Mistake from first 2 lectures:

$$\nu_e d \to epp$$

$$d + u \to W^+ \to \left(\nu_e + e^-, \nu_\mu + \mu^-\right)$$

Sudbury ignores the latter muons as they can't distinguish them from cosmic ray muons, hence why this case was earlier considered to just got to muons.

$$|\psi\rangle = -\sin\theta e^{-iE_1 t}|\nu_1\rangle + \cos\theta e^{-iE_2 t}|\nu_2\rangle$$

$$= \left(\cos^2\theta e^{iE_1 t}....\right)|\nu_\mu\rangle + \left(\sin\theta\cos\theta...\right)|\nu_e\rangle$$

Before, had ν_μ and ν_e the wrong way round.

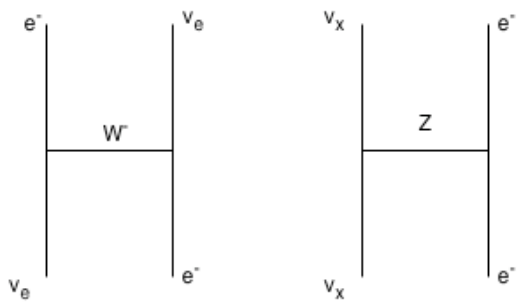
(Handout 1)

So amplitude for $\nu_\alpha \to \nu_b$ $= \sum_i U_{\alpha i} * \exp\left(-im_i^2 \frac{L}{2E}\right) U_{\beta i}$.

The MSW effect

(Mikheyer-Smirnov-Wolfenstein)

Inside the sun, the ν_e interact differently from ν_μ and ν_τ .



First is open only to electrons; the latter is open to all neutrinos. This changes the effective mass of eigenstates and leads to an additional factor

$$Prob\left(\nu_e \to \nu_\mu\right) = \frac{\sin^2 2\theta}{w^2}\sin^2\left(1.27 w \Delta m^2 \frac{L}{E}\right)$$

where

$$w^2 = \sin^2 2\theta + \left(\sqrt{2} G_f Ne\left(\frac{2E}{\Delta m^2}\right) - \cos 2\theta\right)^2$$

Ne is the electron density, G_f is the Fermi constant.

This could also happen in Earth of course, so SNO looks for a 'day/night' effect in Solar data, i.e. neutrinos must pass through the earth at night. A very slight increase in ν_e detection at night is observed.

The state of the art, 2006

- Atmospheric neutrinos: disappearance of ν_μ

  $\Delta m_A^{\ 2} \approx 3 \times 10^{-3} eV^2$

  Mixing angle (A denotes atmospheric): $\theta_A = \frac{\pi}{4}$. Notice that θ_A is ~maximal, i.e. $45°$. So atmospheric data implies $\nu_\mu \to \nu_\tau$ with maximal mixing.

- K2K: ν_μ disappear – consistent with atmospheric neutrinos.
- SNO: Total solar ν flux correct, but $\sim \frac{1}{3}\nu_e$. If atmospheric data is correct, remaining $\frac{2}{3}$ are $\sim$ equal mixture of ν_μ and ν_τ. 'Large mixing angle MSW' provides the best fit. $\Delta m_s^2 \approx 6\times10^{-5}eV^2$, $\theta_s \sim \frac{\pi}{6}$
- Kamland: $\bar{\nu}_e$ disappearing – consistent with the above.

See handout 2.

The Mixing Matrix

$$u = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}}_{Atmospheric} \underbrace{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{Solar} \underbrace{\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}}_{Cross-mixing} \underbrace{\begin{bmatrix} e^{i\frac{\alpha_1}{2}} & 0 & 0 \\ 0 & e^{-\frac{\alpha}{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{Majorana\ \not{CP}\ phases}$$

where c denotes cosine, and s denotes sine.

$$\theta_{12} \approx \theta_{solar} = \theta_\odot = 32°,\ c_{12} \sim \frac{\sqrt{3}}{2},\ s_{12} \sim \frac{1}{2}$$

$$\theta_{23} \approx \theta_{atm} \approx 45°,\ c_{23} \sim \frac{1}{\sqrt{2}},\ s_{23} \sim \frac{1}{\sqrt{2}}$$

$\theta_{13} \le 15°$ ($\bar{\nu}_e$ disappearance)

It seems to be that muon neutrinos oscillate to tao neutrinos, while electron neutrinos mix to muon neutrinos, and not intermixed.

$$u = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \tau_\tau \end{pmatrix} \begin{bmatrix} \nu_1 & \nu_2 & \nu_3 \\ c_\odot & s_\odot & s_{13} \\ -\frac{s_\odot}{\sqrt{2}} & \frac{c_\odot}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{s_\odot}{\sqrt{2}} & -\frac{c_\odot}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \sim \begin{pmatrix} 0.8 & 0.5 & ? \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

See handout 3.

The LSND experiment (Los Alamos)

$\bar{\nu}_\mu$ beam (from $\pi^+ \to \mu^+\nu_\mu$ etc.)

Detects $\bar{\nu}_e$ by $\bar{\nu}_e + p \to e^+ + n$.

Observed $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations such that $\Delta m_{LSND}^2 \sim 1eV^2$.

BUT: for 3 mass eigenstates there are only 3 possible splittings. $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$, which obviously satisfies the relation $\Delta m_{32}^2 + \Delta m_{21}^2 + \Delta m_{13}^2 = 0$. Δm_{LSND}^2, $\Delta m_\odot^2$ and Δm_{atm}^2 do not obey this constraint.

If this result is correct, there must be (at least) 1 more light mass eigenstate. We know that there are only 3 charged leptons (e,μ,τ). So there must be a linear combination of the ν_i (ν_s) that does not couple to W.
Also, from LEP data, $Z \to \nu\bar{\nu}$ decays only involve 3 distinct ν species. So ν_s does not couple to the Z either., e.g. ν_s is "sterile" – it does not interact via EM, strong or weak interactions.
This result is controversial, unlike solar and atmospheric results – will be checked by future experiments.