0. Preliminaries

0.1 Books

Zeilik & Gregory, Introductory Astronomy and Astrophysics

0.2 Size, Mass and Time

0.2.1 Sizes

S. I. Unit: Metre

Three units in common use:

1) AU (Astronomical Unit)
   - Mean Earth-Sun Distance
   ≈ 150mn. km – 1.5×10^{11}m
2) Light Year
   - Distance travelled by light in 1 year
   ≈ 9×10^{15}m
3) Parsec
   - Definition later!
   ≈ 3.26ly≈3×10^{16}m

Some distances:

- Moon: 1.3 Light Second
- Venus: 3 Light Minutes
- Sun: 8-9 Light Minutes
- Pluto: 5-6 Light Hours
- Nearest Star: a few Light Years
- Centre of Galaxy: 30,000 Light Years
- Size of Galaxy: 10^5 Light Years
- Nearest Galaxy: 2×10^6 Light Years
- Visible Universe: 10^{10} Light Years

0.2.2 Masses

SI Unit kg – bit small (Some astronomers use grams!)

Astronomical unit: M_Θ - mass of sun ≈2×10^{30}kg

Mass of galaxy ≈ $10^{11} M_\Theta$

Mass of observable universe: $10^{11}M_{galaxy}$ ≈ 10^{52}kg

Density of universe: few $\times10^{-23}$km^{-1}

0.2.3 Times

SI Unit s – bit small (Astronomers use s or yr)

Time for light to travel around the solar system: few hours

Very massive star – few $\times10^6$yrs

Normal star – few $\times10^9$yrs → 10^{10}yrs

Age of universe: 10^{10}-2×10^{10}yrs

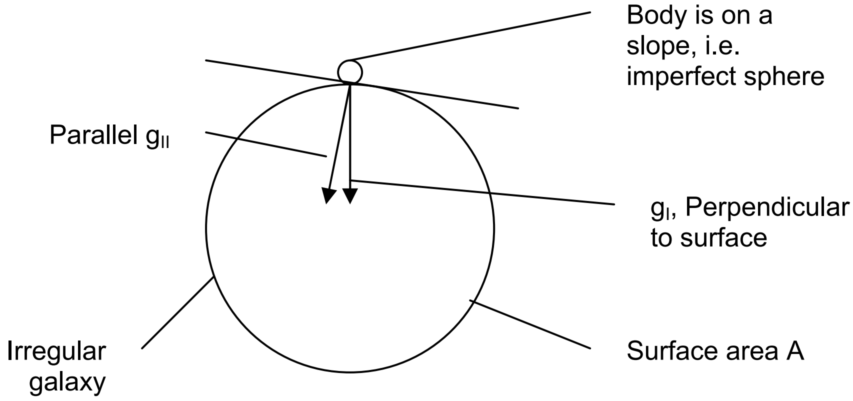
0.3 Some Physics

0.3.1 Gravitation (Classical)

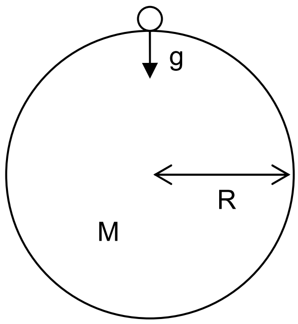
- $-F = \frac{GMm}{r^2}$
- Very weak. In atomic physics, it is always swamped by other forces unless a large mass is involved.
- It is comparable to electric forces – e.g. the coulomb equation $Q = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

  $\frac{1}{4\pi\varepsilon_0}$ is 10^{10} larger than G.
- When a mass is outside a spherical body, then all of the body's mass can be seen as a point in the centre of the body.
- Newton's theory for gravitation is an approximation. It is only valid for weak gravitational fields; when strong ones are involved, general relativity comes into play

Gauss's formulation:



Average perpendicular component (g_1) then $g_I A = 4\pi GM$



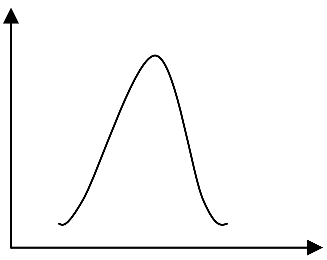
$$g.4\pi r^2 = 4\pi GM$$

$$\therefore g = \frac{GM}{R^2}$$

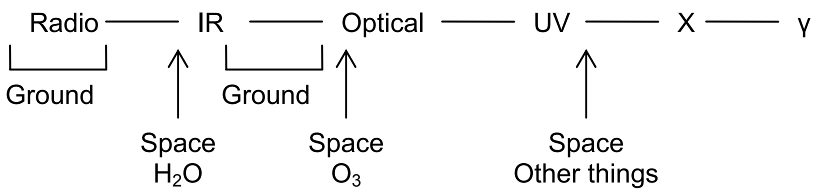
(Compare to Newton's law: $F = \frac{GMm}{R^2} = mg$)

0.3.2 Black body radiation

- Optically think thermal
- Stefan – Power/unit surface area $P_A = \sigma T_{surface}{}^4$ where σ is the Stefan constant.
- Wein – peak power at λ_{max}
  $\lambda_{max} T_{surface} = constant$ (Constant = 2.9mm.K)
- Stefan and Wein come from the plank distribution. Wm^{-2}(unit λ)-1



0.4 Observing Systems

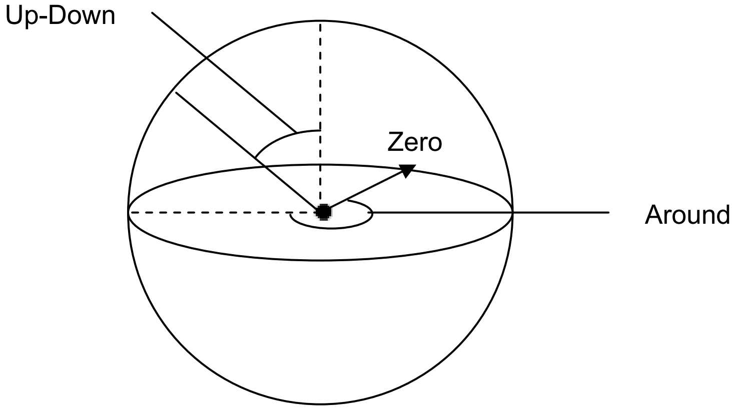


The maximum resolution of a telescope is λ/d radians
Criteria for a telescope site:

- Optical:
  - Transmission (Weather)
  - "seeing" – turbulence in atmosphere. Limits resolution to 1" (>λ/d)
- Infrared:
  - H_2O problem (Weather) – mountain is ideal to avoid this.
  - Telescopes tend to be at 280-290°K → 10μm – which is the desired viewing wavelength. This can be avoided through careful telescope design.
- Both of these need non-turbulent atmospheres → stable airflow. Also, need to be above most of the H_2O. This leads to ideal locations being on islands in mid-ocean, or edge of a continent with wind off the ocean, or on mountains in the mid-latitudes.
- Ideal locations include:
  - Canary Islands
  - California
  - Chile
  - Hawaii
- Radio telescopes also have the problems of interference and water etc. at high frequencies (>20GHz)

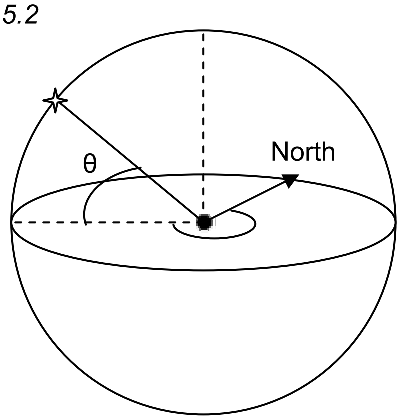
0.5 Astronomical Coordinate Systems

0.5.1 Principles



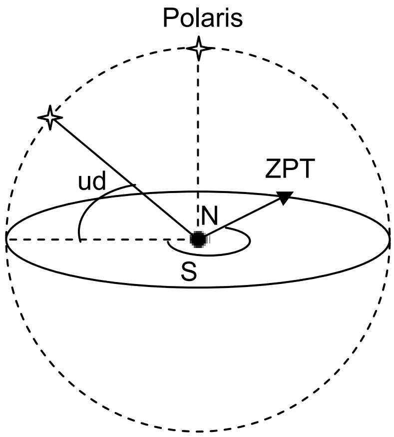
Surface of sphere: need to define:
- Plane where the "up-down" coordinate ≡ 0
- Zero point in plane – "around" coordinate ≡ 0
- (Direction for which way is positive)

0.5.2



Plane ≡ horizon
Zero-point ≡ north
Azimuth-Elevation
Around ≡ Azimuth, Up-Down ≡ Elevation.
This is simple, but has some disadvantages. It is not the same for all observers, and a different part of the sky is showing as the Earth rotates.

0.5.3



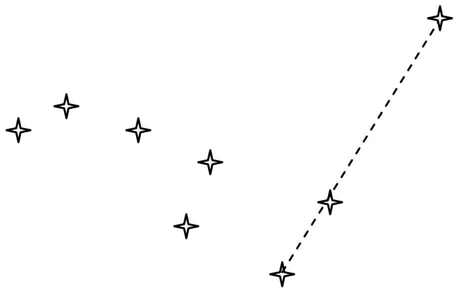
Equatorial coordinates (RA, dec)
Plane = equatorial plane.
ZPT = first point if Aries

Around = Right Ascension (Degrees/hrs)
ud=Declination (Degrees)

Hours = degrees/15

A given star always maintains the same RA and declination.
Disadvantages:
- Different az/el for a given RA and declination depending on time and observer location.
  (az-el)←→(RA,Dec) by trigonometry

Polaris happens to be at dec approx 90°.

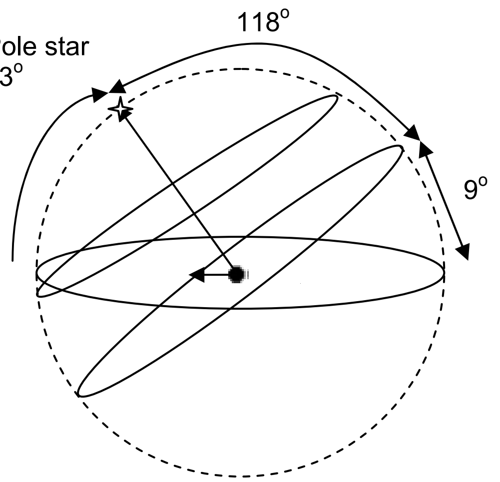


For an observer at the north pole, the elevation of Polaris is always 90°.
Observer in Manchester – lat 53°. Therefore Polaris is always at 53° from horizon regardless of where Earth is in rotation. 37° from zenith.

Any given star rorates around the sky from E→W but it always preserves the same angle away from Polaris (90-dec of star).

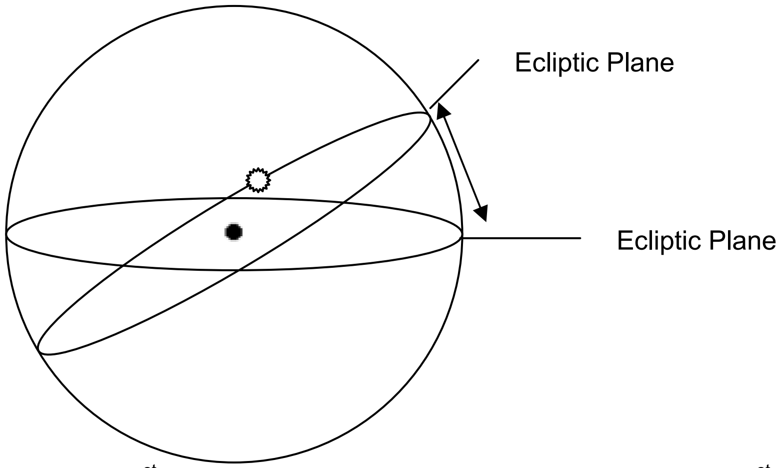
Circumpolar stars are always above the horizon. (Any star with dec > 90-latitude)

Galactic centre: dec-28° → 118° from pole star.
53+118=171° → galactic centre climbs 9° above the horizon once a day.

