1. A bit of history
2. What is the corona and what is the problem of coronal heating?
3. Magnetic coronal heating: the role of magnetic reconnection
4. Nanoflares scenario of coronal heating: how it works and how it can be probed observationally

Reading list:

Easy reading for general interest: … “Nearest Star”, Harvard University Press, 2001

Helium – first discovered in the stellar corona 1868 (?)
1869 – another unknown spectral line discovered, called Choronium. From calculations of the gravitational height, the mass of Choronium should be much less than Helium – a puzzle, when compared with the periodic table. Resolved in 1939; spectral line was actually a line of highly ionized iron. This meant that the temperature in the corona should be very high – a million k or so – which was a new puzzle, as the surface of the sun is much less than this and temperature decreases proportional to R. This problem is still not entirely solved.

Corona:
$T_c \sim 10^6 k$

Quiet areas of the corona (i.e. not many flares):
Number density of particles: $n_c \sim 10^8 cm^{-3}$
Energy supply required: $q_c (erg / cm^2 s) \sim 3 \times 10^5$

Active regions of the corona (flare areas):
Number density of particles: $n_c \sim 10^9 cm^{-3}$
Energy supply required: $q_c (erg / cm^2 s) \sim 10^7$

Solar luminosity: $L_\odot \sim 6.3 \times 10^{10} erg / cm^2 s$.

Due to the solar corona’s temperature, the radiation is given off as IR and X-ray.

“Old” theory of coronal heating: Acoustic heating
Main drawbacks:
- Observed acoustic energy flux is too small
• Strong correlation between the X-ray activity in the corona and the magnetic field $\rightarrow$ magnetic nature of solar coronal activity $\rightarrow$ Magnetohydrodynamics

Magnetic field on the surface of the sun can be measured using the Zeeman splitting of spectral lines. Sun spot – area of cooler gas – caused by the magnetic field holding the hotter gas back.

$\beta << 1$ - dynamics is governed by magnetic field.

$\beta >> 1 \rightarrow$ magnetic field is determined by convective motions.

$B \sim 10^7 G$, $L \sim 10^3 cm \rightarrow v_a \sim 10^3 km / s \rightarrow \tau_A \sim \frac{L}{v_A} \sim 10 s$

$\tau_{ph} \sim \frac{\ell_{ph}}{v_{ph}} \sim \frac{10^3 km}{1 km / s} \sim 10^3 s \gg \tau_A$

(DC currents) quasi-static evolution – force free magnetic field: $j \parallel B$

Magnetically open region (coronal hole) $\rightarrow$ generation of … waves propagating upwards (AC currents)

A viable energy source – the maximum Poynting flux

$S_{max} \sim v_{ph} \frac{B^2}{4\pi} \sim 10^8 erg / cm^2s \gg q_e$

Magnetohydrodynamics:

\[
\vec{j} = \nabla \times \vec{B} = \mu_0 \vec{j}
\]

$\rightarrow j = \frac{1}{\mu_0} (\nabla \times B)$

\[
\rho, \nu, p
\]

$\frac{\rho d\vec{v}}{dt} = -\nabla P + \left( \frac{j}{\mu_0} \times \vec{B} \right)
\]

\[
= -\nabla P + \frac{1}{\mu_0} \left( \nabla \times B \right) \times B - \frac{1}{\mu_0} B \times \left( \nabla \times B \right)
\]

\[
= -\nabla P - \frac{1}{\mu_0} \left( \nabla \frac{B^2}{2} - \left( B \cdot \nabla \right) B \right)
\]

\[
= -\nabla \left( P + \frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} \left( B \cdot \nabla \right) B
\]

where $\left( \frac{j}{\mu_0} \times B \right)$ is the magnetic force, $\nabla \frac{B^2}{2}$ is the magnetic pressure and $\left( B \cdot \nabla \right) B$ the magnetic tension. Call the last version of this equation (1).

$\beta = \frac{P}{B^2 / 2\mu_0}$

$\beta << 1 \rightarrow$ magnetic force dominates
\( \beta \gg 1 \rightarrow \) thermal pressure dominates.

We need another equation which governs how \( B \) evolves with time – magnetic induction equation.

Ohm’s law: \( E = \eta \cdot j \rightarrow I = \frac{V}{R} \)

Electric force = resistivity \( \times \) current

Add magnetic term: \((v \times B) + E = \eta \cdot j\)

Maxwell’s equation: \(- (\nabla \times E) = \frac{\partial B}{\partial t}\)

Hence, we have:

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \frac{\eta}{\mu_0} \nabla^2 B
\]

(NB: we have assumed \( \mu_0 = 1 \))


Resistive diffusion?

