CP Violation & BaBar, Continued
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Purpose of BABAR: Precision measurements of CP violation in B-mesons.
PEPII collider – 9GeV electrons and 3.1GeV positrons.
CM energy $= M(\Upsilon(4s))$ - decays to B^+B^- or $B^0\overline{B}^0$.
Asymmetric beam energies → B decay vertices are separated in space by a measurable distance.

BABAR Detector:

- SVT – tracks charged particles & locates decay vertices
- DCH – tracks charged particles & measures track momentum via curvature in magnetic fields, $p_r = 0.3Br$, and the angle
- DIRC – identifies charged hadrons via the Cerenkov angle $\cos\theta_0 = \frac{1}{n\beta}$.
  $p = \beta\gamma m$.
- EMC – detects and measures energy of neutrals i.e. γ. Also identifies electrons and muons through E/p.
- IFR – identifies muons.

Mixing

Leptons: $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$

Charged weak interactions can give transitions within generations, e.g. $e^- \to \nu_e W^-$.
Transitions between generations are not allowed, e.g. $\mu^- \to \nu_e W^-$.

Quarks: $\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix}\begin{pmatrix} t \\ b \end{pmatrix}$

Kaons decay, therefore $s \to u$ allowed. $s \to uW^-$.

Cabibbo: the s-quark and d-quark take part in weak interactions as a mixture of each other; $d' = d\cos\theta_c + s\sin\theta_c$ and $s' = -d\sin\theta_c + s\cos\theta_c$. Note that θ_c here is not related to the Cerenkov angle – it is the Cabibbo angle and is around $12°$.

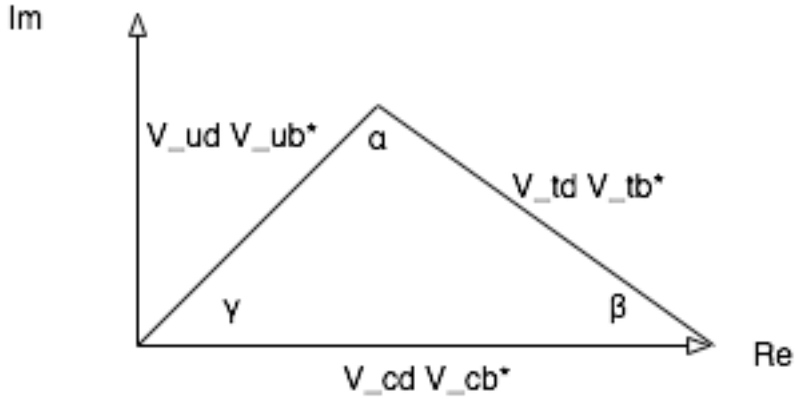
Kabayashi & Maskowa: 3 generations.
Relation between flavour eigenstates (b, s or d quarks) and the weak eigenstates is now held in the CKM matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM}\begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Note: It is convention to write this using the d, s and b quarks; it could quite easily be done with the u, c and t.

- CKM matrix is a 3x3 Unitary matrix - $V^\dagger V = I$.
- Can write Unitarity relations, e.g.:
  $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

- Can represent this as a triangle on the complex plane.



This is a unitary triangle. The area of the triangle is proportional to the amount of CP violation in SM.
Goal of BABAR: overconstrain U.T. by measuring sides and angles as accurately as possible.

General treatment of mixing in neutral mesons
Schrödinger equation for a single particle in its rest frame is:

$$i\hbar \frac{\partial|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle = E|\psi\rangle$$

Rearrange this:

$$\frac{\partial|\psi\rangle}{\partial t} = -\frac{imc^2}{\hbar}|\psi\rangle$$

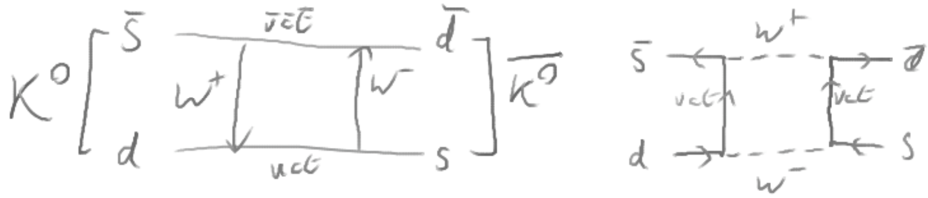
Take $\hbar = 1$ and $c = 1$. So we just have

$$\frac{\partial|\psi\rangle}{\partial t} = -im|\psi\rangle$$

Solution: $|\psi(t)\rangle \propto e^{-imt}$
Wave function is oscillating with a frequency of the rest mass. Now say that the particle can decay with rate Γ (the same as the particle's natural width).

$$\frac{\partial|\psi\rangle}{\partial t} = -im|\psi\rangle - \frac{\Gamma}{2}|\psi\rangle$$

Solution: $|\psi(t)\rangle \propto e^{-imt}e^{-\frac{\Gamma}{2}t}$
The first part is the oscillating term, while the second part is an exponential decay. Now say that we have two neutral mesons (particle / antiparticle) which are close together in mass (if they're particle and antiparticle, they have to be identical in mass), they can "mix" into each other. E.g. the Kaon sytem.



