^2}{2m_e} n^{2/3}$$

where n is the number density. This is almost the Fermi energy, due to the pressure of the electrons through the Pauli exclusion principle, however the derivation was slightly inaccurate - it is actually $E_{fermi} = \frac{h^2}{2m_e}\left(\frac{3n}{8\pi}\right)^{2/3}$.

A temperature can be equated to this.

$$E_{fermi} = kT_{fermi}$$

So for a gas with a certain number density, you can obtain its' Fermi temperature. Hence for a star with a certain physical temperature, you can compare it to its' Fermi temperature, and hence find that a star is degenerate if $T < T_{Fermi}$.

$$n \sim \frac{\rho}{\mu_e m_p}$$

where μ_e is the number of protons and neutrons per electron. i.e.

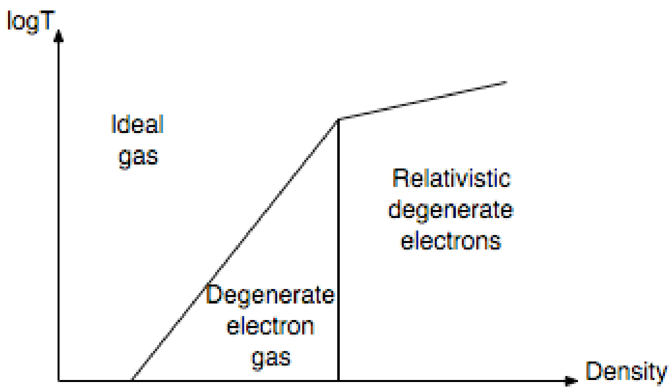
$$\mu_e = \frac{N_p + N_n}{N_e}$$

For a helium white dwarf, $\mu = 2$, hence N can be calculated, put into the first equation to find T. This $T_{fermi} = 6x10^9 k$ - this is the temperature at which the star will begin radiating thermally. Also, $E_F \approx 8.7x10^{-14} J$.

For the sun, the Fermi temperature is around 1000k - and the sun's a lot warmer than that. So the electrons act as a gas.

There are three different states of matter.
- Non-degenerate (normal)
- Degenerate, but non-relativistic
- Degenerate and relativistic

The corresponding state depends on temperature and density.



For the relativistic degenerate mass, the pressure for a given density is smaller.
If the density is too high, the gas becomes relativistic, the Fermi pressure drops and gravity can exceed the Fermi pressure. There is no way back, and the core collapses further.

The largest mass that can be sustained by Fermi pressure is the Chandrasekhar mass limit;

$$M_{ch} = \frac{5.836 M_\odot}{\mu_e^{\,2}}$$

where μ_e is again the number of particles with neutron / proton star mass per electron.

6.3 Evolution of stars with $M > 8M_\odot$

Stars with around $M > 8M_\odot$. These stars are massive enough to start burning Helium when the Hydrogen runs out, and forms heavier elements. $He_{core} \rightarrow C$, $H_{shell} \rightarrow He$. As the sizes of the atoms are larger, the cross sections are larger, so the process speeds up. This section of the process takes between $10^3 \rightarrow 10^4$ years. After this, you end up with an onion shell model, with layers of H, He, C, Ne, Mg. This happens in 10^2 years. After this, Ne → O, Mg (1 year), O → Si (1 month). Si → Fe (1 day). After this, elements can no longer fuse. The star is no longer stable, and gravity takes over - it will explode (when the outer layers of the star hit the core, they are reflected out into space. Time is around 1 second.)

6.4 Supernova Explosion Type II

This is the free-fall of the star once it has run out of nuclear fuel, when the mass is greater than $8M_\odot$. There is no force any more that is preventing the core from collapsing. There will eventually be a Fermi pressure due to the electrons. However, the densities become so high that an inverse β^- decay happens;

$$p + e^- \rightarrow n + \nu$$

This produces a large number of neutrons and neutrinos. (first observed during the supernova 1987A with ~ 11 neutrinos detected in Japan).

Star compacts to make a neutron star. The outer layers bounce off the surface of this - at the beginning, the incoming energy is greater than the outgoing hence the process stalls for a short time before the energy escapes to form the supernova. The release of energy is around $10^{46} J$. 99% of that is carried away by neutrinos. The other 1% is in EM radiation.

The energy given off is now sufficient to make the heavier elements, inputting the energy into the process. One specific one is ^{26}Al, which can be detected using gamma rays.

The magnitude of the explosion tends to vary, depending on the mass of the star, and is typically much lower than that from type 2.

$1/e >> 50$ days.

Hydrogen lines are present in the spectra.

6.5 Supernova Type I;

If you have a binary system consisting of a white dwarf in orbit around a MS star, the MS star will eventually change to be a red giant. The WD starts to attract matter from the MS, and a rotating accretion disk can form in a plane around the white dwarf - increasing the mass of the WD. The frictional processes heat up the accretion disc, sufficient enough to emit radiations - so we see them as X-ray binaries.