Estimate: \( \nabla \times (v \times B) \sim \frac{vB}{L} \) and \( \frac{\eta}{\mu_0} \nabla^2 B \sim \frac{\eta B}{\mu_0 L^2} \)

Hence:

\[
\frac{vB}{L} \frac{\mu_0 L^2}{vB} \sim \frac{\mu_0 vL}{\eta} \rightarrow \text{Magnetic Reynold’s Number } R_m
\]

(\( R = \frac{vL}{\nu} \rightarrow \text{ Reynold’s Number} \))

Typically, \( R_m \gg 1 \).

If we neglect the last term of the equation, we get

\[
\frac{\partial B}{\partial t} = \nabla \times (v \times B) \quad (2)
\]

This can be seen as a frozen-in magnetic field: the magnetic field flows with the fluid.

It cannot change its geometrical structure / topology. This is a very strong constraint.

Called “Ideal MHD” (Ideal MagnetoHydroDynamics)

Return to (1). Take the LHS and the first part of the RHS.

\[
\rho \frac{dv}{dt} = -\nabla P
\]

\[
\rho \frac{\Delta v}{\Delta t} = -\frac{P}{\mu}
\]

\( v \sim \frac{P}{\sqrt{\rho}} \)

This is the case when \( \beta << 1 \). When \( \beta >> 1 \), then we have:

\[
\rho \frac{v}{\Delta t} \sim \frac{B^2}{\mu_0 \Delta L}
\]

\( v = \frac{B^2}{\mu_0 \rho} \sim \frac{B}{\sqrt{\mu_0 \rho}} \rightarrow \text{Alfven Velocity} \)
\((j \times B) = 0\) means that the current is flowing along the magnetic field lines, \(\rightarrow\) called force-free magnetic fields.

**A viable energy source?**

We know that we need to provide \(10^7 \text{ erg/cm}^2 \text{s}\), or \(10^4 \text{ J/m}^2 \text{s}^{-1}\), to the surface for active regions. Does magnetic heating provide this?

Use the Poynting vector.

\[ S = \frac{1}{\mu_0} (E \times B) \]

Assume that the magnetic field is frozen, and there is no \(\eta\) - so we have \(E = -(v \times B)\). Hence:

\[ S = -\frac{1}{\mu_0} (v \times B) \times B \]

\[ = \frac{1}{\mu_0} B \times (v \times B) \]

\[ = \frac{1}{\mu_0} \left[ vB^2 - B(v \cdot B) \right] \]

\[ \approx \frac{vB^2}{\mu_0} \]

Put in some numbers…

\[ S \sim 10^3 \left( \frac{\text{ms}^{-1}}{4\pi \times 10^{-7}} \right) \times \left( 10^{-2} [T] \right)^2 \sim 10^5 \text{ J/m}^2 \text{s}^{-1} \]

So we have enough energy. Only a fraction of this will actually be turned into heat, but there is enough available to do this.

**Magnetic Reconnection**

Remember that we have

\[ \frac{\partial B}{\partial t} = \nabla \times (v \times B) + \frac{\eta}{\mu_0} \nabla^2 B. \]

How do we increase the dissipation?

Dissipation in a wire is \(Q = I^2 R\). In this case, we have \(Q = \eta j^2\). So if we have a small \(\eta\), we need to get a large \(j\).

From \(\nabla \times B = \mu_0 j\), we have \(j \sim \frac{B}{\mu_0 e}\).

If we use the full time-derivative equation, then the second term allows the breaking of magnetic field lines. This allows the reconnection of magnetic field lines.
Take a volume of magnetic field. The total energy will be \( W_M \sim \frac{B^2}{2\mu_0} L^3 \). The dissipation power will be \( \dot{W}_M \sim \eta j^2 L^3 \sim \eta L^3 \frac{B^2}{\mu_0^2} \frac{1}{L^2} \sim \eta L \frac{B^2}{\mu_0^2} \). So the time to dissipate all the energy will be:

\[
\tau_\eta \sim \frac{W_M}{\dot{W}_M} \sim \frac{L^2 \mu_0}{\eta}
\]

For the solar corona, this time turns out to be circa a million years. Reconnection can make this much quicker.

\[
u \cdot B \sim \frac{L B}{\mu_0 \Delta}
\]

\[
u L \sim v_A \Delta \Rightarrow u = \frac{v_A \Delta}{L}
\]

\[
\Delta = \sqrt{\frac{\eta L}{\mu_0 v_A}}
\]

\[
u \sim \frac{v_A}{\mu_0 v_A} \sqrt{\frac{\eta}{L \mu_0 v_A}}
\]

\[
S = L \mu_0 u B^2 \sim L \mu_0 B^2 v_A \sqrt{\frac{\eta}{L v_A \mu_0}}
\]

Total magnetic energy is \( U_B \sim \frac{B^2}{2\mu_0} L^3 \)

So the energy dissipation through reconnection is \( \tau_B = \frac{U_B}{S} \approx \frac{L/2\mu_0}{\mu_0 v_A \sqrt{\frac{\eta}{L v_A \mu_0}}} \ll \tau_\eta \).