Now wavefunction for one particle has a contribution due to the presence of the other particle.

Say $|\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|\bar{\psi}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

A general linear combination is $p|\psi\rangle + q|\bar{\psi}\rangle = \begin{pmatrix} p \\ q \end{pmatrix}$.

Write transitions as:

$$M_{11} \equiv \langle\psi|\hat{H}|\psi\rangle$$
$$M_{22} \equiv \langle\bar{\psi}|\hat{H}|\bar{\psi}\rangle$$
$$M_{12} \equiv \langle\psi|\hat{H}|\bar{\psi}\rangle$$

etc.

So we have:

$$i\frac{\partial}{\partial t}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$$

i.e. we have a pair of coupled modes – start with pure ψ, end up with a mixture of ψ and $\bar{\psi}$.

We want to diagonalise this matrix, and hence decouple the domes. Unfortunately, CPT symmetry means that all particles and antiparticles have the same mass and width. Hence $M_{11} = M_{22}$.

If CP symmetry was exact, then $M_{12} = M_{21}$.

$$i\frac{\partial}{\partial t}\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}$$

We can solve this, and get eigenvalues $(A+B)(A-B)$.

Therefore the decoupled modes are:

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(|\psi\rangle + |\bar{\psi}\rangle\right) \quad \text{(1a)}$$
$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\left(|\psi\rangle - |\bar{\psi}\rangle\right) \quad \text{(1b)}$$

$|\psi_1\rangle$ and $|\psi_2\rangle$ are mass eigenstates – i.e. they are real particles which we can detect – and also CP eigenstates.

<u>Evolution with time</u>

We can rearrange the equations for $|\psi_1\rangle$ and $|\psi_2\rangle$ (1):

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right)$$
$$|\bar{\psi}\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle - |\psi_2\rangle\right)$$

Start with pure $|\psi\rangle$ state. After time t:

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-iM_1 t}e^{-\frac{\Gamma_1}{2}t}|\psi_1\rangle + e^{-iM_2 t}e^{-\frac{\Gamma_2}{2}t}|\psi_2\rangle\right)$$

We can use equations (1) to get this in terms of flavour eigenstates $|\psi\rangle$ and $|\bar{\psi}\rangle$:

$$|\psi(t)\rangle = \frac{1}{2}\left[\left(e^{-iM_1 t}e^{-\frac{\Gamma_1}{2}t} + e^{-iM_2 t}e^{-\frac{\Gamma_2}{2}t}\right)|\psi\rangle + \left(e^{-iM_1 t}e^{-\frac{\Gamma_1}{2}t} - e^{-iM_2 t}e^{-\frac{\Gamma_2}{2}t}\right)|\bar{\psi}\rangle\right] \quad \text{(2)}$$

Start with $|\psi\rangle$, probability that it is still $|\psi\rangle$ after time t is mod square of the first term of (2).

$$\begin{aligned} Prob(not\, mix) &= \frac{1}{4}\left| e^{-iM_1 t} e^{-\frac{\Gamma_1}{2}t} + e^{-iM_2 t} e^{-\frac{\Gamma_2}{2}t} \right|^2 \\ &= \frac{1}{4}\left(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-\frac{(\Gamma_1+\Gamma_2)}{2}t} \cos\left((M_1 - M_2)t\right) \right) \end{aligned} \quad (3)$$

Hence we have two terms with different lifetimes. There is a term oscillating with frequency of mass difference between $|\psi_1\rangle$ and $|\psi_2\rangle$.

Conversely, the probability to mix into $|\bar{\psi}\rangle$ is the mod square of the second term of (2).

$$Prob(mix) = \frac{1}{4}\left(e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-\frac{(\Gamma_1+\Gamma_2)}{2}t} \cos\left((M_1 - M_2)t\right) \right) \quad (4)$$

Application to K and B system:

- Kaons are very light – there are not many decay modes.
  $K_1 \to 2\pi$ - K-short (K_S) $(\tau \sim 2.7cm)$
  $k_2 \to 3\pi$ - there is not much phase space so it has a long lifetime. It is called K-long (K_L) $(\tau \sim 15.5m)$
- B-mesons are much heavier, many more decay modes – lifetimes of B_1 and B_2 are similar.
  $\Gamma_1 = \Gamma_2 = \Gamma$
  Mass difference $\Delta m_B = M_1 - M_2$ is much bigger, therefore B-heavy (B_H) and B-light (B_L)
  Therefore for B systems, equations (3) and (4) simpify to:
  $Prob(mix\, or\, not) = \frac{\Gamma}{2} e^{-\Gamma t}\left(1 \pm \cos(\Delta m_B t)\right)$ (5)
  Γ at the start is a normalization factor. $+$ for not mixing, $-$ for mixing.
  Experimentally, look at asymmetry.
  $\frac{N_{unmixed} - N_{mixed}}{N_{unmixed} + N_{mixed}} = \cos(\Delta m_B t)$.