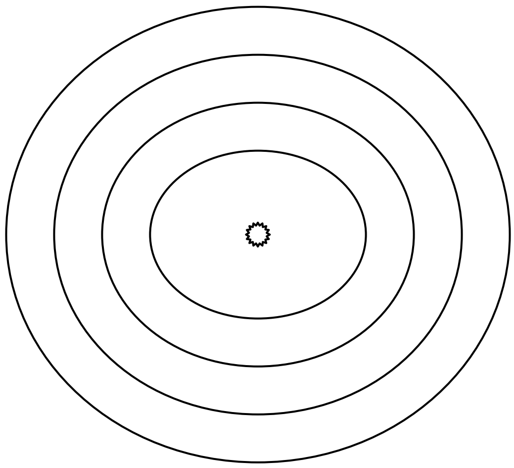


0.5.4 The Sun

- Not constant RA, Dec – sun goes round once a year.
- RA varies with one full cycle every year due to Earth's orbit.
- Declination +23.5 (June) → 0 → -22.4 (December) → 0 …

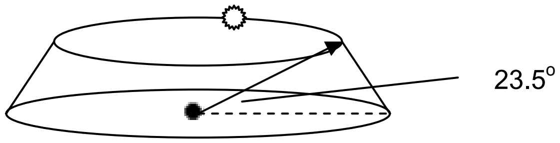


On June 21st, at the North Pole there is always sunshine. On 21st December, there is always darkness.

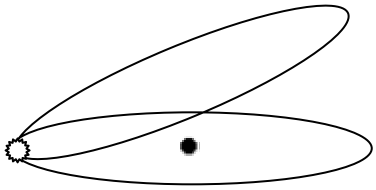


Planets orbit in the elliptic plane.

The lowest point north where it stays light for 24 hours:
At 90° latitude (North pole) on June 21st:

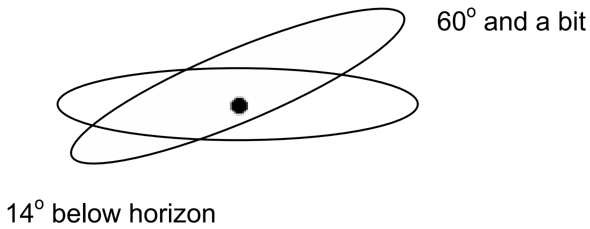


At 66.5° North, there is a 23.5° tilt from the diagram:



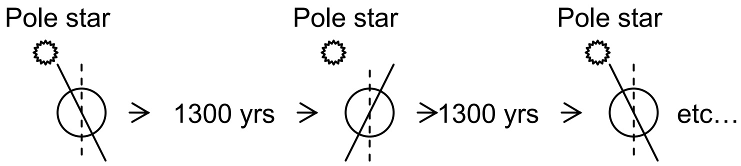
At this latitude, the sun just touches the horizon on June 21st, giving 24-hour sunlight. this is around the Arctic Circle, where you just get perpetual daylight in summer.

At 53°N (37° tilt)



0.5.5 Precession

RA and Declination of stars is not exactly fixed. The reason for this is that the Earth's axis precesses.



This is called the precession of the equinoxes. It is customary to quote the RA and Declination as they were in either 1950 or 2000.

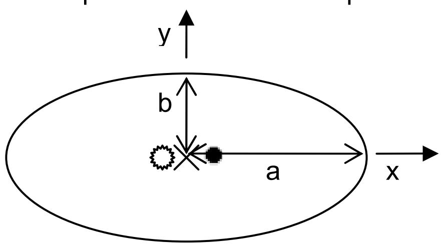
1. The Solar System

1.1 Keppler's Laws

Johannes Keppler (1571-1630)

1.1.1 First Law

"Each planet moves in an elliptical orbit with the sun at one focus."



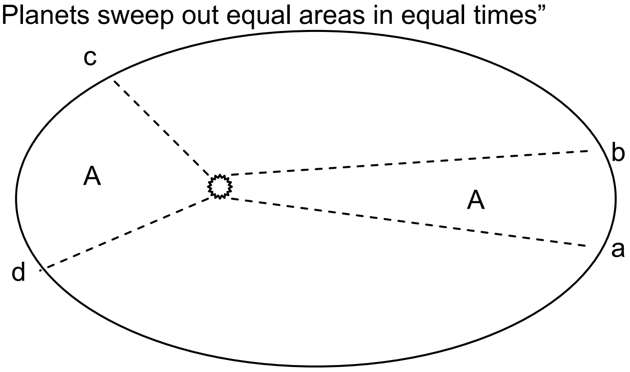
Semi-Major axis = a
Semi-Minor axis = b

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Ellipticity $b^2 = a^2(1-e^2)$ for e<1. Bigger e → more elliptical.

1.1.2 Second Law

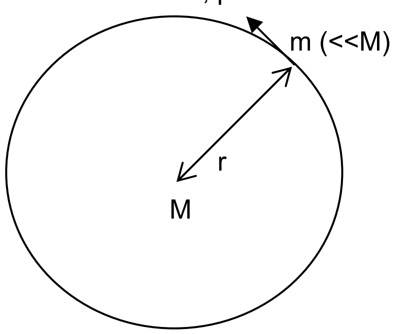
"Planets sweep out equal areas in equal times"



Equal areas → same time a→b as to go c→d. Therefore, planets move faster the closer they get to the sun.
This follows from conservation of angular momentum mvr.

1.1.3 Third Law

"If a planet has a circular orbit of radius R and an orbital period of T, then $T^2 \propto R^3$"
For circular case, provable directly from Newton.



Gravitational fore:

$$\frac{GMm}{r^2}=\frac{mv^2}{r}$$

$$\Rightarrow v^2 = \frac{GM}{r}$$

But circumference of orbit $= 2\pi r$, orbital period = T.

$$\Rightarrow v = \frac{2\pi r}{T}$$

$$v^2 = \frac{4\pi^2 r^2}{T^2} = \frac{GM}{m}$$

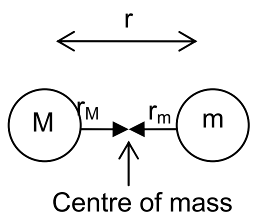
$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM}$$

→ If you want to find the mass of something, look at something which orbits it and measure T and R.

Complications:

i. For non-circular orbits, take R as a (Semi-major axis).

ii. Two similar masses



Each mass orbits around a centre-of-mass of the system.

$$Mr_M = mr_m$$

$$r = r_m + r_M$$

$$r_m = \frac{Mr}{M+m}$$

Apply Newton's laws to small body:

$$F_a = \frac{GMm}{r^2} = \frac{mv^2}{r_m} = \frac{mv^2(M+m)}{Mr}$$

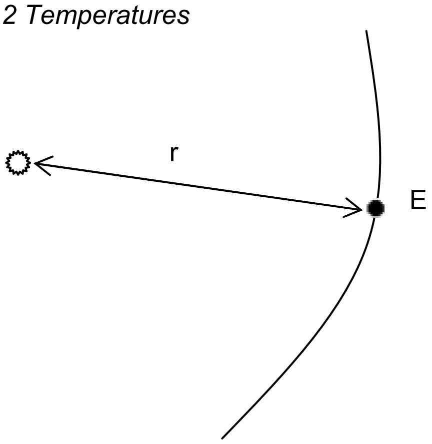
$$v^2 = \frac{4\pi^2 r_m{}^2}{T^2}$$

$$\frac{GM}{r^2} = \frac{4\pi^2 r_m{}^2}{T^2}\left(\frac{M+m}{M_r}\right)$$

$$\frac{GM}{r^2} = \frac{4\pi^2}{T^2}\frac{M_c{}^2}{M+m}$$

So to convert between the small & large system and 2 large system, you simply substitute M+m for M.

1.2 Temperatures



Radius r, area (Cross-section) πr^2

Heat intercepted by Earth = heat radiated.

The sun radiates L_o Watts into an area of $4\pi r^2$, L_o joules cross every second.

Earth gets $\frac{\pi r^2}{4\pi R^2}$ fraction of these photons.

➔ heat intercepted = $L_o \frac{r^2}{4R^2}$

A factor A is reflected immediately (Albedo)

➔ heat intercepted/sec = $\frac{(1-A)L_o r^2}{4R^2}$

Heat radiated;

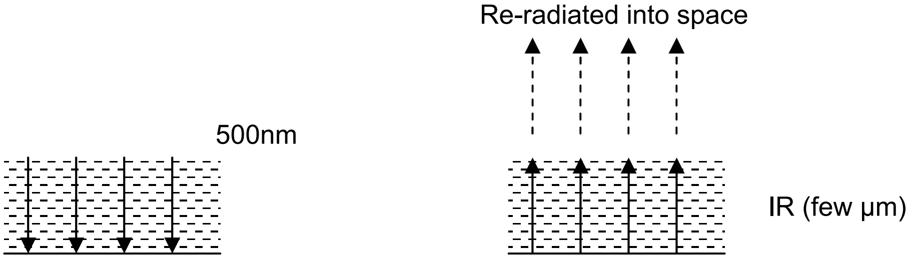
Stefan's Law: σT^4 per unit surface area.

→ total heat radiated/sec = $4\pi r^2 \sigma T^4$

$$4\pi r^2 \sigma T^4 = \frac{(1-A)L_o r^2}{4R^2}$$

$$T = \frac{1}{2\sqrt{R}} \sqrt[4]{\frac{L_o(1-A)}{\pi\sigma}}$$

This works for planets like mars – gives the temperature to the right value. However, for the Earth it gives a result which is 30K too low, and for Venus the value is 400-550K too low This is due to the planet's atmosphere (The Greenhouse Effect)



Heat is absorbed by the atmosphere, so the atmosphere warms up.

Absorbers of IR photons include CO_2, SO_2, H_2O, …

Heat absorbed from sun = heat re-radiated

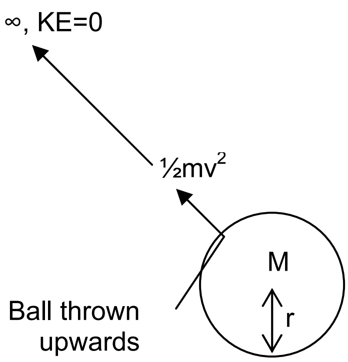
Balance broken: Earth will heat up until balance is restored.

Venus has a lot of CO_2, SO_2 etc… → 70 atmospheres. Increases Venus's temperature by about 400K.

Earth has a similar amount of CO_2, but most of it is locked up in carbonate rock.

1.3 Planetary Atmospheres

Crude approximation:



V_{mol} given by $\frac{1}{2} m v_{mol} = \frac{3}{2} kT$

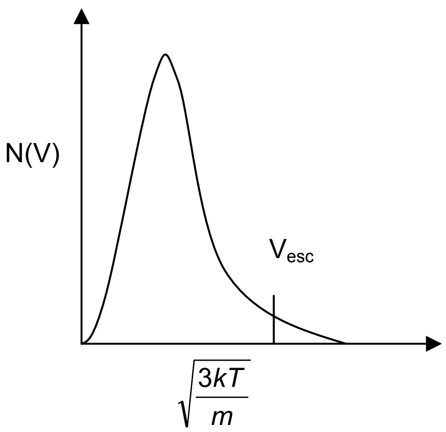
→ $V_{mol} = \sqrt{\frac{3kT}{m}}$

Escape velocity of a planet:

Conservation of Energy: $\frac{1}{2} m v_{ext}{}^{2} - \frac{GMm}{m} = 0 + 0$

→ $V_{escape} = \sqrt{\frac{2GM}{r}}$

If M is small, the planet will loose atmosphere.



The particles beyond the line V_{exc} escape, but if V_{esc} is large enough you don’t need to worry. (Small fraction)

Not all the particles have the same velocity.

Mercury: Low mass, high temp (500k). Low V_{esc}, high V_{mol}. Therefore looses atmosphere.
Jupiter: Low mass, low temp. Therefore keeps atmosphere.

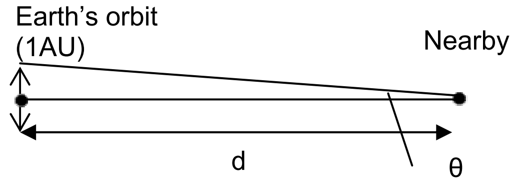
Galileo Galilei discovered the moons of Jupiter – another piece of evidence for the heliocentric solar system.

2. Stars

2.1 Nearby stars - distances

Earth – Sun: 1.5×10^{11}m → 8 light minutes
Sun – outer planets: few light hours
Nearest star, Proxima Centauri, few light years.
How do we know?

To us, the nearby star appears to move back and forth once a year, while the distant star remains the same.