When the mass on the white dwarf exceeds the Chandrasekhar mass the white dwarf will be annihilated. The MS star will get a "kick" from this explosion. It will lose its' outer layers, probably turn into a white dwarf itself, and will fly out into the galaxy to cool for eternity.

A strongly peaked light curve of -19 absolute magnitude is a characteristic of this explosion, and indicates that the energy given off is typically around the same.

$1/e \sim 50$ days.

There is typically no H in the spectrum.

6.6 Supernova Remnants

The evolution of the remnant can be described in four phases;

Free expansion;

There is generally a compact object in the middle, and an expanding shell (the ejecta) going out into space. Typical velocities are around $10^4 kms^{-1}$. The time limit for this phase is around 100 → 1000 years. The surrounding ISM has a number density of atoms of around $n \sim 0.3cm^{-3}$, and this is carried away by the ejecta. The time limit is when the swept-up mass is the same as that of the ejecta, and indicates when the slow-down caused by this extra mass is becoming significant. The size at the end of this process is around 3 parsecs.

Adiabatic expansion;

$M_{swept} > M_{ejecta}$

Here, the expansion speeds have dropped dramatically, and this stage can last around 10^4 years. The shell is now getting irregular due to the differences in the number density of the ISM.

This phase is also called the "Blast-wave" phase or "Sedov-Taylor" phase.

Remnant is expanding adiabatically into the surroundings.

$$R = 14pc\left(\frac{E_{SN}}{10^{44}J}\right)^{1/5}\left(\frac{n}{cm^{-3}}\right)^{-1/5}\left(\frac{t}{10^4\,years}\right)^{2/5}$$

and $T_{shock} = 10^{10}k\left(\frac{E_{SN}}{10^{44}J}\right)\left(\frac{n}{cm^{-3}}\right)^{-1}\left(\frac{R}{pc}\right)^{-3}$

Further Expansion;

Expansion continues but temperature drops to below $10^6 k$ in shock front. Cooling via emission lines at X-rays and UV. A prominent shell structure is formed as material is being swept up and radiates (100,000 years or so). This phase is also called "Radiative phase".

Sub-sonic;

Eventually the shell slows down, and speeds are sub-sonic. It may continue to expand with velocities $v \leq 20km.s^{-1}$ due to conservation of momentum, but it is not observable any more.

Typical examples of the above SNR's are Shell-like SNR's which make up about 90% of all remnants.

A second class of "filled SNR"'s is also called Plerions. Plerions have a central pulsar which provides the SNR with energy and energetic particles, causing emission in the centre. The archetype is the Crab pulsar.

Only five Supernova Type II have been observed in our galaky in the last 1,000 years;

SN1006: probably having a magnitude of -9, almost 10,000 times brighter than the brightest stars!

SN1054 - the explosion created the Crab nebula. The explosion was observed by Chinese astronomers.

SN1572 - Tycho's supernova

SN1602 - Kepler's Supernova

Cas A - probably happened about 250 years ago. Was not observed, presumably it was too faint and perhaps obscured.

Observations in radio due to synchrotron emission, in the optical, UV and X-ray due to emission lines, and in soft X-rays due to hot gas and bremsstrahlung.

Despite the apparent low rate during the last 100 years, there are still estimated to be about one SN every 30 to 50 years. Not all may be observed, but obscured from view.

6.7 Neutron star

Predicted by Baade and Zwicky in 1934. First theoretical calculations were made in 1939 by Oppenheimer (of Manhatten Project fame) and Volkov. It was discovered in 1967 when Joycelyn Bell and Antony Hewish as pulsars.

Since neutrons are fermions, you can do the same analysis for neutron stars as for white dwarfs $(\mu_e = 1)$. → $M_{NS} \le 5.836 M_\odot$

Effects of general relativity and rotation modify this result. Modern calculations $M_{NS} \le 3M_\odot$. Measured values tend to be around $1.4M_\odot$. Typical size is $R \sim 10km$.

Average density; $\rho = \frac{M}{V} = 6x10^{17}\, kg\, m^{-3}$

Compare this to the nuclear density $\rho \sim 2.7x10^{17}\, kg\, m^{-3}$. Can view the neutron star as a giant atomic nucleus with around 10^{60} neutrons.

Real structure is more complicated; solid crust and liquid, super-conducting interior.

Region	R (km)	ρ (kg m^{-3})	Type
Surface layers	10	10^9	^{56}Fe, elongated atoms, crystal structure
Outer crust	$9.7 \to 10$	$10^9 \to 10^{14}$	WD matter, neutron rich nuclei, degenerate electrons
Inner crust	$9.1 \to 9.7$	$10^{14} \to 10^{17}$	Neutron rich nucleus, degenerate relativistic electrons
Neutron liquid	< 9.1	$> 10^{17}$	Super-fluid neutrons, some superconducting protons and electrons
Core	??	$> 10^{18}$??	Bare quarks??

