Photon Decoupling
This is when Thompson scattering (the reaction between photons and electrons) stops. The interaction rate is

$$\Gamma_T = n_e \sigma_T ,$$

where n_e is the number density of electrons, and $\sigma_T = 6.65 \times 10^{-25} cm^2$ is the Thompson scattering cross-section. Decoupling occurs when the interaction rate drops below the Hubble rate, when

$$\Gamma_T = H .$$

In the matter era, $\Gamma \propto a^{-3}$ (due to the number density decreasing) while $H \propto a^{-3/2}$. By working this out more precisely (i.e. getting the exact relation between H and a, and equating it to Γ_T), we get the time of decoupling to be

$$T_{dec} \approx 0.25 eV$$

This is also known as the time of last scattering of the CMB.

For $T >> T_{dec}$, the photons and baryons (electrons) are tightly coupled, and the mean free path of the photon is effectively zero. This is known as the *tight-coupling regime*.

For $T << T_{dec}$, the mean free path of the photon is effectively infinite and the photons 'free stream' towards us.

6.4 Density and Velocity Fluctuations

The universe is not isotropic and homogenous at decoupling. Since $H \propto \rho^{1/2}$ (Friedman equation), decoupling occurs at different times in different places, creating temperature anisotropies due to density fluctuations. This means that the surface of last scattering is closer/further away from us than the median.

However, the radiation fluid is moving at this time, and hence the velocity fluctuations create temperature fluctuations via the Doppler effect. This creates differences in any direction (depending on the velocity directions) of regions of different temperature.

6.4.1 Dynamics of the Tight Coupling Regime
In the tight coupling regime, the photons and baryons can be represented by a photon gas / radiation fluid, with $p = \frac{1}{3}\rho$. Hence modeling this regime involves just the dynamics of matter and radiation.

Recall EOM:

$$\delta_r ' = \frac{4}{3}\left(\delta'_m - \theta_r\right)$$

$$\theta_r ' = \frac{1}{4} k^2 \delta_r \quad \text{(velocity perturbations)}$$

Differentiating,

$$\delta_r '' + \frac{1}{3} k^2 \delta_r = \frac{4}{3} \delta_m ''$$

$$\theta_r = \delta_m ' - \frac{3}{4} \delta_r '$$

On super-horizon scales,

$$\delta_r = \frac{4}{3}\delta_m = \frac{4}{3}A(k)k^2\eta^2$$

On sub-horizon scales, there are two possibilities; $\delta_m \propto \eta^2$ is the matter era, and $\delta_m \propto 1$ in the radiation era.

We will consider solutions which come inside the horizon during the radiation era, for simplicity. We will assume that photon decoupling takes place within the radiation era. In fact, $T_{dec} < T_{eq}$, though not by much.

For $\eta < \eta_H = \frac{2\pi}{k}$, i.e. before the horizon crossing,

$$\delta_r = \frac{4}{3}A(k)k^2\eta^2,$$

and for $\eta < \eta_H$, i.e. after the horizon crossing,

$$\delta_r'' + \frac{1}{3}k^2\delta_r = 0,$$

and hence

$$\delta_r = \alpha\cos\left(\frac{k}{\sqrt{3}}(\eta - \eta_H)\right)$$

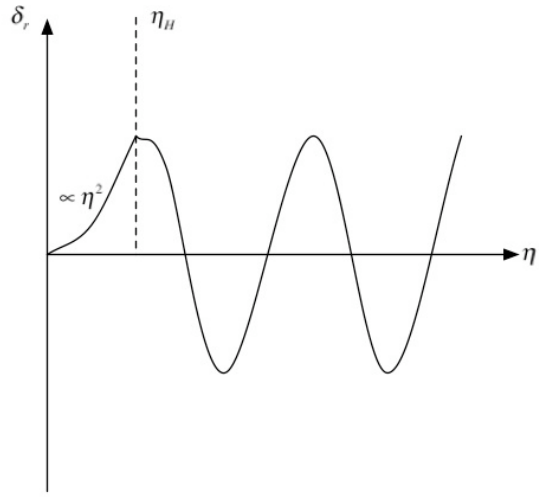
where α is a constant. If we match the solution at $\eta = \eta_H$ then

$$\delta_r = \begin{cases} \frac{4}{3}A(k)k^2\eta^2 & \eta < \eta_H \\ \frac{4}{3}A(k)(k\eta_H)^2\cos\left[\frac{k}{\sqrt{3}}(\eta - \eta_H)\right] & \eta > \eta_H \end{cases}$$

For $\eta > \eta_H$,

$$\delta_r = \frac{16\pi^2}{3}\cos\left[\frac{k\eta}{\sqrt{3}} - \frac{2\pi}{\sqrt{3}}\right]$$

See diagram, right.



One can also compute $\theta_r = \delta_m' - \frac{3}{4}\delta_r'$

$$\theta_r = \begin{cases} 0 & \eta < \eta_H \\ -\frac{4\pi}{\sqrt{3}}k\sin\left[\frac{k\eta}{\sqrt{3}} - \frac{2\pi}{\sqrt{3}}\right] & \eta > \eta_H \end{cases}$$