1AU=θxd => d=1AU/θ
(Where θ is in radians)

Define a unit of distance "parsec" such that P=1" $\left(\frac{1}{3600}"\right)$ ←→ d of 1pc

Suppose p=1" → $d = \frac{1.5x10^{11}}{1/208000}$ =3.1×10^{16}m ≈ 3.3 light years
(1/20800 is the conversion from arcseconds to radians)

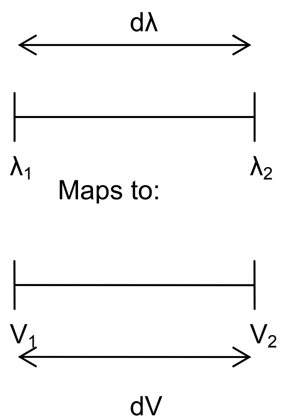
Effect was observed by Bessel in 1887, 61 Cygni p=0.293 arcseconds = 3.4 parsecs.
Proxima Centauri (Nearest star) p=0.765 arcseconds = 1.3 parsecs = 5 light years.
- This is a small shift (Recall atmosphere seeing of most telescopes ≈ 0.5 arcseconds).
Measurable for stars out to around 100pc (But quite hard
Works for the nearest few hundred stars.

2.2 Stellar Brightness

2.2.3 The SI Unit of Brightness

Want to know "Brightness"
- How much energy do I get from the star
  - per second
  - Per unit of collecting area
  - Per unit wavelength or frequency

F_λ units are $Wm^{-2}m^{-1}=Wm^{-3}$
F_v units are $Wm^{-2}Hz^{-1}$.
Radio astronomers unit 1 Jansky (Jy) = $10^{-26}Wm^{-2}Hz^{-2}$.
What is the relationship between these units?
Take the bandwidth that you are sensitive to, and consider a small range of λ, dλ, and V, dV.



You must get the total energy/sec/unit area whether you work with F_λ or F_v.

$$F_\lambda . d\lambda = F_V . dV$$

$$F_\lambda = F_V \frac{dV}{d\lambda}$$

but $v = \frac{c}{\lambda}$

➔ $\frac{dV}{d\lambda} = \frac{-c}{\lambda^2}$

2.2.2 The Optical Astronomy Method

System is called the Magnitude Scale. It is based on Hipparcos's work:

– Brightest stars = 1^{st} magnitude
– Less bridge star = 2^{nd} magnitude
– Smallest star (he could see with his eye) = 6^{th} magnitude

The difficulty is making this into a standard; the eye is a logarithmic detector. The difference in the magnitudes is a factor of 2 each time.

If the eye sees a difference between A and B which it thinks is the same as difference B and C, then:

$$\frac{Brightness(C)}{Brightness(B)} = \frac{Brightness(B)}{Brightness(A)}$$

(Definition of logarithmic)

The system was formalised in the 19^{th} century by Poyson.

He defined the magnitude scale so that:

1^{st} magnitude star 100x bright as 6^{th} magnitude star

2^{nd} magnitude star $\sqrt[5]{100}$ bright as 6^{th} magnitude star

3^{rd} magnitude star $\sqrt[5]{100^2}$ bright as 6^{th} magnitude star

$\left(\sqrt[5]{100} \approx 2.51\right)$

– Difference of a magnitude → brightness factor $100^{\frac{2}{5}}$
– Brightest star has the smallest magnitude
– Logarithmic

Generally, if two stars with magnitudes m_1, m_2, and brightness B_1, B_2:

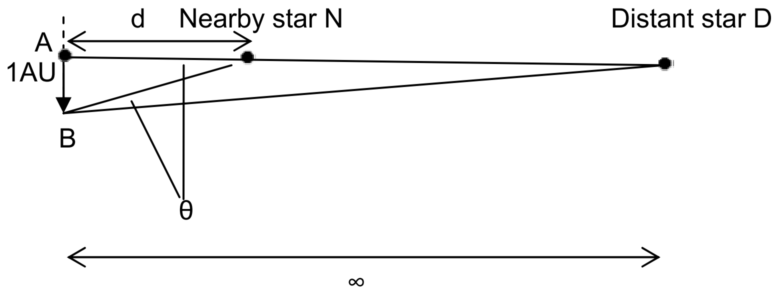
$$\frac{B_1}{B_2} = 100^{\frac{1}{5}(m_2 - m_1)}$$

Take $\log_{10}$:

$$\log_{10}\left(\frac{B_1}{B_2}\right) = \frac{1}{5}(m_2 - m_1) x \log_{10} 100$$

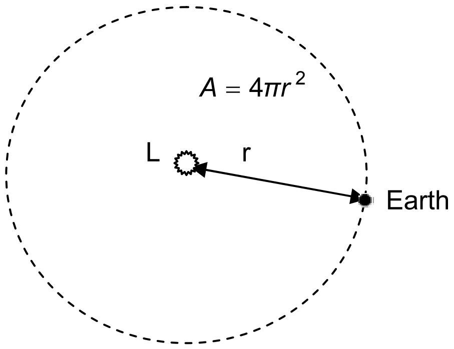
$$\log_{10}\left(\frac{B_1}{B_2}\right) = -0.4(m_2 - m_1)$$

Relative brightness



Not yet established as an absolute scale.
Need extra definition to establish how bright magnitude 0 is.
Defined as:
Mag=0: for any filter is the brightness of the star Vega in that filter.
Standard filters:
e.g. V has λ of about 550nm, Δλ of about 80nm (510→590nm)
m_v=0→Vega.

Relating luminosity to apparent brightness:
Luminosity is the actual power of the star in W (Or W.Hz^{-1}). Apparent brightness is in Wm^{-2} (Or Wm^{-2}Hz^{-1}). It obeys the inverse square $\frac{L}{4\pi r^2}$.



In the magnitude system;
The analogue of the luminosity is <u>absolute magnitude</u> (M). It is defined as the apparent magnitude that a star would have if it were at a distance of 10pc.
Relation of M and m:
Suppose a star is at a distance D (>10pc)

$$\frac{B_1}{B_2} = \frac{Brightness(Observed)}{Brightness(10pc)} = \left(\frac{10}{dpc}\right)^2$$

Take $\log_{10}$ $\log_{10}\left(\frac{B_1}{B_2}\right) = 2\log_{10}\left(\frac{10}{dpc}\right) = -2\log_{10}\left(\frac{d_{pc}}{10}\right)$

But: $\log_{10}\left(\frac{B_1}{B_2}\right) = -0.4(m_1 - m_2) = -0.4(m - M)$

$\therefore m - M = 5\log_{10}\left(\frac{d_{pc}}{10}\right) = 5(\log_{10} d_{pc} - \log_{10} 10)$

$\Rightarrow m - M = 5\log d_{pc} - 5$

m = Apparent magnitude → brightness
M = Absolute magnitude → luminosity
d_{pc} = Distance

(m-M known as "distance modulus")

2.3 How stars shine

In the 19th century, this was wrongly thought to be due to gravitational potential energy.

e.g. sun: Total available $E_p \approx \frac{GM_o^2}{R_o}$ Joules

But the sun radiates at L_o Joules/sec (About 4×10^{26}W)
Lifetime of the sun = energy available / rate of radiation = few $\times10^7$ yrs
But we know that the Earth is older than this (Geology) → gravitational PE is not the major energy source of the sun.
20th century: nuclear fusion

${}_1^1H + {}_1^1H \rightarrow {}_1^2H + e^+ + \nu$ (Slow)

${}_1^2H + {}_1^1H \rightarrow {}_2^3He + \gamma$ (Fast)

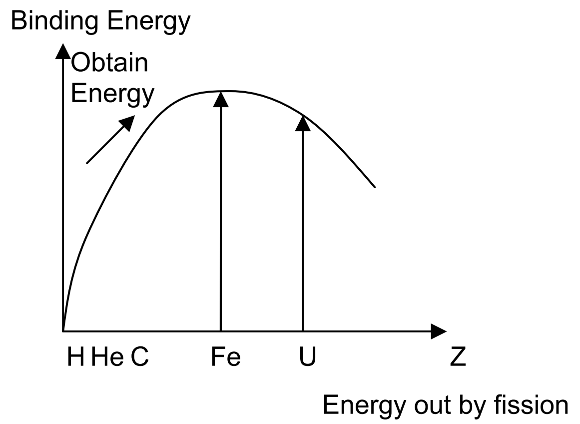
${}_2^3He + {}_2^3He \rightarrow {}_2^4He + 2{}_1^1H$

Net result: $4{}^1H \rightarrow {}^4He + e^+ + \nu + \gamma$

Obtain energy: γ → collides →More photons

Mass deficit: $\Delta M({}^4He, 4{}^1H) \rightarrow \Delta mc^2 \rightarrow energy$

Consistent with sun lasting ~10bn years at current rate of 4×10^{26}W.
It is also possible to fuse Helium → C ← depends on the mass and therefore the internal temperature.
It is possible to fuse nuclei up to Fe while giving energy out.

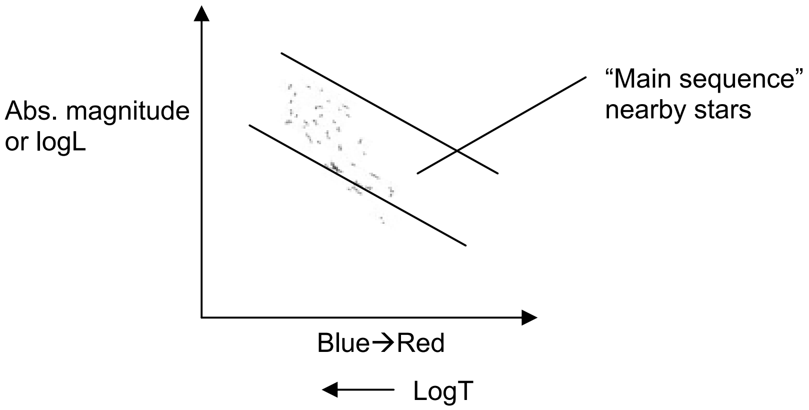


Through this, you can find the limit on stellar fusion.

2.4 Hertzsprung-Russell Diagram

2.4.1 Basic Method

Useful to plot absolute magnitude ($\propto \log L$) vs. colour (Related to temp due to Wien's law $\lambda_{peak} T = K_W$



Remember:
Red: Peak 700nm → $T \approx 4{,}000K$
Blue: Peak 400nm → $T \approx 8{,}000K$
Now remember Stefan's law:
(Power/unit surface area of a star with radius R)

$$\frac{L}{4\pi r^2} = \sigma T^4$$

$$\log L = \log 4\pi r + 2\log r + 4\log T$$

Therefore, if you know where a star is on the HR diagram, you can calculate its' radius.

Example Questions:

5. Star A radiates twice as much power as Star B, but lies 3 times further away. Calculate the difference in their absolute magnitudes and in their apparent magnitudes.

   Abs. magnitude: A 2x more powerful than B.
   Therefore: $-25\log_{10}\frac{B_A}{B_B} = M_A - M_B$
   → $M_A - M_B = -2.5\log_{10} 2 = -0.75$

   Apparent magnitude: A 3x further away → 9x less bright (Apparent)
   But A is twice as powerful → appears 2/9 as bright as B (i.e. 4.5x fainter)
   $M_A - M_B = -2.5\log\left(\frac{1}{4.5}\right) = 1.63$

6. A is red, B is blue. A appears 2 magnitudes fainter than B, but lies at 3x the distance. Estimate the ratio of their radii.
   $L = 4\pi R^2 \sigma T^4$

   A 2 mag fainter than B → apparent brightness ratio is $100^{\frac{2}{5}}$ =6.31 x fainter.
   A lies at 3x distance, which reduces the apparent brightness by a factor of 9.
   If it was the same brightness → 9x fainter. But is 6.31xfainter → brighter by a factor
   $\frac{9}{6.31} \approx 1.43x$

   $\frac{\lambda_{\max,A}}{\lambda_{\max,B}} \approx \frac{700nm}{400nm} = \frac{T_B}{T_A}$

   $\frac{T_B}{T_A} = \frac{7}{4}$

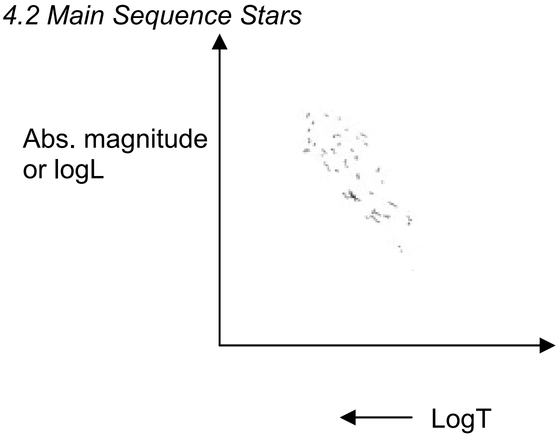
Stefan's law:

$$\frac{L_A}{L_B} = \frac{4\pi R_A{}^2 \sigma T_A{}^4}{4\pi R_B{}^2 \sigma T_B{}^4}$$

$$\frac{R_A{}^2}{R_B{}^2} = \frac{L_A}{L_B}\frac{T_B{}^4}{T_A{}^4}$$

$$\frac{R_A}{R_B} \approx 2$$

2.4.2 Main Sequence Stars



Main sequence – band of stars from top left to bottom right
Stars spend most of their loves at <u>one</u> point on the MS.
Fusing H→He

Stellar physics tells you that $L \propto M^4$ (approx).
Radiation rate $\propto M^4$ because higher mass stars have a higher central temperature and pressure → nuclear reactions in stellar centre go faster.