There is probably also a layer of atmosphere made of H or He with a height of a few cm.

Important feature; neutron stars are highly magnetized.

Collapse of progenitor star with $R_* \sim 10^9 m$. $B_* \sim 10^{-2} T$. Concentrate on the flux through a sphere around the star. $\phi = B_* A_* = B_{NS} A_{NS}$ as flux is conserved.

$$B_{NS} = B_* \frac{A_*}{A_{NS}} = 10^{-2}\left(\frac{10^9}{10^4}\right)^2 = 10^8 T .$$

This is 1000 billion times stronger than the B field on earth. It is still around a million times stronger than the best magnets created on earth.

Observational evidence from X-ray cyclotron features and from pulsars.

6.8 Black Holes

A neutron star is the last stable form of a star (assuming quark stars don't exist). Beyond $2 \to 3M_\odot$ the gravity of the star is too much, and the Fermi pressure of the neutrons is not sufficient enough - it will collapse to a black hole.

Black holes are general relativistic objects but were already expected from classical physics.

Consider a light ray trying to escape the gravitational potential of a star of mass M and radius R.

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}mc^2 \ge \frac{GMm}{r}$$

$$\boxed{R = \frac{2GM}{c^2}}$$

No light can escape from an object with radius smaller than this value.

In general relativity, the same radius is derived from considering that all light is red-shifted to zero frequency for such radius.

The relation is called the Schwarzschild radius.

$$r_g = \frac{2GM}{c^2} \approx 3\left(\frac{M}{M_\odot}\right) km$$

This radius can be considered as the radius of a black hole.

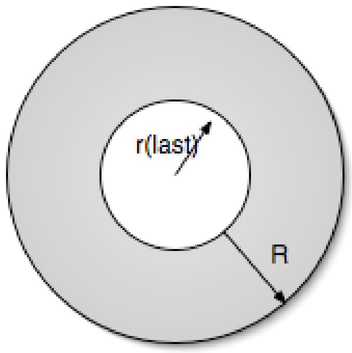
For a rotating black hole (a Kerr Black Hole);

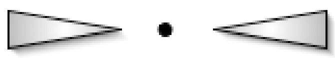
$$r_+ = \frac{r_g}{2} = \frac{GM}{c^2}$$

Near a black hole, "everything" has to rotate with the black hole, even space-time. This is called frame dragging (see information on Gravity Probe B, launched to measure frame-dragging around the earth.)
For any orbital motion about a black hole, there is a minimum radius you can be at before you are dragged into the black hole. This is the last stable orbit, with a radius of r_{last}. For r greater than this, the orbit is possible. Less than this, the mass falls into the black hole.

For a non-rotating black hole, $r_{last} \approx 3p_g = 3\frac{2GM}{c^2}$

Important for accretion process;





Disk of mass M

r_{last} = the inner radius of the accretion disk.

For maximally rotating black hole, $r_{last} = r_+$ for co-rotating. $r_{last} = 9r_+$ for counter-rotating orbit.

7. Accretion

Some of the most powerful events in high energy astrophysics include accretion, which is the addition of mass onto a central object - either a neutron star or a black hole - under the influence of gravity. The gain in gravitational energy is converted into radiation.

7.1 Accretion Efficiency

During a fall onto the central mass M, the mass m acquires kinetic energy that is taken from the gravitational energy.

$$E = \frac{1}{2}mv^2 = \frac{GMm}{r}$$

At the "surface" of M, i.e. $r = R$, kinetic energy is converted into heat and hence radiation.

If $\dot{m} = \frac{dm}{dt}$ is the mass accretion rate, assume that all the energy is radiated. Then the luminosity is;

$$L = \frac{dE}{dt} = \frac{1}{2}\dot{m}v^2 = \frac{G\dot{m}M}{R}$$

Use Schwarzschild Radius $r_g = \frac{2GM}{c^2}$ to write $GM = \frac{r_g c^2}{2}$.

$$L = \frac{1}{2}\dot{m}c^2\left(\frac{r_g}{R}\right)$$

Write this as $L = \xi\dot{m}c^2$

with ξ as the efficiency $\xi = \frac{1}{2}\frac{r_g}{R}$

Compute efficiency, white dwarf, neutron star.

$$\xi = \frac{1}{2}\frac{r_g}{R} = \frac{1}{2}\left(\frac{3}{R(km)}\right)\left(\frac{M}{M_\odot}\right)$$

For a white dwarf; $M = 1.4M_{\odot}$. $R = 5000km$. $\xi = \frac{3x1.4}{2x5000} = 4x10^{-4}$.

For a neutron star; $M = 1.4M_{\odot}$. $R = 10km$. $\xi = \frac{3x1.4}{2x10} = 0.21$

For the black hole, use the last stable orbit around it as the radius. $R = r_{last}$.

$$\xi = \frac{1}{2}\frac{r_g}{R} = \frac{1}{2}\frac{r_g}{3r_g} = 0.17$$

Efficiency for a black hole and a neutron star is roughly the same, around 0.2.
Compare this to the efficiency of the pp chain in the sun, which is $\xi_{pp} = 7x10^{-3}$.