At BABAR, pairs of B-mesons produced in coherent state. Although both mesons are continuously evolving superposition of B_L and B_H, the wavefunctions are always exactly out of phase such that when one is a B^0, the other is $\bar{B}^0$ at that exact time.

Suppose one B decays into a state that can only come from a B^0 - at that point, we know the other was $\bar{B}^0$ at that time - we "start the clock".
The other B might decay as $\bar{B}^0$, or it might mix into B^0 then decay.

- classify an event as "mixed" if we have $B^0 B^0$ or $\bar{B}^0 \bar{B}^0$, or "unmixed" if we have different flavours.

In order to measure Δm_B we need to measure the time difference Δt between B-decays, and identify whether they were B^0 or $\bar{B}^0$ when they decayed – "flavour tagging".

Measuring Δt

- In $\Upsilon(4s)$ frame, B's have very small momentum, and the decay vertices would be in the same place.
- Asymmetric beam energies → $\Upsilon(4s)$ frame is boosted with Lorentz factor $\beta\gamma = 0.56$ with respect to the lab frame. Therefore we can measure the distance along this boost direction (the z-direction) Δz between decay vertices using SVT.
  $$\Delta t = \frac{\Delta z}{\beta\gamma c}$$

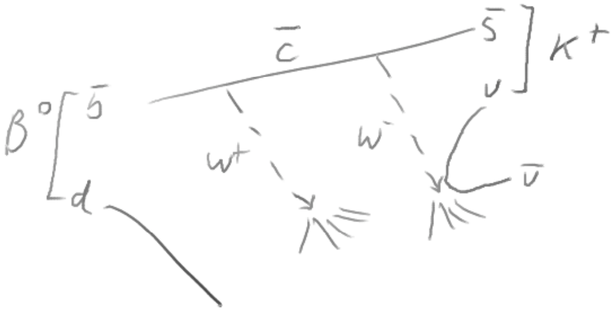
Flavour Tagging

- Could try to reconstruct the whole B-decay from the decay products, but if we did this then we would loose a lot of statistics because many decays involve neutrinos which we can't detect, also we often don't detect one or more tracks or γ's (e.g. if the particle is going straight down the beam pipe)
- Instead, use "inclusive" technique. Look for tell-tale signs of B^0 or $\bar{B}^0$ decay. Use neural nets to look for these signs.

2 examples:
Lepton tagging:
B^0 contains $\bar{b}$, and a $\bar{B}^0$ contains a b quark.

Negative lepton is an indication of $\bar{B}^0$.
Positive lepton is indication of B^0.

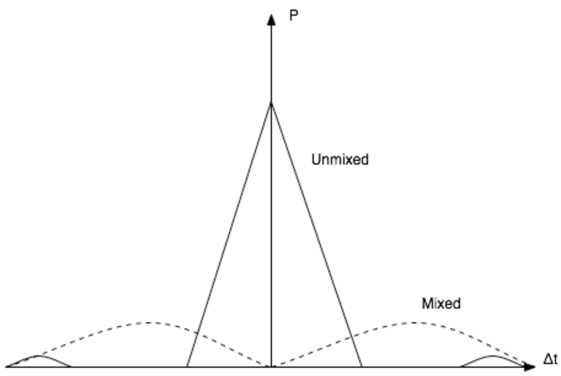
Kaon tagging:



Positive Kaon is an indication of B^0
Negative Kaon is an indication of $\overline{B}^0$.
May have other Kaons in the event, but if you look at $N(K^+)$ vs. $N(K^-)$, it is still a useful discriminant.
Another good example of use of the DIRC.