Lifetime of a star (How long on one point) = $\frac{Energy(Available)}{Radiation(Rate)} \propto \frac{M}{M^4} \propto M^{-3}$

- 10 M_Θ lives 1/1000 as long as the sun – few x10^6 years.
- Stars which spend their lives high up on the MS are there because they are massive.
- If you look at a random collection of stars, then most of them will be on the MS.

2.4.3 How stars begin life (Get onto the main sequence)

Collapsing gas cloud consisting of H atoms, with a mass M_r, radius R, temp T, and an individual mass m.
Condition for the star to collapse is g.p.e./particle > thermal energy.

$$\frac{GMm}{R} > \frac{3}{2}kT$$

$$R = \frac{GMm}{{}^3\!/_2 kT}$$

(If it just collapses)

Critical Density $\rho = \frac{M}{{}^4\!/_3 \pi R^3} = \frac{M}{{}^4\!/_3 \pi}\frac{\left({}^3\!/_2 kT\right)^3}{(GMm)^3} = \frac{81MK^3T^3}{32\pi G^3M^3m^3} = \frac{0.8k^3T^3}{G^3M^2m^3}$

Typical interstellar cloud T~10K
Mass ~ mass of star to be formed ~ 1 M_Θ
m = mass of H atom.
Plug numbers in → 2×10^{16}Kgm^{-3} → 10^{11}G atoms.metre^{-3}
This is high by galactic medium standards.

Stars can be formed by shock compression. As shocks like those needed are given off by other star formations, star formation begets star formation. Shocks can also come from supernova explosions.

2.4.4 Evolution of the main sequence: low mass star (~1 M_Θ)

During the main sequence life of the star:
- Gravity → trying to induce contraction
- Balanced by pressure → nuclei fusion reactions in core.

This is a stable equilibrium.
e.g. suppose the star contracts, then the release of GPE causes the temperature in the core to go p. This leads to nuclear reactions running faster. Therefore, there is more pressure, pushing the star back to its original size.

When H runs out in the core (After a few billion years) as it is converted into H1 (H will still be in the outer section of the star, but it is not hot enough there to fuse.)

- Core starts to contract, looses GPE
- GPE → heat → higher T in shell surrounding the core (Burns H) → more photons
- Photons diffuse into the outer regions of the star on the way (heating them up). To the first approximation, the outer regions can be treated as an ideal gas. $V \propto T$ → T increases, V increases.
- Outer regions of the star expand → R increases for given L → λ_{peak} goes up → red giant.
- Meanwhile, back in the core…
  - Core contracts until something else balances gravity
  - This is degeneracy pressure of the gas – squash matter until e orbitals overlap – doesn't like it (Pauli exclusion principle – quantum mechanics)
  - Surrounding area continues to get hotter – may or may not start the He burning as well (1 M_Θ, will burn. < M_Θ, won't)
  - If the He burning does not start, loses outer surface, leaving core only.
  - If He burning starts, new energy source → all processes reverse → star shrinks again ("helium flash" – takes merely hours or days)
- Final state, 1 M_Θ
  - He runs out, core collapsing again.
  - PE release is not enough to trigger burning of heavier elements.
  - Core collapses to a degenerative state → white dwarfs.
  - Outer layers expand again → blown off into outer space.

  White dwarf mass << previous stellar mass

  Fred Hoyle 1915-2001 (Worked in cosmology)
  Major contribution to stellar evolution.

2.4.5 Evolution of main sequence – high mass

- Main sequence life is shorter – few million years.
- Once H is exhausted, the core collapses. GPE is released → much more than for small stars. Temperature induced is much higher → He burns before core becomes degenerative.
- Temp. so high that He, C, O, … can fuse. end up with onion skin layer – different layers fuse different matter. Hotter going in (Middle so hot it can fuse Si)
- Finally an iron core forms in the centre → that's it (No more energy out of iron)
- Sudden collapse of core
- Huge release of GPE all at once:
  - Tears some of iron apart
  - Drips neutrons into Fe – heavier elements
  - Thermal energy in envelope, neutrinos etc. → supernova explosion
- Massive stars form nearly all of the C, N, O, Fe, N, …
  (75%H 20%He produced in big bang)

– Core has collapsed – very high density. So high that electron degeneracy pressure is insufficient t hold the ore up (Electrons go relativistic) → e's fored into protons → neutrons.
– Core formed from neutrons, held up by neutron degeneracy pressure → neutron star
– Mass much less than original state (1.4 $M_\odot$ → original 5, 10, 20 $M_\odot$)
– More massive stars yet – remnant cannot even be supported by n degeneracy pressure → mass crushed without limit → black hole. (S. Chandrasekha, 1910-1995). General relativistic effect. Infinitely dense middle.

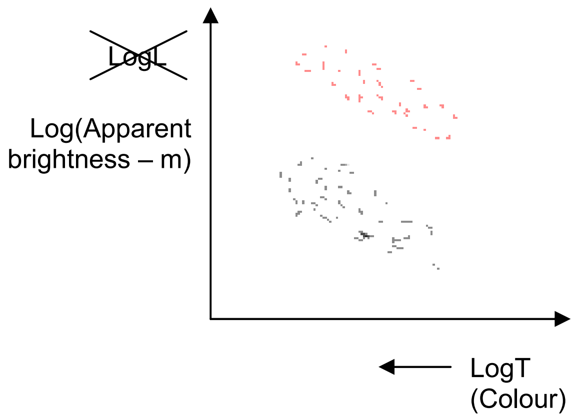
2.5 Stellar Distances again

2.5.1 Parallax

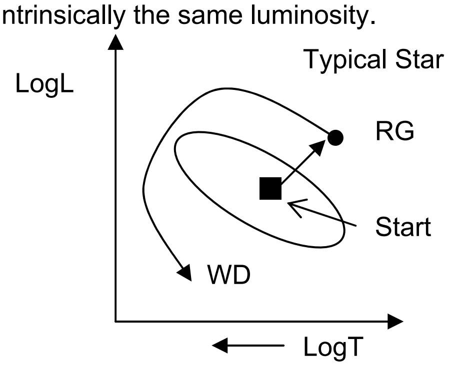
Already seen.
- Only useful for nearby stars (<100pc) (accuracy)

2.5.2 Main Sequence Fitting

- Occasionally get clusters of stars
  - Globular (10^6 oldish stars)
  - Open clusters (Younger, fewer)
- Useful because you know they are all the same distance.
- Suppose you have two such clusters:
  - Plot the HR diagram for each



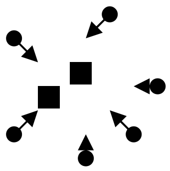
- Know stars of the same temperature <u>which are on the main sequence</u> have intrinsically the same luminosity.



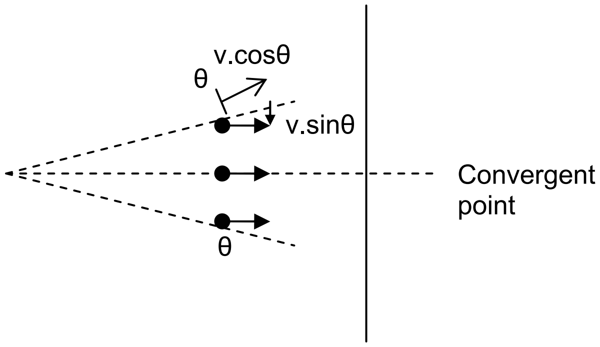
- Therefore, stars in each cluster must have different brightness because they are at different <u>distances</u>: $m-M=5.\log_{10}d_{pc}-5$.
- Hence can get <u>ratio</u> of distances to cluster 1 and 2 from the inverse square law.
- Hence the name "main sequence fitting" – gives distance ratio by moving one HR diagram till it les on top of the other.
- Disadvantages:
  - Stars in clusters – can't get around this.
  - Ratio of distances – need a cluster with a well-known distance.

2.5.3 Finding one cluster distance - Hyades

- Some clusters are known as "moving clusters". It appears to converge.



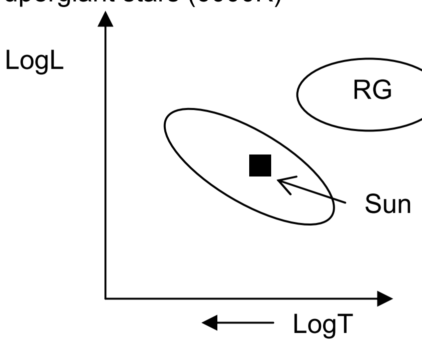
- It is an effect of the motion.



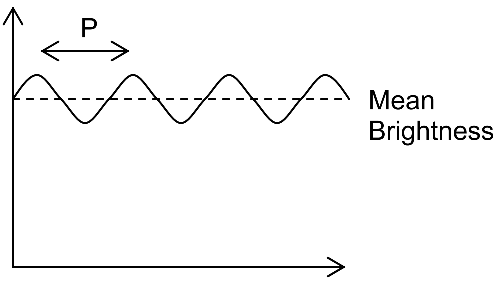
- v.sinθ is proper motion on sky – can be calculated.
  Angular motion $\dot{\theta} = \frac{v \sin\theta}{R}$ (v=ωr)
- v.cosθ is measurable as a Doppler shift.
  Spectral line at (lambda) has a shift $\frac{\Delta\lambda}{\lambda} = \frac{v \cos\theta}{c}$ ← v<<c
  Hence if we can measure (theta-dot) and (delta-theta), we know θ so we can eliminate v, and calculate R.
- Some nearby clusters are quite large and close – 20^{o} – 30^{o} away from convergence point.
- Can also use the method to see which stars are physically associated.
- Method (MC + MSF) extends to much greater distances than parallax method.
- Hyades is used normally as it is the nearest one to us.

2.5.4 Cepheid Variables

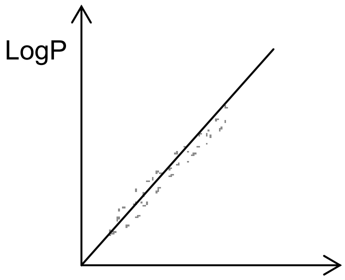
- Supergiant stars (6000K)



- Unstable – entire star pulsates (Variable in brightness with a characteristic shape)

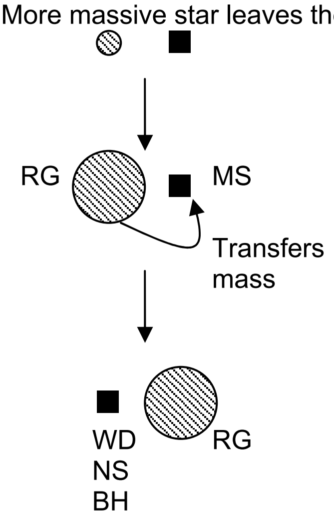


- Plates of Magellanic Clouds.
  All of the stars in MC at approx. the same distance.
  Magellanic clouds are satellite galaxies orbiting a normal galaxy. The ones we see easily are orbiting our galaxy, hence staying at approx. same distance.
  Cepheids found in MC – but with correlation between period and the mean brightness.



  Hence cepheids have correlation between the period and intrinsic luminosity.
- Find a Cepheid anywhere else:
  - Can measure the period P → infer intrinsic luminosity M.
  - Can measure the apparent magnitude m.
  - Put into $m-M=5.\log_{10}d_{pc}-5$, and distance comes out.
- Disadvantage: Cepheid only.
- Advantage: huge distances.