7.2 Eddington Luminosity

From $L = \xi \dot{m} c^2$ one could expect an infinitely large luminosity if we make $\dot{m}$ large enough. However, the more luminosity we have the more photons and these photons produce a radiation force which will eventually balance the gravitational force pulling in the matter. As a result, the luminosity will be limited. The maximum value is the "Eddington Luminosity" and is obtained when;

$$F_{grav} = F_{radiation}$$

Consider an electron-proton pair falling onto the central object. Write down the gravitational force on the pair.

$$F_{grav} = \frac{GM}{r^2}\left(m_p + m_e\right) \approx \frac{GMm_p}{r^2}$$

The radiation force F_{rad} is provided by Thompson scattering, where photons exert the pressure mostly on electrons.

$$F_{rad} = \frac{\Delta P}{\Delta t}$$

where ΔP is the momentum change in time interval Δt.

Each single photon has an energy $h\nu$ and momentum $p_1 = \frac{h\nu}{c}$.

Total number of photons N_{ph} per time interval,

$$\frac{N_{ph}}{\Delta t} = \frac{L}{h\nu} f$$

The factor f describes the fraction of photons interacting with electrons. It has to decrease with the radius r due to the density of photons falling as you go out. It also has to depend on the size of the electron, hence the Thomson cross-section.

$$f = \frac{\sigma_T}{4\pi r^2}$$

$$F_{rad} = \frac{\Delta p}{\Delta t} = \frac{N_{ph} p_1}{\Delta t} = \frac{Lh\nu\sigma_T}{4\pi r^2 ch\nu} = \frac{L\sigma_T}{4\pi cr^2}$$

$$F_{grav} = F_{rad}$$

$$\frac{GMm_p}{r^2} = \frac{L\sigma_T}{4\pi cr^2}$$

Solve for L

$$L_{edd} = \frac{4\pi GMm_p c}{\sigma_T} = 1.3x10^{31}\left(\frac{M}{M_{\odot}}\right)W$$

This is the maximum value for the assumptions of steady and spherically symmetric accretion.

Summery;

Accretion produces photons. The more luminosity, the more photons, which produce radiation pressure and hence limiting the mass flow. Balance gives the max possible value.

7.3 Applications of the Eddington Luminosity

1) Accretion rates

The maximum value is given by the rate producing L_{edd}. But not all objects equally efficient.

$$L_{edd} \geq L_{acc} = \xi \dot{m} c^2$$

$$\dot{m} \leq \frac{L_{edd}}{\xi c^2}$$

Use typical efficiency $\xi \sim 0.2$ and $M = 1M_\odot$

$$\dot{m} \leq \frac{1.3x10^{31}}{0.2c^2} \approx 5x10^{14}\, kgs^{-1} = 2x10^{22}\, kg\, yr^{-1}$$

Or $\dot{m} \leq 10^{-8} M_\odot yr^{-1}$

This is for a steady case.

2) Mass estimate

Compare L_{obs} with L_{edd}. Assume typical ξ → estimate for M.

Consider an X-ray binary with $L_{obs} = 4x10^{30} W$.

Given $L_{edd} = 1.3x10^{31}\left(\frac{M}{M_\odot}\right)W$ and $\xi = 0.2$

$$L_{\exp} = 0.2L_{edd} = 2.6x10^{30}\left(\frac{M}{M_\odot}\right)$$

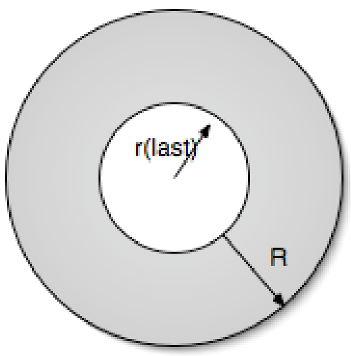
Hence;

$$\frac{M}{M_\odot} = \frac{4x10^{30}}{2.6x10^{30}} = 1.53$$

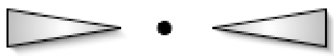
This is in the range of a neutron star.

3) Radius of accretion disk

Neglect r_{last}. Let R be the radius of a thin accretion disk.



Disk of mass M



$$L_{obs} = 2x\pi R^2 + \sigma T^4$$

The first part is for the top and bottom of the disk.

If we get the spectrum, we know from Wein's law

$$\lambda_{\max} T = 3mm$$

→ $T = \sigma T^4$

→ radius of disk.

4) Temperature / expected spectrum

Assume black body.

$L_{acc} = 4\pi R^2 \sigma T_{BB}{}^4$

(assuming spherically symmetric)

First estimate; $T_{bb} = \left(\dfrac{L_{acc}}{4\pi R^2 \sigma} \right)^{1/4}$

Assume that all gravitational energy is converted into thermal energy.

$$\frac{G\left(m_p + m_e\right)M}{R} = \frac{Gm_pM}{R} = 2x\frac{3}{2}kT_{th}$$

The final part has a 2 in it due to there being both a proton and an electron.

