For sections 1.1, 1.2, see handouts.

1.3 Particle Horizons

The FRW universe has a particle horizon which represents the distance over which particles are in causal contact.

First, consider the metric (or proper distance) between $r = r_1$ and $r = r_2$ at constant t, θ, ϕ.

$$ds = \sqrt{g_{rr}}\, dr$$

$$d_m(t) = a(t)\int_{r_1}^{r_2} \frac{dr}{\sqrt{1-kr^2}}$$

Now consider a light ray emitted at $r = r_H$ at $t = 0$, and observed at $r = 0$ at time t_0. r_H represents the horizon of points in causal contact from $t = 0$.

Null geodesic, hence $ds = 0$ (also $d\theta = d\phi = 0$).

$$\rightarrow dt^2 = a^2 \frac{dr^2}{1-kr^2}$$

$$\rightarrow \int_0^t \frac{dt}{a} = \int_{r_H}^0 \frac{dr}{\sqrt{1-kr^2}}$$

The proper distance to this horizon at time t is

$$d_H(t) = \int_0^{r_H} \sqrt{g_{rr}}\, dr = a(t)\int_0^{r_H} \frac{dr}{\sqrt{1-kr^2}}$$

$$= a(t)\int_0^t \frac{dt'}{a(t')} = a(t)\eta(t)$$

If $a \propto t^p$, then

$$d_H(t) = t^p \int_0^t \frac{dt'}{a(t')^p}$$

$$= t^p \left[\frac{(t')^{1-p}}{1-p}\right]_0^t$$

which is finite if $1-p > 0$, i.e. $p < 1$. If $0 < p < 1$ then $d_H(t) \propto t$.

NB: $p = 2/3$ in the matter era, and $p = 1/2$ in the radiation era.

Define the Hubble Radius to be $H^{-1}(a)$, and the comoving Hubble Radius to be $H^{-1}a^{-1}$, then the power law cosmologies

$$\rho \propto a^{-3(1+w)}$$

$$\rightarrow d_H(t) \propto H^{-1}$$

1.4 A Brief History of Time

1.4.1 Equal Matter and Radiation

Defined by $\rho_r = \rho_m$.

$$\rightarrow \frac{\rho_m}{\rho_r} = \frac{\Omega_m}{\Omega_r} a = 1$$

$$\rightarrow a_{eq} = \frac{\Omega_r}{\Omega_m}$$

$$1 + z_{eq} = \frac{1}{a_{eq}} = \frac{\Omega_m h^2}{\Omega_r h^2}$$

$$= 23980\left(\Omega_m h^2\right)$$

This is approximately 3000 for $\Omega_m h^2 = 0.012$.

$$T_{eq} = \frac{T_\gamma\left(t_0\right)}{a_{eq}}$$

$$= 65417\left(\Omega_m h^2\right)k$$

$$= 5.64\left(\Omega_m h^2\right)eV$$

1.4.2 Recombination

$$T_{rec} \approx 0.25eV$$

$$\rightarrow 1 + z_{rec} \approx 1000$$

This is the epoch when $p + e^- \rightarrow H$, and is also the epoch of last scattering of the Cosmic Microwave Background (CMB).

1.4.3 Nucleosynthesis

$$T_{nuc} \sim 1MeV$$

$$1 + z_{nuc} \approx 4 \times 10^8$$

This is when hadrons aggregate into nuclei (i.e. formation of $n + p \rightarrow D, {}^3He, {}^4He, {}^7Li$)

[NB: this is also the neutrino disassociation energy]

1.4.4 Quark-Hadron Transition

Also called the confinement transition. It is when quarks form into hadrons.

$$T_{QCD} \approx 200MeV$$

$$1 + z_{QCD} \approx 8 \times 10^{11}$$

[NB: these all happen in the radiation era. Using the redshift, and the relationship with time of $t^{1/2}$, we can calculate when these events happened]

1.4.5 ElectroWeak Phase Transition

This is when the ElectroMagnetic and the Weak force become distinct $\left(SU(2) \times U(1) \rightarrow U(1)\right)$

$$T_{EW} \approx 200GeV$$

$$1 + Z_{EW} \approx 8 \times 10^{14}$$