2.6 Binary Star Systems
- Born individually – met? No. Chances are very small.
- Born together out of the same gas cloud – evolve together.
- More massive star leaves the main sequence first.



- Normally see an evolved star and a normal star as a binary.
- Clearest evidence for solar mass (1-10M(sun)) black holes.
  Cygnus X-1 – X-rays emitted. Very variable on a short time scale Δt. Small size cΔt.
  Infer mass of compact object (Keppler's laws + observation of orbiting normal star)
  Small volume, large mass (6M(sun)). Therefore, there is something very large and very dark – can only be a black hole.

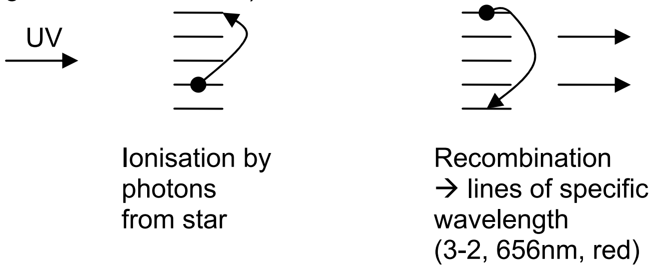
3. The Galaxy

3.1 Constituents of the Galaxy

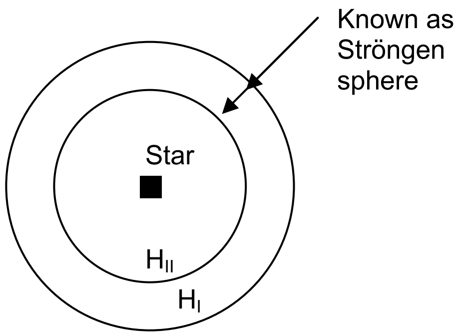
3.1.1 Stars and the gas around them

Dominate the observable matter – make up 95% of the (luminous) mass of the galaxy. (Excluding dark matter – only stuff which emits EM radiation)
Types of nebula associated with stars:

– H_{II} Regions:
(Jargon: H_I is neutral hydrogen → p+e. H_{II} is ionised hydrogen → p (Exists in hot / highly ionising areas). H_2 is not the same – this is molecular hydrogen (H—H) (Exists in dense, cold clouds))
Also known as thermal emission nebulae. Exist near to sources of energy, e.g. close to bright, hot stars – forming regions.
Near to the star, the gas is ionised (This gas is basically the collapsing gas cloud that gave birth to the star). This shines in atomic lines.



Further away from the star, gas is not so ionised → H_1



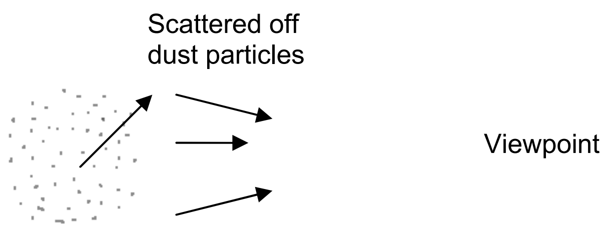
Stromgren sphere – the end of the H_{II} region. Within this sphere:
The total rate of ionization = total rate of recombination.
Number of atoms ionised per second = number of photons leaving the star/sec.
Number of recombinations per second per unit volume = (alpha)$n_e n_p$
where n_e is the number density of electrons, and n_p is the number density of protons.
Overall gas is neutral, so $n_p = n_e$ → alpha.n_e^2
Stromgren sphere radios, r:

$$\alpha n_e^2 . \frac{4}{3}\pi r^3 = N$$

$$\Rightarrow r = \sqrt[3]{\frac{3N}{4\pi \alpha n_e^2}}$$

Typically for young bright stars [N high] → few x10pc (Observable)
For stellar-type stars – less important.

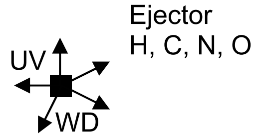
– Reflection nebulae:



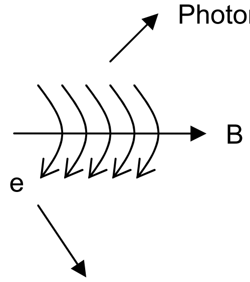
Nebula is illuminated in scattered light. NB this is not line radiation.

Scattering more efficient for blue photons. (Similar to sky). Therefore, nebulas will appear blueish.
Depends on the dust not being too thick, and also the size of the dust particles.

- Death throes of normal stars: "planetary nebulae" (Nothing to do with planets)
  Recall that red giant outer envelopes get expelled. Can happen in several ways; spherical ejection, and non-axisymmetric ejection (We don't know why)

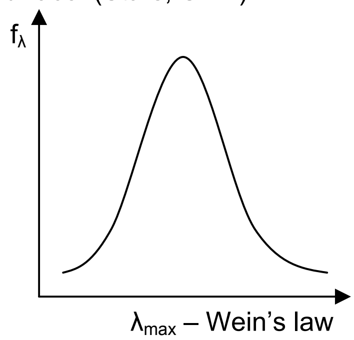


  Ionisation and recombination occurs, giving off emission lines – lots of colours
- Death throes of massive stars
  Massive stars end in supernova explosions, leaving extremely dense neutron stars behind.
  See supernova explosion (very bright) and the expanding shells.
  Physical process include non-thermal processes: Synchrotron process



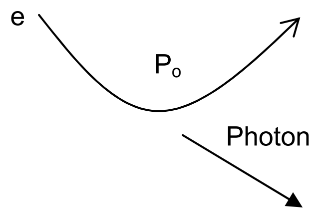
Summery of emission mechanisms:

- Blackbody radiation "optically thick thermal" – only photons you see are those at the surface. (Stars, CMB)



$T\lambda_{max} = k_w$

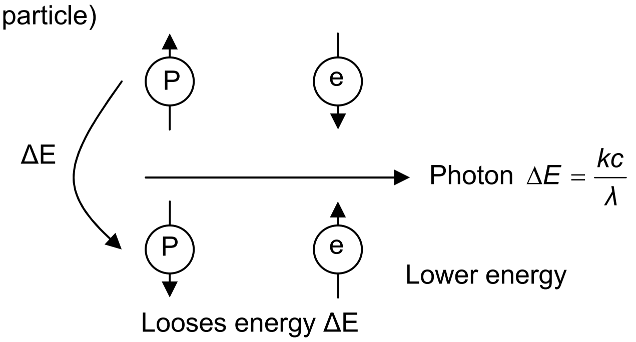
- Optically thin? Extremely complicated. Have to worry about the microprocesses. Happens in hot gases – radiation known as "bremsstrahlung". ~10^8K gases in galaxy clusters → X-rays.



- Non-thermal emission
  Synchrotron radiation –Supernova remnants, active galaxies.
- Line emission
  Ionization / recombination.

Happens in H_{II} regions (thermal emission nebulae) and in planetary nebulae, as well as active galaxies.
- Dust
  - Absorbs → therefore annoying, although it's only a few 1/10 of % of total galaxy mass.
  - Irritatingly effective at absorbing optical photons. (Also reradiates as a black body emitter). Average optical photon propagates only around 1 kpc through the galactic plane. (compared to 8kpc to the centre of our galaxy)

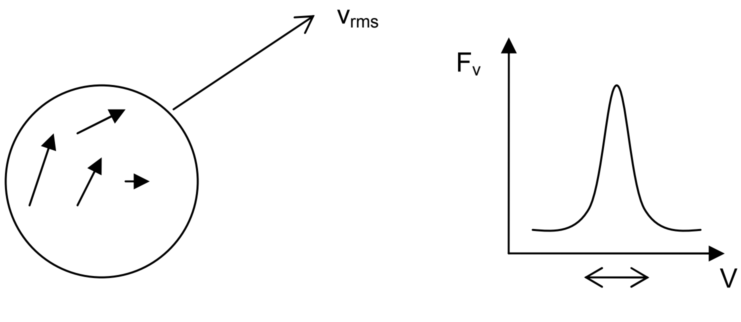
3.1.2 H_I regions (Neutral hydrogen)
- Quite a lot of this, but it is very difficult to see in the optical. Due to most of it being in the galactic plane, therefore is obscured by dust. Also it is cold (50k) → doesn't emit much black body radiation. ($\propto T^4$)
- Radio rides to the rescue again…
  Quantum mechanics → particles have spin (Effective angular momentum of the particle)



Two possible spin states for the H atom.
Very low energy given off: $\lambda = 20cm \leftrightarrow 1420MHz$ - too high for optical, but handy for radio.
Very unlikely: (once every few x100yrs), but there are lots of H in the galaxy.
First postulated in 1945 (van der Hulst)
What can we do with this?
- Frequency will change due to the velocity of the hydrogen.:

  $$\frac{\Delta v}{v(1420MHz)} = \frac{v_{lineofsight}}{c}$$

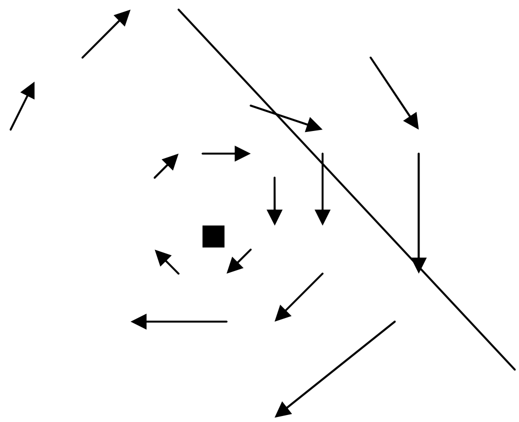
  Therefore can measure velocities.
- Temperature of a single cloud:



$$\frac{1}{2}mv^2{}_{rms} \approx kT$$

Width of line Δv proportional to the range of velocity, $v_{rms} \propto \sqrt{T}$
- Lots of clouds, structure of galaxy can be shown
  Along any direction, you intercept many clouds each with their own line-of-sight

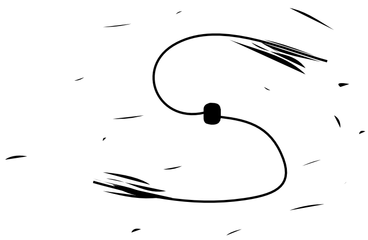
velocity component.

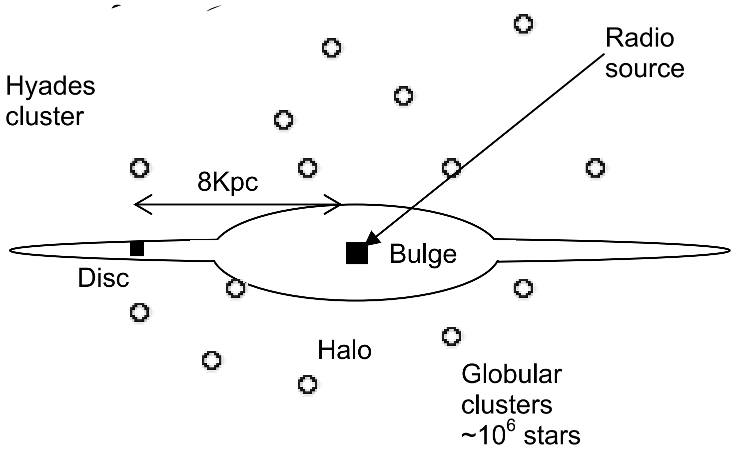


- H_I line at 21cm allows:
  - Investigation of neutral H
  - Velocity measurements → dynamics of galaxies.
  - Structure of galaxy
- Free of obscuration

3.2 Structure of the Galaxy

Spiral.





How do we know?
- Milky way → ring in sky → disk / flattened distribution.
- Turns out that although globular clusters don't have Cepheids, they do have other stars (RR Lyrae – old red stars) which have similar properties (Period-Luminosity relationship → can measure the distance to globular clusters).
  Centre of the globular cluster distribution → centre of the galaxy.

3.3 Stellar populations

Two different types of stars:
"Population I" → high heavy-element abundance. (Over He, e.g. C, N, O)
"Population II" → Low heavy-element abundance.

Physical difference Population II stars are formed from primordial clouds. (H, He).
Population I stars are formed from recycled material (Previous stars eject heavy element-

rich material into space → gas collapses again to form new stars.) – Heavy element abundance results from the previous stellar evolution.