Second estimate;

$$T_{th} = \frac{Gm_pM}{3kR}$$

Third estimate;

$h\nu = kT_{rad}$

$T_{bb} \le T_{rad} \le T_{th}$

For a white dwarf; $M = 1M_{\odot}$. $R = 10^7 m$. $\xi = 4x10^{-4}$

$L_{acc} = \xi L_{edd} = 5x10^{27} W$

$\rightarrow T_{bb} = 8x10^4 k$

Also $T_{th} = 5x10^8 k$

$8x10^4 \le T_{rad} \le 10^8 k$

→ $7eV \le h\nu \le 43keV$

optical ≤ ≤ X-ray

NS; $1KeV \le h\nu \le 70MeV$

X-ray Gamma ray

7.4 Accretion disks

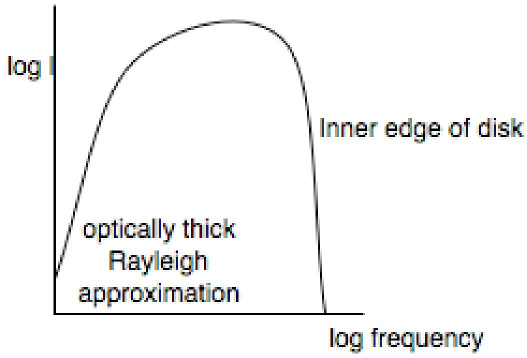
Accretion can occur in two kinds;

- Bondi-Hoyle accretion: often in high mass x-ray binaries, when the central object is located in the wind region of the mass donor star, Eddington luminosity is a good approximation since matter falls in from all sides.
- Accretion disks; if the companion star evolves it will eventually overflow the "Roche-Lobe" (the line of constant gravitational potential that is common to both stars, where matter can flow force-free from one star to the other. The point of interception between the two fields is called "Langrengion point" L1.)

Infalling matter has angular momentum $\underline{L} = \underline{r}x\underline{p}$ or $L = mv_{\perp}$.

If angular momentum is conserved, $E_{rot} = \frac{1}{2}\frac{L^2}{I} \propto r^{-2}$.

→ matter forms a rotating disk where E_{rot} (and L) is dissipated as head.



The inner radius, r_{in} of this disk is the last stable orbit in the case of black holes.
Currently, treatment of only thin disks is possible analytically. Thick disks cannot be solved yet.
The luminosity of an annulus of size Δr in distance r;

$$L_{disk}(r) = \frac{3G\dot{m}M}{2r^2}\left[1-\left(\frac{R}{r}\right)^{1/2}\right]\Delta r$$

Estimate temperature of the disk;

$$T = \left(\frac{3G\dot{m}M}{8\pi\sigma r^3}\right)^{1/4} \propto r^{-3/4}$$

so that it is hottest at the inner edge of the disk.
Assuming that the emission is optically thick, we obtain total spectrum;

$$I(\nu) \propto \int_{r_{in}}^{r_{out}} 2\pi r B(T,\nu)\,dr \propto \nu^{1/3}$$

for frequencies corresponding to the inner and outer edges of the disk.
For smaller frequencies, we expect the normal Rayleigh approximation of an optically thick plasma, while at higher frequencies we see a cut-off, since the frequencies at the inner edge are the highest we observe from the disk.

8. X-Ray Binaries

8.1 Types of X-ray binaries
Wide variety; High mass X-ray binaries (HMXB's) and low mass X-ray binaries (LMRB's). There are also X-ray pulsars, X-ray bursters, X-ray transients, … In all of these cases; central object (usually neutron star or black hole) that accretes matter from a companion.

Binary periods are typically a few hours, but it can be less.

8.1.1 HMXB's
The companions are massive O- or early B-star. They are young with lifetimes of around 10^7 years. We have no X-ray bursters in this category.

8.1.2 LMXB's
Companions are "light" and cool. Main sequence stars or white dwarfs. Evolving slowly → old population, e.g. found in Globular clusters.

8.2 X-Ray Pulsars
8.2.1 Neutron Stars as central objects

In addition to steady emission from accretion disk, some LMXB's and HMXB's are also showing X-ray pulses, periods from seconds to around 15 minutes.

X-ray pulses are due to the rotation of the neutron star. The neutron star has a strong magnetic field, which prevents matter from falling straight onto the surface → the matter is channeled along the field lines onto the magnetic poles. At this point, it produces the x-rays.

Binary motion can be detected in x-ray periods → leads is to mass estimates using Keppler's laws, in particular if the orbit inclination is known from eclipses.
Sometimes, additional variation occurs due to the contribution of one-sided heating of the companion from the x-rays.

