6. Microwave Background

6.1 Statistics of Temperature Fluctuations

We will study the observed fluctuations in the black-body spectrum of the Cosmic Microwave Background (CMB). Effectively, we can think of the anisotropies on the sphere. We will decompose the temperature into spherical harmonics;

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{\ell=2}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta,\phi).$$

We will ignore $\ell = 0,1$, as $\ell = 0$ is the monopole, i.e. the uniform background, and $\ell = 1$ is the dipole due to the Earth's relative motion with respect to the CMB – it is not of primordial origin. $\ell = 2$ is known as the quadrupole, and is the first primordial mode.

The spherical harmonics are defined as

$$Y_{\ell m}(\theta,\phi) = \sqrt{\frac{2\ell+1}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}} e^{im\phi} P_\ell^m(\cos\theta),$$

where $P_\ell^m(\cos\theta)$ are the associated Legendre functions.

$$\int Y_{\ell m}(\theta,\phi) Y_{\ell' m'}{}^*(\theta,\phi)\, d\Omega = \delta_{\ell\ell'}\delta_{mm'},$$

where the $d\Omega$ represents the surface element of the sphere, running over θ,ϕ.

→ $$a_{lm} = \int d\Omega\, Y_{\ell m}{}^*(\theta,\phi) \frac{\Delta T}{T}(\theta,\phi).$$

One can define the angular correlation function

$$C(\theta) = \left\langle \frac{\Delta T}{T}(\psi) \frac{\Delta T}{T}(\psi+\theta) \right\rangle_\psi$$

$$= \frac{1}{4\pi}\sum_{\ell=2}^{\infty}\left(\sum_{m=-\ell}^{\ell}|a_{\ell m}|^2\right)P_\ell(\cos\theta)$$

$$= \frac{1}{4\pi}\sum_{\ell=2}^{\infty}(2\ell+1)C_\ell P_\ell(\cos\theta)$$

where

$$C_\ell = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}|a_{\ell m}|^2,$$

which is effectively an azimuthal average.

[NB: the angular brackets represent an integration over ψ, sort of like an average.]

If the temperature fluctuations are Gaussian, as suggested by inflation, then the entire distribution is specified by knowledge of the C_ℓ's, with

$$a_{\ell m} \sim N\left(0,\sqrt{C_l}\right)$$

i.e. the $a_{\ell m}$'s have a statistical distribution, rather than an exact known distribution.

We will usually plot

$$\left\langle \Delta T_\ell^{\,2} \right\rangle = \frac{\ell(\ell+1)}{2\pi} C_\ell T_{CMB}^{\;\;2}$$

in units of $(\mu K)^2$ (micro-Kelvin squared). This corresponds to temperature anisotropy on an angular scale $\theta \approx \frac{180°}{\ell}$, that is $\ell = 180$ corresponds to $1°$.

On small scales, we can treat the curved sky as if it were flat, and we can use the approximate relation

$$k \approx \frac{\ell}{\eta_0}$$

to convert from ℓ, the dimensionless angular wave number, to k, the flat space wave number, where η_0 is the conformal time at the present day.

6.2 Basic features of the angular power spectrum

(See handout for illustration – note that the one drawn in the lecture was logarithmic)

1. The SW (Sachs-Wolfe?) plateau is due to potential fluctuations, and is almost flat from $\ell = 5 \to 100$.
2. The ISW (Integrated Sachs-Wolfe?) effect is due to the decay in the gravitational potential during $\Lambda -$ domination.
3. Equally spaced peaks and troughs are due to acoustic oscillations in the radiation fluid prior to last scattering, and are imprinted at recombination, i.e. $\delta_r'' + \frac{1}{3}k^2\delta_r = 0$.
4. The damping envelope is due to photon diffusion and the coupling to baryons.

Note that what we actually observe is

$$C(\theta)_{obs} = \frac{1}{4\pi}\sum_{\ell=2}^{\infty}(2\ell+1)W_\ell^{\,2} C_\ell P_\ell(\cos\theta)$$

where we have multiplied by

$$W_\ell = \exp\left[-\frac{\ell^2\sigma^2}{2}\right]$$

the window function of the telescope beam with full-width half-max ($1.22\frac{\lambda}{d}$) $\Omega_{FWHM} = 2.35\sigma$.

6.3 Recombination and Photon Decoupling

At very early times, photons thermalise into the black-body spectrum. The important reactions are:

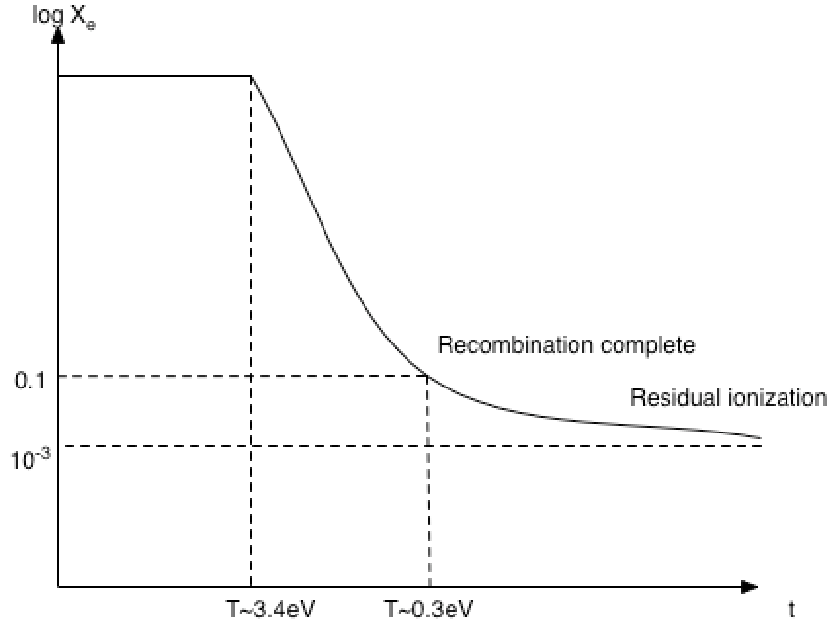
$p + e^- \leftrightarrow H + \gamma$ - atomic recombination

$e^- + \gamma \leftrightarrow e^- + \gamma$ - Thomson scattering

<u>Recombination</u>

Ionization energy of hydrogen is $I = 13.6eV$, but recombination cannot occur directly to the ground state, and the transition from the $n = 2$ state with $I \approx 3.4eV$ is the relevant one.

When the temperature of the universe $T < 3.4eV$, the recombination reaction moves to the right and the number of free electrons decreases.



where X_e is the fraction of free electrons. We say that recombination has finished at $T \approx 0.3eV$, although the process continues afterwards.