Generally speaking, population I stars live in regions of high stellar density, and lots of gas (i.e. in the disk of the galaxy)

In halo, stars tend to be formed from primordial gas – population II.

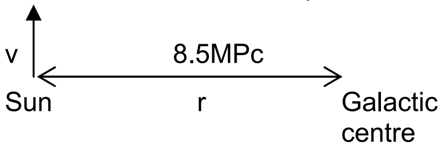
In the disk of the galaxy, there is more star formation and more young stars.

3.4 Stellar motions

Motions in the disk and the halo are very different.

3.4.1 Disk stars

The predominant motion is <u>rotation</u> around the galactic centre. Our sun travels are $250 kms^{-1}$ and is about 8.5kpc from the centre.



Using $\frac{GMm}{r^2} = \frac{mv^2}{r}$ gives us 10^{11} solar masses.

However, there is also a slight motion perpendicular to the disk (Oscillation), which allows us to measure the mass in the disk.

Change form PE→KE→PE. Conservation of energy means that PE+KE=constant.

PE: mgh cannot be used, as g does not remain constant. Therefore have to integrate:

$\Delta PE = mg(z)dz$

$$\frac{1}{2}mv_z^2 + m\int g(z)dz = \frac{1}{2}mv_0^2$$

If you measure lots of stars at different z, v_z→g(z) you can get the mass distribution "easily".

Gauss formulation $< g_\perp > A = 4\pi GM_{enclosed}$

Tells you the mass per unit area within any z.

Compare this with mass of the stars and gas → exceeds by about 30-50% (Experimental error ~ 10-20%) – weak evidence for dark matter. In the disk – not that much. At larger distances in the disk, more evidence is shown.

(Further evidence in external galaxies – provided by measuring v as a function of radius with the 21cm line → internal mass of galaxy (measured by dynamics) > mass in stars. Stronger evidence comes from clusters of galaxies, where you can measure the velocities of the individual galaxies, and how much mass they should have. Velocities are inconsistent with just luminous matter.)

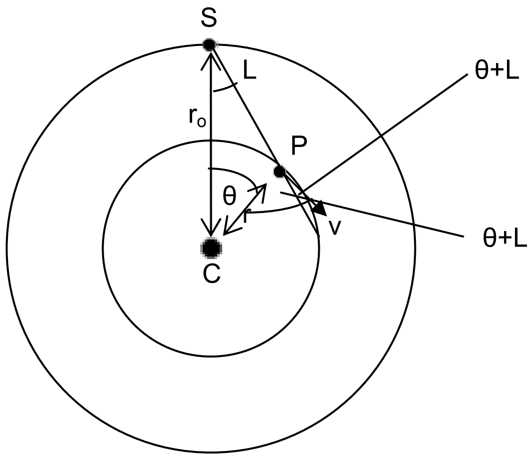
3.4.2 Bulge and halo stars

Very different:

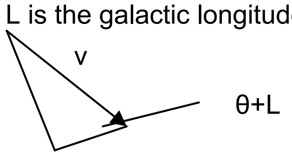
Motion is essentially in random orbits, in any plane. We don't know why this is – we need to know how galaxies are formed first.

3.5 Use of HI for galactic centre

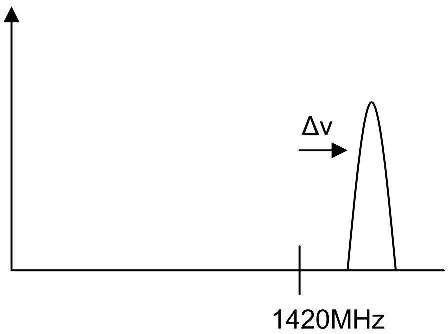
Can use the 21cm line to map the galaxy.



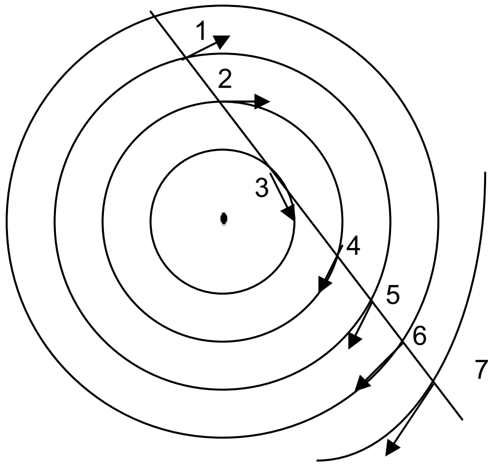
Sun stationary (Get this by subtracting an angular momentum to correct movement)
Angular velocity of gas cloud at p
$\Omega(r)-\Omega(r_o)$
Hence velocity v is $r(\Omega(r)-\Omega(r_o))$
L is the galactic longitude of P

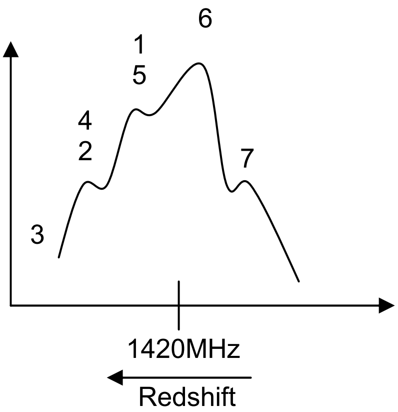


Component of v along the line of sight
$v_{//} = v\sin(\theta + L)$
→ $v_{//} = r[\Omega(r)-\Omega(r_o)]\sin(\theta + L)$
Consider triangle SPC and use sine rule.
$$\frac{r}{\sin L} = \frac{r_o}{\sin(180-(\theta+L))} = \frac{r_o}{\sin(\theta+L)}$$
Substitute in:
$v_{//} = r_o[\Omega(r)-\Omega(r_o)]\sin L$
Measure by $\frac{\Delta v}{v} = \frac{v_{//}}{c}$



Problem: any line of sight intercepts a lot of clouds.

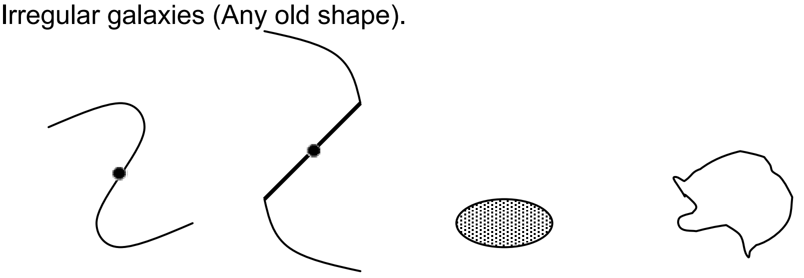




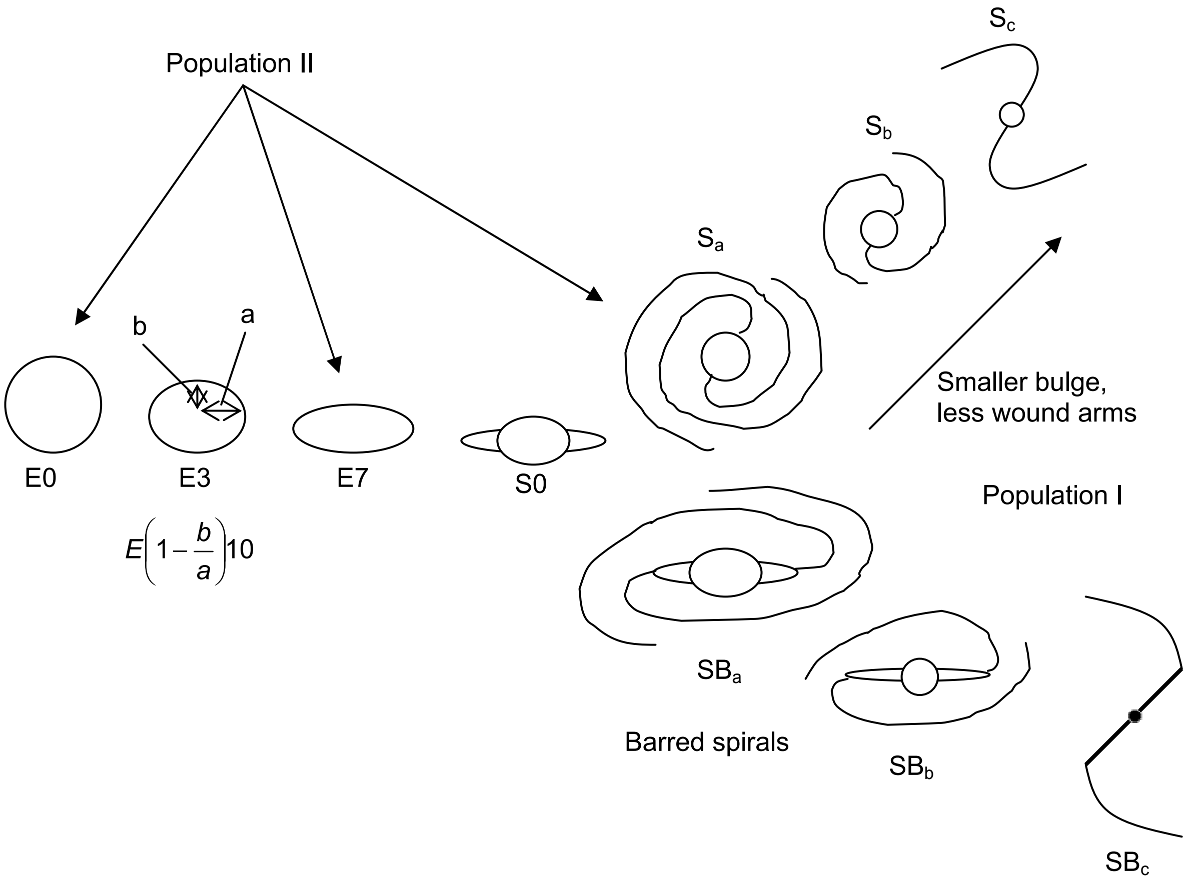
But:
measure lowest v. Unambiguously measures gas fro point 3 → $v_{II(3)} \rightarrow \Omega(r)$
“tangent point analysis”

4. External Galaxies

- In 1918, we knew the size of the galaxy to about 40kpc.
  There were some "fuzzy patches" in the sky which were known as Nebulae (planetary nebulae, thermal emission nebulae, reflection nebulae, etc….). Some of these were due to gas around stars, but some were external galaxies, with around 10^{11} stars (unresolved from Earth). That they were galaxies was unknown until around 1920.
  Arguments about this went on for a while until Hubble resolved some stars in external galaxies. In particular, he resolved Cepheids → can measure absolute and apparent magnitudes of these, hence distances. He found these were much larger than 40pc, therefore couldn't be stars in our own galaxy → must be external galaxies.
- Nearest galaxy is the Andromeda galaxy (used to be known as the Andromeda nebula). Has an absolute magnitude of about -19.9 (25x more luminous than the sun).
- Hundreds of billions of other galaxies, which can be classified:
  Regular galaxies (Easy to classify): Spirals (70%), of which there are ordinary and barred, and ellipticals (~30%).
  Irregular galaxies (Any old shape).



- Hubble resolved these into the "tuning fork" diagram.



– Differences between spirals and ellipticals:

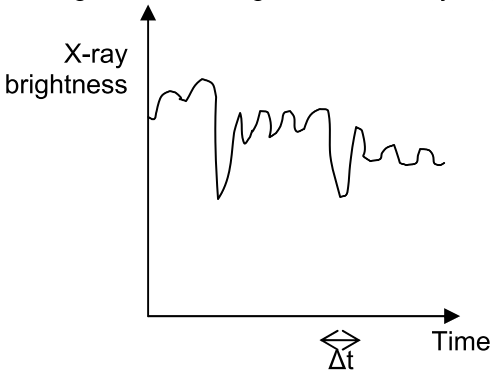
	Spirals	Elliptical
Velocities of stars	Random (bulge) Rotation (disk)	Random (Ellipticals look like large spiral bulges)
Gas / dust	Lots (disk / spiral arms) Little (bulge / halo)	Little
Star formation	Lots (disk / arms) Little (bulge / halo)	Little (Mostly old red stars)
Stellar population	population **I** in disk	population **II**

– Physical connection? Merges seem to take place between galaxies – thought of as how spirals end up as ellipticals. Even without merges, in about 10^{10} years all the gas will be used up in existing spirals.