8.2.2 Black hole as central object
If compact object with $M \geq 3M_{\odot}$, no hard surface, no B-field structure. No distinct period should be observed. We do however often observe a flickering in the x-rays outputted that is not just noise. We can estimate the expected time scale for flickering from light-travel time arguments across the accretion disk with the last stable orbit.

For non-rotating black hole, the last stable orbit was given by $r_{last} = 3r_g$. So $t \geq t_{\min} = \frac{r_{last}}{c} = \frac{3r_g}{c} = 2x10^{-5}\left(\frac{M}{M_{\odot}}\right)s$.

For say $M = 15M_{\odot}$, $t \geq 0.5ms$ e.g. observed for Cygnus X1.

For super-massive black holes, say $10^8 M_{\odot}$ → $t \geq 3x10^3 s$, ~ 1 hour.

8.3 HMXB
Accretion can occur in two kinds: one is via the discussed accretion disk, the other is called Bondi-Hoyle accretion: the companions in MHXB often have strong stellar winds an the central object may simply be located in the wind, accreting matter.
Accretion disks;
If the companion star evolves, it may overflow its' Roche-Lobe feeding matter into accretion disk.
Roche lobe;
If we look at lines of gravitational potential in binary system we find one potential line where both potentials become connected. Matter moving on this particular potential line can move from one companion to the other without additional force. This line is called Roche-Lobe. The point of intersection is called the Lagrangian point L_1.
When the star evolves, it may expand to its Roche lobe and matter can be transferred to companion.
Roche-Lobe overflow is also important for LMXB's.
HMXB's are progenitors of double neutron star systems.

8.4 LMXB
Similar Roche-Lobe overflows as in HMXB's.
Some LMXB's show X-ray bursts
- If matter flow is weak, i.e. accretion rate is low, and magnetic field is low, then matter may not need to be channeled onto poles.
- Slowly, a layer of hydrogren builds.
- Eventually, hydrogen is ignited in a thermonuclear explosion on the surface.
- The sudden increase in X-ray luminosity lasts a few seconds or less and disappears if all H is burnt.
- Eventually the process repeats itself.

For an X-ray burst to happen, magnetic field must be low. Hence X-ray bursts are never observed in X-ray pulsars. Also, the accretion rate much be low - hence they aren't observed in HMXB's.
In some cases, a relative velocity of neutron star (where it accretes) and accretion disk (where matter is supplied from) leads to quasi-periodic oscillations.
LMXB's are the projenitors for fast-spinning millisecond pulsars.

9. Pulsars

9.1 The Discovery

Neutron stars were predicted by Baade & Zwicky in 1934. In 1967, Jocelyn Bell-Burnell was observing in Cambridge for her PhD with Anthony Hewish when she discovered pulsating radio sources. A journalist later called them pulsars.
Pulsars are rotating neutron stars with a radio beam along the magnetic axis which is inclined to the rotational axis.
Whenever the beam crosses our line-of-sight, we observe a pulse; rotational period = pulse period.
The energy of the radiation is taken from the rotational energy, hence the pulsar is slowing down.
The Crab pulsar was the fastest rotating pulsar known until 1982, when a peculiar pulsar was discovered that rotated with a period of 1.56 milliseconds.
Today, we know more than 100 pulsars with millisecond period: millisecond pulsars.

9.2 Evolution

Pulsars are born with a period of 10-30ms or so, and spin down fast. We describe the spin-down by measuring the chance in period

$$\dot{P} = \frac{dP}{dt}$$

Most of the pulsars' period are around 1 second, with period derivatives of 10^{-15}. The longest period known is 8.5 seconds; if the period becomes too large the radio emission ceases.
We assume that the energy lost is mostly radiated as magnetic dipole radiation.

$$\frac{dE}{dt} = -\frac{\mu_o \left|\ddot{\underline{P}}\right|^2}{6\pi c^3}$$

$$\underline{P} = \underline{P}_o \sin\omega t$$

Related to magnetic field (at the surface).

$$B \approx \frac{\mu_o P_o}{4\pi R^3}$$

Normally take $R \approx 10km$, the standard radius of a neutron star.

$$\ddot{\underline{P}} = -\omega^2 \underline{P}_o \sin\omega t$$

$$\frac{dE}{dt} = -\frac{\mu_o \omega^4 P_o^{\,2}}{6\pi c^3}$$

This power is taken from rotational energy $E_{rot} = \frac{1}{2} I\omega^2$ where I is the moment of inertia $I = \frac{2}{5} mr^2$ for a sphere.

$$\frac{dE}{dt} = \frac{dE_{rot}}{dt} = \frac{d}{dt}\left(\frac{1}{2} I\omega^2\right) = I\omega\dot{\omega}$$

$$\dot{\omega} = -\frac{\mu_o P_o^{\,2}}{6\pi c^3 I}\omega^3$$

→ $\dot{\omega} = -k\omega^3$

with $k = \frac{\mu_o P_o^{\,2}}{6\pi c^3 I}$ and $n = 3$ the "braking index".

$$\frac{d\omega}{dt} = -k\omega^n$$

$$\left(k = -\frac{\dot{\omega}}{\omega^n}\right)$$

Through separation of variables;

→ $t = \int dt = \frac{1}{k}\frac{1}{n-1}\left(\omega^{-(n=1)} - \omega^{(n-1)}\right)$

$n \neq 1$

Assume that $P_{birth} << P_{today}$.

$\omega_o >> \omega$

Hence we can neglect the last tem.

$t = \frac{1}{k}\frac{1}{n-1}\omega^{-(n-1)}$

Use $k = -\frac{\dot{\omega}}{\omega^n}$ → $t = -\frac{1}{n-1}\frac{\omega}{\dot{\omega}}$

with $\dot{\omega} = -\frac{2\pi}{P^2}\dot{P}$ $\omega = \frac{2\pi}{P}$

→ $t = \frac{1}{n-1}\frac{P}{\dot{P}} =_{n=3} \frac{P}{2\dot{P}}$

→ $\tau = t = \frac{P}{2\dot{P}}$

Example;

$p \sim 1s$

$\dot{P} = 10^{-15} s.s^{-1}$

→ $\tau = 5x10^{14} s = 15x10^{6} years$

$\tau = \frac{P}{2\dot{P}}$ is the "characteristic age".

Loss in rotational energy;

$\dot{E} = \frac{dE}{dt} = 4\pi I \frac{\dot{P}}{P^3}$

This is sometimes called the spin down luminosity

With $P \sim 1s$ and $\dot{P} = 10^{-15}$ → $\dot{E} = 4x10^{23} W$!

So with $I = \frac{2}{5}mR^2 = 10^{28} kg.m^2$;

$P_o = \sqrt{\frac{3c^3 mR^2}{5\pi\mu_o}P\dot{P}}$

→ insert into B;

$B = \frac{\mu_o P_o}{4\pi R^3} = \sqrt{\frac{3c^3\mu_o MR^2}{80\pi^3 R^3}}\sqrt{P\dot{P}}$

$M = 1.4M_{\odot}$ $R = 10km$

$B = 3.2x10^{-15}\sqrt{P\dot{P}}$

With $P \sim 1s$ and $\dot{P} \sim 10^{-15} s.s^{-1}$

→ $B \sim 10^{8} T$

as seen through measurements.

9.3 Millisecond pulsars

These are born in X-ray binaries when matter is accreted and "dead" pulsars are recycled by being spun up to millisecond periods.

For pulsar B1987+21; $P = 1.56ms$, $\dot{P} = 10^{-19}$

→ $\tau = 2.5x10^{8} years$, $B = 3.2x10^{15}\sqrt{p\dot{p}} = 3.7x10^{4} T$

→ millisecond pulsars are old pulsars. B field is reduced due to accretion ("burial of B-field under matter")

~80% of all MSP's are in binary orbits (in contrast to only about 5% for non-recycled pulsars).

Matter cannot flow onto magnetized neutron star easily (see discussion about X-ray pulses).

If the B-field pressure is too strong, no accretion.
Equate B-field pressure "ram pressure" with pressure from the in-falling matter.

$P \geq P_{\min} = 2B_5^{6/7} ms$

where B_5 is the B-field in units of $10^5 T$ limiting period.

→ $\dot{P} \leq 2x10^{-15} P^{4/3}$
for all recycled pulsars

All MSP's are below the "spin-up line".

Summary;
- Pulsars are highly-magnetized neutron stars. They emit a coherent radio beam along the magnetic axis. Emission mechanism is not understood.
- Estimate the magnetic field assuming that the rotational energy lost due to magnetic dipole radiation.

$B = 3.2x10^{15} \sqrt{p\dot{p}}$

where p is the period in seconds, and $\dot{P} = \frac{dP}{dt}$ $\left(s\,s^{-1}\right)$.

$\dot{P} > 0$ slows down.
→ age "characteristic age".

$\tau = \frac{P}{2\dot{P}}$

Evolution;
$P\dot{P}$ diagram.
L-1
Recycling; spinning-up of neutron star by accretion.
Two types.
LMXB; slow evolution of the companion → long mass transfer → short final periods. Result is usually a pulsar with a white dwarf companion. "1-15ms"
HMXB: Fast evolution of the companion. Short mass transfer. Long-ish periods "30-50ms". Companion eventually explodes in a supernova, usually; binary system disrupted. Sometimes the binary system survives, as a double-neutron star-system.

The spin-up line gives a relationship between P and B, or (using $B \propto \sqrt{P\dot{P}}$), P - $\dot{P}$. It can be shown that all MSP must lie below the spin-up line because it's derived from condition of max-possible mass transfer given by a magnetic field.

(9.3.1, 9.4, 9.5 here?)