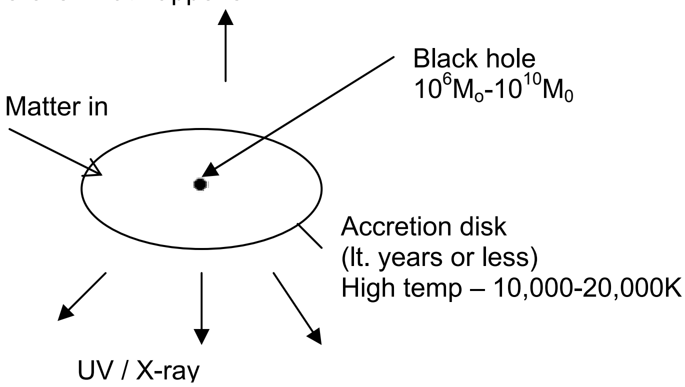
5. Active Galaxies

5.1 Basic principles

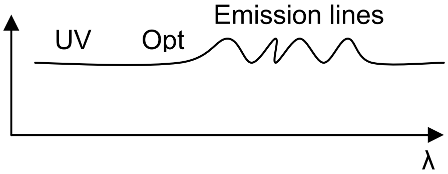
- Most galaxies are normal (90%) – most of the light is from stars.
- 10% of galaxies are known as "active" galaxies.
  Significant fraction of optical radiation is from non-stellar processes occurring in central region. Some relatively near active galaxies, 10-20% of optical radiation. Distant objects, many times that.
- Central regions of active galaxies are very bright, and very small. How do we know?



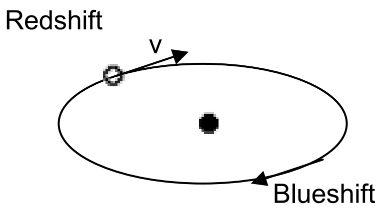
  Large variations
  Size of emitting region must be < cΔ (otherwise it can't all vary at once)
  Light days or smaller
- Stellar processes cannot generate so much light in such a small space.
- Here is what happens:



  Optical line emission from UV rays hitting gases surrounding diagram.
  (Photoionisation & recombination)
  Spectrum:



  Also radio jets (ejection of very fast +β fields – synchrotron radiation) coming out of poles of BH.
- How do we know about the BH?
  Observe gas in the vicinity of the centre



Velocity v will produce a shift in the emission line from that has with respect to gas in the rest of the galaxy.

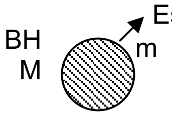
$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

Newton: $\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow M = \frac{v^2 r}{G}$

$r = \theta D$ as r<<D

Central mass (BH) $10^6 M_o$ (our galaxy) → $10^9 M_o$.

- How big is the central black hole?



Escape velocity $\frac{1}{2}mv^2 = \frac{GMm}{r}$

$2GM$

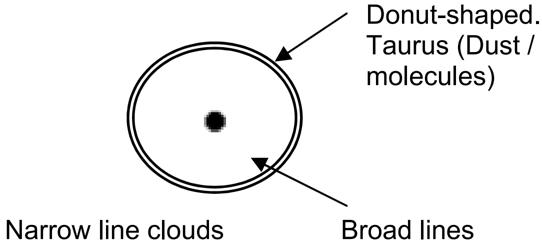
Wrong, as a non-relativistic formula has been used for KE, and Newtonian theories of gravity which only work for small gravitational fields. Need general relativity. However, answer is right.

Radius tends to be in the AU to Parsec range. (Try it!)

5.2 Different types of AGN (Active Galactic Nuclei)

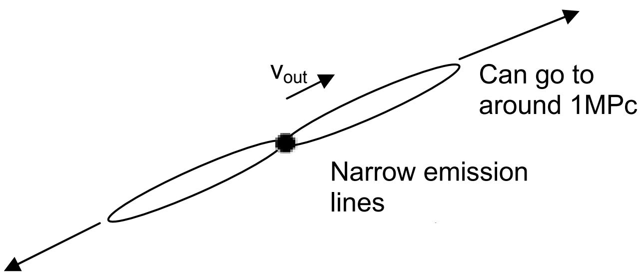
5.2.1 Seyfert Galaxies

- Spiral galaxies with bright non-stellar core with optical emission lines.
- Radio jets are weak.
- Two types of optical emission lines
  - Broad – widths fewx10^3kms^{-1}→10^4kms^{-1}
    Widths produced by fast gas motions in the central 10-20 light-days region.
  - Narrow – widths $\leq 10^3\ kms^{-1}$ - gas further out and moving slower.
- Some have no broad lines.



5.2.2 Weak Radio Galaxies

- Mostly in elliptical host galaxies
- Look in radio like:



- Radio outflows are very fast → if you look down the jet, the jet appears brighter due to relativistic effects.
  (v_{out} significant fraction of speed of light)

5.3 Distant AGN

More luminous → central processes outshine all of the stars.("Quasar")

5.3.1 Radio quiet quasars

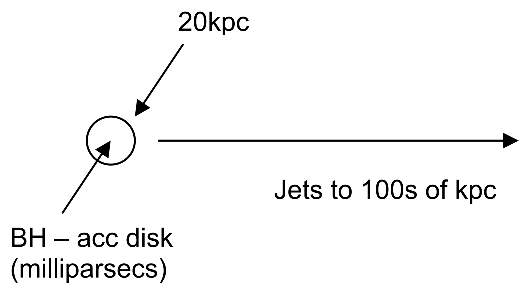
- Like Seyferts, but a lot brighter. Host galaxy is hard to see.
- Recent (1999) evidence: host galaxies are elliptical.
- Usual range of central emission: UV / X-Ray / Optical / Line Emission but no radio jets. (Why? Not known.)

5.3.2 Radio loud quasars

Exactly the same system: If favourable angle (down the axis) then you see the narrow lines plus the broad lines, plus a lot of optical, UV and X-ray radiation. If it also emits radio jets, then it is a radio-loud quasar.

If you are looking in an unfavourable angle, then the torus blocks optical, UV and broad lines.

In objects which do have radio jets, these propagate hundreds of kPc into intergalactic space.

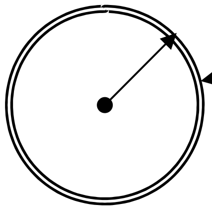


6. Cosmology

6.1 The expanding Universe

6.1.1 Olber's Paradox

Assume space is infinite, static, and uniform. How much light should we get from stars? Consider shells at distance r with a constant number N of stars / unit volume.



Number of stars in the shell $= Nx4\pi r^2 dr$

Apparent brightness of these stars $= \frac{k}{r^2}$ (Inverse square law)

→ Total brightness observed from shell $= N.4\pi kdr$

r has cancelled out. Total brightness of universe $= \int_0^\infty 4\pi kNdr = \infty$????

- Sky is not this bright, therefore some assumptions must be wrong:
  1. Not static, but expanding → Doppler shifts of radiation.
  2. Age of universe is finite → observable universe is finite (Section 6.3 – no boundary) in extent (Light from anything else in the universe has not got to us yet)

First clues to cosmology.

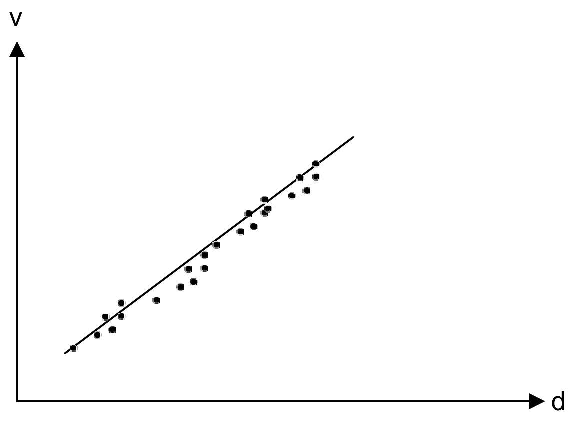
6.1.2 Hubble's Law

– Measured spectral lines in nearby galaxies
From shift in wavelengths, the galaxy is moving away with speed

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda_{emitted}} = z_{redshift}$ (Only valid for low Redshift)

This was not new.

– Found Cepheid variables in nearby galaxies. Measured distance d to the nearby galaxies.

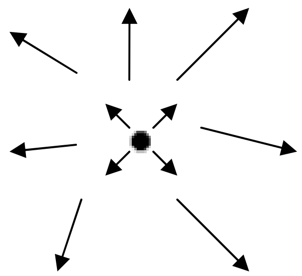


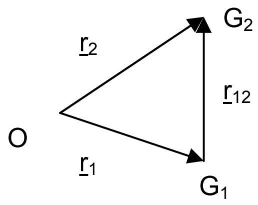
$v = H_o d$ - H_o is Hubble constant.

Substitute:

$d = \frac{cz}{H_o}$

– All galaxies are moving away from us with speed proportional to distance.
[Doesn't this mean we're something special? No! Consider us at 0





$\underline{\dot{r}_2} = H_o \underline{r_2}$

$\underline{\dot{r}_1} = H_o \underline{r_1}$

From G_1's point of view, recession of G_2 happens at speed

$\underline{\dot{r}_{12}} = \underline{\dot{r}_2} - \underline{\dot{r}_1} = H_0(\underline{r_2} - \underline{r_1}) = H_o \underline{r_{12}}$

G_1 sees exactly the same thing!

Therefore, no matter where you are in the universe, galaxies are flying apart according to Hubble's law. → universe is expanding. Expansion is isotropic.

[Hubble's law is the only one which does this - $v = H_o d^2$ doesn't.]

– H_o is thought to be ~ 60-70kms^{-1}Mpc^{-1}.

What happens at high Redshift?

$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$ ← True for low v.

At high velocity, use special relativity formula:

$1 + \frac{\Delta\lambda}{\lambda} = \sqrt{\frac{c+v}{c-v}}$ ← Don't need to remember this.

$(1+z)^2 = \frac{c+v}{c-v}$

$\Rightarrow \frac{v}{c} = \frac{(1+z)^2 - 1}{(1+z)^2 + 1}$

$z << 1 \rightarrow \frac{v}{c} = z$

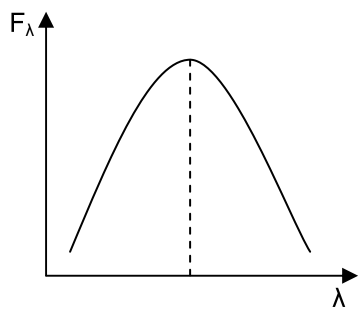
$z = 5 \rightarrow \frac{v}{c} = \frac{35}{37}$

This is strictly correct only for a universe with no matter!

Matter gives you gravitational fields, which are properly handled by general relativity.

6.1.3 Cosmic microwave background (CMB radiation)

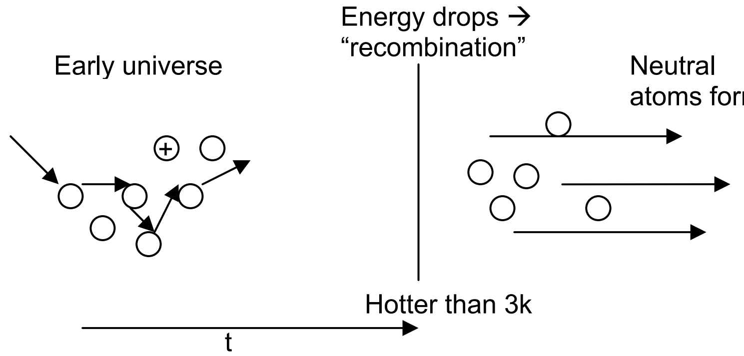
– Discovered in 1965 by Penzias & Wilson (Bell labs)
  Found some radiation at around 3k as extra noise on other measurements.
– This is black body radiation



$\lambda_{peak} T = 2.9x10^{-5} m.k$

→ $\lambda_{peak} = 1mm$ short microwave / far IR

– Relic radiation from the early history of the universe when the universe was much smaller and hotter.
Early universe is ionised (too much energy for electrons to stay around atoms)



Optically thick thermal radiation.
Wavelength of photons increases due to / with expansion of universe → lowers their effective temperature.

– Slight temperature anisotropies (at tens of µk level). Varies ever so slightly with direction → tells about structure forming at the time of recombination. Very sensitive to the cosmology / matter density of the universe.

6.1.4 "Big Bang"

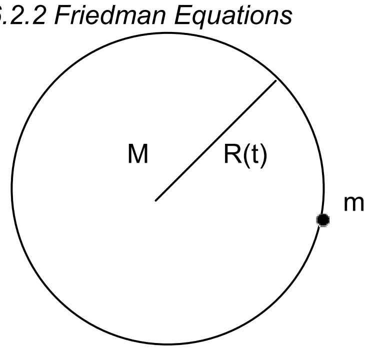
– Universal expansion and CMB are very strong evidence.
– Not a conventional "explosion" – space / time started then.
No "outside" to stand in watching the big bang explode. Did not explode "into" anything.
– In the 1960's an alternative theory called the "steady state". Proposed that the universe expanded, but matter created in the gaps to leave everything the same with time ("perfect cosmological principle") → wrong.
1) violates general relativity as matter cannot be created out of nothing (but requires new physics)
2) CMB radiation – Steady State can't do it.
3) Quasars are more common at high Redshift → violates PCP.

6.2 Newtonian Cosmology

6.2.1 Warning!

Equations derived in this section are (almost) correct. The ideas about space and time (methods) assume Newtonian 3D space and are wrong!

6.2.2 Friedman Equations



Universe expands → R(t) increases.
Conservation of energy PE + KE constant.

$$-\frac{GMm}{R} + \frac{1}{2} m\dot{R}^2 = C$$

Write density $\rho = \frac{M}{\frac{4}{3}\pi R^3}$

→ $\frac{1}{2}\dot{R}^2 - \frac{G}{R}\rho\frac{4}{3}\pi R^3 = c'$

$$\boxed{\dot{R}^2 - \frac{8\pi\rho G}{3}R^2 = c''} \quad \text{(F1)}$$

Can also see that if the total energy > 0 → universe expands forever. < 0 , universe is bound (recollapses)

Can also write $F = ma$. $\frac{GMm}{R^2} = m\ddot{R} \rightarrow \boxed{\ddot{R} = -\frac{4}{3}G\pi\rho R}$

$H = \frac{\dot{R}}{R}$ varies with time, define $H_o = \frac{\dot{R}_{now}}{R_{now}}$ (F2)

6.2.3 Solutions; age of universe

Suppose c''=0 → universe just expanding.

$\dot{R}^2 = \frac{2GM}{R}$ → $\dot{R} = \frac{(2GM)^{1/2}}{R^{1/2}}$

$R^{1/2}\frac{dT}{dt} = (2GM)^{1/2}$

$\frac{2}{3}R^{3/2} = (2GM)^{1/2}t$

t = age of universe.

$\frac{2}{3}R_{now}\frac{(2GM)^{1/2}}{\dot{R}_{now}} = (2GM)^{1/2}t$

$t = \frac{2}{3}\frac{R_{now}}{\dot{R}_{now}} = \frac{2}{3}H_o^{-1}$

$H_o = 65kms^{-1}Mpc^{-1}, 2x10^{19}km = 1Mpc$

$t = \frac{2}{3}\frac{1x3x10^{19}}{65}\sec \rightarrow 15 billion.years$

6.2.4 Comments, Fate of Universe

Constant = 0, "critical universe"

Critical density $\rho_{crit} = \frac{3\dot{R}}{8\pi GR^2} = \frac{3H^2}{8\pi G}$

$H = \frac{\dot{R}}{R}$

$H_{o,now} = \frac{\dot{R}_{now}}{R_{now}} = 50 \sim 70kms^{-1}Mpc^{-1}$

If $\rho > \rho_{crit}$, universe recollapses.

Now, $\rho_{crit} = \frac{3H_o^{\ 2}}{8\pi G}$

But ρ appears to be even smaller!

What we think:

$\rho_{baryonic}$ is basically a few percent of ρ_{crit}

Why?:

- $\rho_{luminous} << \rho_{crit}$
- Detailed models of nucleosynthesis in BB model – cosmology in 3rd year) (1970's/80s)

- Affects CMB fluctuations

Most of the matter is in a non-baryonic form. (Total matter $\approx 0.3\rho_{crit}$) ???
90% of matter is in as-yet undetected form (WIMPs)
Why:-
- Many systems in the universe with evidence for gravitating "dark matter"
- Rotation curves v(r)
- Clusters of galaxies – dynamics

Further amount of energy in "dark energy" aka the "cosmological constant"
→ General Relativity modification to Friedman equations
$\rho_{mass} + \rho_{energy} = 1$ - universe still expanding forever.
Why:-
CMB fluctuations (Recent results - 2001)

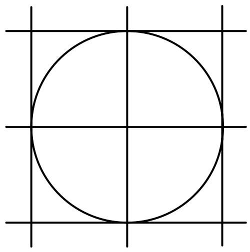
6.3 Curvature

6.3.1 Newtonian Picture

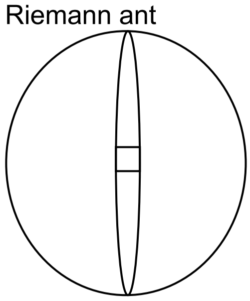
Wrong:
- Assumes some version of absolute 3D space, and time as a separate coordinate.
- Gives awkward questions (e.g. where's the edge of space?)

6.3.2 Positive Curvature Illustrated

$(\rho_{mass} + \rho_{dark} > 1)$
- Imagine 2D ants → 3D humans
- Euclid ant

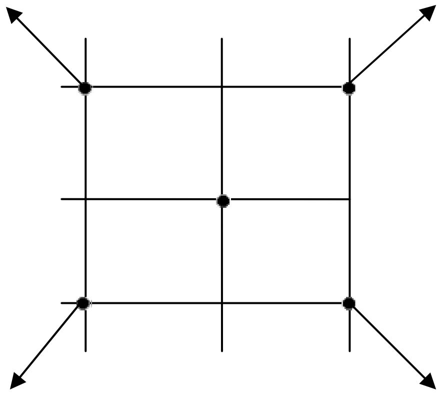


Can work out that the angles of triangle = 180^o, circumference = 2πr
- Riemann ant → Space might be curved into a 4th dimension



- "sphere"
- Small region, Euclid applies.
- Globally, angles > 180^o
- Globally, the circumferences of circles < 2πr
- Einstein ant – really does describe the geometry of space as we live on it.
. → Curvature is caused by mass / energy

- Hubble ant



Locally $v = H_o d$
Globally ants living on surface of expanding sphere.

Negative curvature – imagine something curved the other way. $(\rho_{mass} + \rho_{dark} < 1)$
(Similar to saddle)

LOCALLY things are simple – use normal dynamics, and Special Relativity.
GLOBALLY need the full apparatus of curvature and general relativity (GR)

"Paradoxes"

Where did the big bang go off?
Everywhere. (Positive curvature [4D hypersphere]
Space is just on the surface of a small hypersphere.

Universe is expanding, so does everything expand with it?
No – locally (us and nearby galaxies), space curvature is negligible.
Nearby galaxies stay the same size – use normal dynamics and SR.
Anything that is gravitationally bound together will stay gravitationally bound together.
Influence of this is much greater than anything to do with the expansion of the universe.

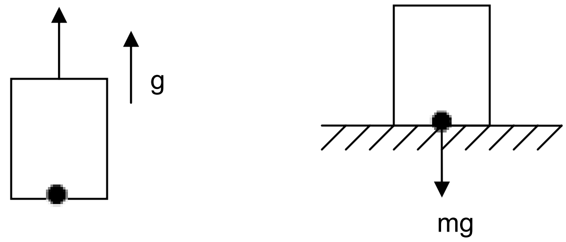
Where is the edge of the universe?
Not a good concept – for positive curvature, the universe is finite but has no boundaries and no centre.
Negative curvature can be an infinite universe, but again has no boundaries and no centre. (Universe will appear finite as light from the rest of the galaxies won't have reached us yet)

What about local dense matter, black holes etc?
Locally, flat local universe is occasionally disturbed (puckered) by mass.
Around a black hole, non-flat enough to need general relativity. Although in a small area around the gas, space and time will be locally flat so dynamics and SR can be used.

6.4 General Relativity

6.4.1 Principle of Equivalence

The laws of physics are the same in an accelerating frame and a <u>uniform</u> gravitational field.



Inertial mass $\frac{F}{a}$ and gravitational mass $\frac{F}{\left(\frac{GM}{r^2}\right)}$ are the same.

In Newton's laws, this is an assumption. Einstein defined this.

This is really quite unexpected: e.g. accelerating space ship:
Second flash has a shorter time to get to the bottom of the space ship – you "catch up".
Flashes as "ticks of a clock", more closely spaced at bottom of spaceship.
Therefore clock runs slower at the bottom of the spaceship.
Clocks run at different rates at different points in a uniform gravitational field!

6.4.2 Transition to Curvature

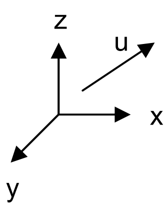
In a small region of space-time, the curvature is negligible. Therefore, space-time is flat in small regions. Maths of Special Relativity can be used.
≡ (principle of equivalence)
In a small region of space-time the gravitational field is uniform. Space-time is describable by special relativity.

BUT

If you go to a region of space-time where the gravitational field is not uniform (Produced by masses), this is equivalent to curvature. Curvature must be caused by mass → General Relativity.

Curvature → $A \neq 4\pi r^2$ → $r_{earth} \neq \sqrt{\frac{A}{4\pi}}$ (Off by a few mm – for a neutron star this would be much bigger).

6.4.3 Scalar Products in 3D



Length $u.u = \Delta x^2 + \Delta y^2 + \Delta z^2$ ← Invarient.

For fun, $\begin{pmatrix}\Delta x & \Delta y & \Delta z\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta x\\\Delta y\\\Delta z\end{pmatrix}$ u.g.u

Scalar product $\sum g_{\mu\nu}u_{\mu}u_{\mu}$

$$\left[gu = \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}\begin{pmatrix}\Delta x\\\Delta y\\\Delta z\end{pmatrix} = \left(g_{ll}u_l + g_{l2}u_2 + g_{l3}u_3\right) = \left(\sum g_{i\nu}u_{\nu}\right)\right]$$

6.4.4 'Scalar Products' in SR

Suppose there are two events separated by distance Δt, Δx, Δy, Δz

For fun, $(\Delta t \quad \Delta x \quad \Delta y \quad \Delta z)\begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$

$-c^2\Delta t^2 + \Delta x^2 + \Delta y^2 + \Delta z^2$ ← Invariant. Take any frame; you will get the same answer even though Δt, Δx, ... may be different.
This is the maths of Special Relativity. The matrix is known as the "SR Metric", or "Minkowski metric".

Invariant quantity with momentum and energy.
$p \rightarrow (E, p_x c, p_y c, p_z c)$
$-E^2 + p_x c^2 + p_y c^2 + p_z c^2$ invariant.

6.4.5 Curvature and GR

Curvature of space-time is described by the 2nd derivatives of the space-time metric. Take a person standing on a 4d hypersphere, or a normal sphere. The first derivative tells you about the straight tangent, while the second will describe the curve of the sphere.
$G(2nd.derivative.of.g) = mass / energy$
$\mu\nu^{th}$ component of thing on RHS is the flow of p_μ over surfaces of constant u_ν.
00th component → flow of E over constant time → energy.
10th component → Flow of p_x over constant t → momentum
11th component → flow pf p_x over constant t → pressure.

Therefore Mass/Energy consists of mass, energy, momeutm, pressure → $T_{\mu\nu}$ ← Energy-Momentum tensor.

$$G(g) = \frac{8\pi G}{c^4}T$$
$$G_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$
→ mass etc → causes space-time curvature.

No questions on General Relativity in the exam!!!

Examples Questions:

Sources:
- Examples sheet
- Past exam papers
- Zeilik & Gregory

Question 3:

Pole star: 90° at pole, 53° here in Manchester.
Declination 45 star will always be 45° from pole star.
Therefore at highest point, 53°+45°=98° from horizon, 8° from zenith.
Elevation is 82° at highest point.

Light fades faster in the tropic:
At the tropics, the sun goes down almost vertically therefore fades faster than at Manchester.

Question 4

$r = \theta d$ (θ is in radians)

Kepler's 3rd law: $\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$ (M = mass of central object, if M>>m, T=time in seconds)

Conversion between arcseconds and radians is $\frac{1}{206000}$.

Question 8

Frequency of H: 1420MHz

Question 10

$v = H_o d$

$\frac{v}{c} = \frac{\Delta\lambda}{\lambda} = z$