10. Active Galaxies

- See lecture course about galaxies

Usually, you have a confusing number of observations, presentation is driven historically. Here we will start with the model, and look at some of the observations.
Milky way: $L \sim 10^{37} W$.
Active galaxy: $L \sim 10^{36} - 10^{42}$.
It turns out that the luminosity of the active galaxies is dominated by a bright core, therefore we talk about Active Galactic Nuclei (AGN) in particular.

10.1 AGN-Uniformed Scheme

(L-2)
- Supermassive black hole with $10^6 \rightarrow 10^9 M_{\odot}$
- Accretion disk
- Obscuring dust Taurus
- Fast-moving gas clouds near accretion dish. These will produce broad Doppler-shifted lines of emission → Broad-line region (BLR)

- Slow-moving gas clouds further away. These will produce narrow Doppler-shifted lines. → Narrow-Line Region (NLR)
- Radio jet (emitting, fast-moving particles, collimated by the magnetic field of the black hole). The formation of this jet is not really understood.
→ many different "types" of AGN's are simply a geometric effect, i.e. they depend on the observable parts of the AGN from our direction.

Quasars;
These were the first identified members of the AGN class. They appear to be a blue star, as they have a lot of UV emission (we basically see the high-energy emission from the accretion disk. They are very far away with the largest redshift (up to $Z \geq 6.4$). The host galaxy is barely detectable, as the jets are so bright they out-shine the rest of the galaxy. "Quasar fuzz".
Continuous non-thermal radiation from synchrotron-self compton (i.e. synchrotron photons that are inverse-Compton scattered to high energies by the same electrons producing them in the first place.
Thermal emission from accretion disk ("blue bump").
Only 10% have detectable radio emission. Both the broad and narrow emission lines can be seen.
Examples; 3C273, 3C48.

Seyfert I;
Quasar-like nuclei, but here we can clearly detect the host galaxy as a spiral galaxy. We also see the broad and narrow line regions.

Seyfert II:
As Seyfert I galaxy, but only see narrow-line regions.

Radio galaxies;
These are similar to radio-loud Seyfert galaxies. They have very prominent jets (due to synchrotron emission) that extend to huge distances (several Mpc).
Examples: Cygnus A, Centaurus A, Virgo A.

BL Lac's:
Also called Blazars. First source BL Lacterae. First thought to be a variable star, but it turned out that they were looking directly into a jet. Spectral peak in the IR $\left(10^{14} Hz\right)$, which can be inverse-Compton scattered up to γ -ray.
Variation on time scales of hours or less. increase by a factor of 100 within weeks (an outburst).
Hosts are elliptically galaxies.

10.3 Jets and Superluminal motion
- Jet emission = synchrotron emission (polarised).
- Observed intensity is too high.
- Intensity is boosted by relativistic beaming.
- In some AGN jet components ("blocks") seem to move with an observed velocity $v_{obs} > c$, or $\beta_{obs} > 1$. "Superluminal motion". → not real but a consequence of the relativistic motion and geometry.

$$v_{obs} = \frac{v \sin\theta}{1-\beta\cos\theta}$$

$$\beta_{obs} = \frac{\beta \sin\theta}{1-\beta\cos\theta}$$

$\beta = \frac{v}{c}$ =real velocity

What is $\beta_{obs}{}^{max}$?

Compute $\frac{d\beta_{obs}}{d\theta}=0$ → $\cos\theta=\beta$ for maximum.

What is the value for $\cos\theta=\beta$?

$$\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\beta^2}$$

$$\beta_{obs}{}^{\max}=\frac{\beta\sqrt{1-\beta^2}}{1-\beta^2}=\frac{\beta}{\sqrt{1-\beta^2}}=\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}\beta=\gamma\beta$$

When does $\beta_{obs}{}^{\max}=\gamma\beta>c$?

$$\gamma\beta>c$$

$$\beta>\sqrt{\frac{1}{2}}$$

$$v>0.7c$$

Superluminal motion is observed if $\beta=\cos\theta$ and $v>0.7c$.

If superluminal motion is observed, then the geometry is constrained → helps with interpreting the data.

In geometry where we may look towards the jet (θ is small), the other side's jet ("counter-jet") may be boosted away from us, and hence will be much fainter.

Questions;

1) Basics

a) $E=h\nu$, $1eV=1.6x10^{-19}J$

b) Black-body radiation $B(T)=\frac{2h\nu^3}{c}\frac{1}{e^{h\nu/kT}-1}$

c) $S=\sigma T^4$, $L=4\pi R^2\sigma T^4$ $\sigma=5.67x10^{-8}Wm^{-2}k^{-4}$

d) Maximum in Planck function $T\lambda_{\max}=0.0029k.m$

e) Optical depth τ. $I_\nu=T_oe^{-\tau_\nu}$. Optically thick if $\tau>>1$, optically thin $\tau<<1$.

f) $E=\gamma m_oc^2$. $E_{kin}=(\gamma-1)m_oc^2$.

g) Doppler boosting is when everything is beamed forward into a cone of angle γ^{-1}. Intensity Is enhanced.