<u>Putting it all together</u>
Remember equation (5).

$$Prob(mix\,or\,not) = \frac{\Gamma}{2} e^{-\Gamma\Delta t}\left(1 \pm \cos(\Delta m_B \Delta t)\right)$$

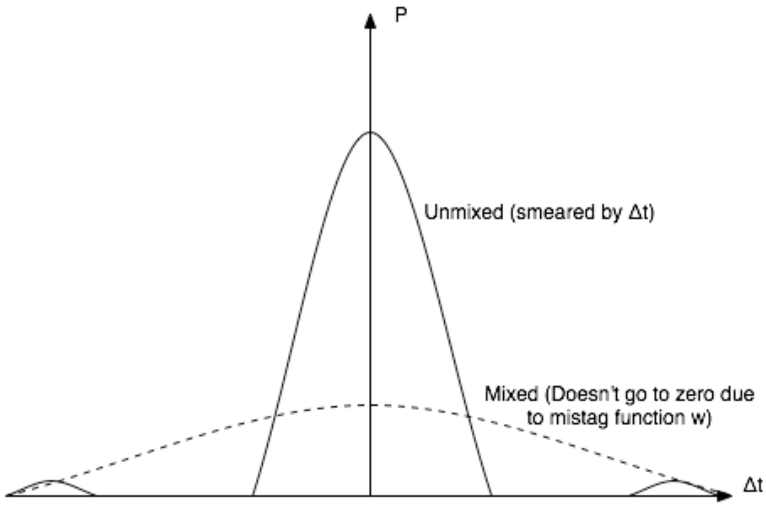


But:

- We can't measure Δt perfectly – resolution function $R(\Delta t)$.
- Sometimes get flavour tag wrong – get a factor $(1-2w)$ where w is the mistag fraction.

$$Prob(mix\,or\,not) = \frac{\Gamma}{2} e^{-\Gamma\Delta t}\left(1 \pm (1-2w)\cos(\Delta m_B \Delta t) \otimes R(\Delta t)\right)$$

NB: $\otimes$ is the convoluted symbol.



BABAR 2001:

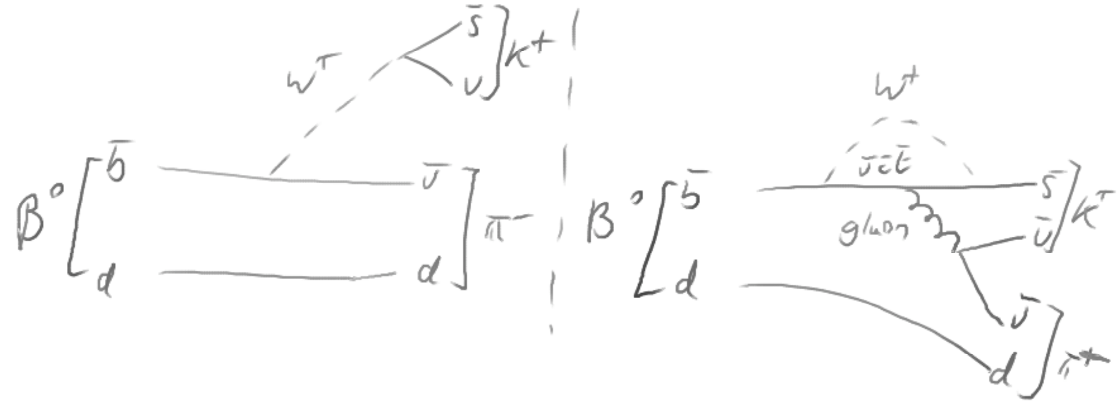
$$\Delta m_B = (0.499 \pm 0.010 \pm 0.015) \times 10^{12} \hbar s^{-1}$$

First error is statistical, second is the systematic error.

<u>CP violation in B-mesons</u>

There are three types:

- CP violation in mixing (indirect CP violation)
  - Mass eigenstates cannot be chosen to be CP eigenstates
  - E.g. Kaon system $K_1 \neq K_S$, $K_2 \neq K_L$, so $K_L \to 2\pi$ seen (first observation of CP violation)
  - Can only occur in neutral mesons.
- CP violation in decay (direct CP violation)
  - $\Gamma\left(B^0 \to f\right) \neq \Gamma\left(\bar{B}^0 \to \bar{f}\right)$, where f and $\bar{f}$ are final states.
  - Can occur in charged or neutral mesons.
  - Happens when a decay can take place through more than one Feynman diagram → interference.
  - E.g. $B \to K\pi$



First is "Tree" diagram; second is "Penguin" diagram.

- CP violation in interference between decays with and without mixing
  - Both B^0 and $\bar{B}^0$ can decay to final state f_{CP}, which is a CP eigenstates.

    $B^0 \to f_{cp}$

    $B^0 \to \bar{B}^0 \to f_{cp}$

    Interference can lead to time-dependent CP violation.
  - Only occurs in neutral mesons.

Can define quantity λ:

$$\lambda = \eta_{f_{cp}} \left(\frac{q}{p}\right)\left(\frac{\bar{A}}{A}\right)$$

$\eta_{f_{cp}}$ is the CP eigenvalues of final state f_{cp}. $= \pm 1$

$\left(\frac{q}{p}\right)$ is related to B-mixing. E.g. $\left|B_L\right\rangle = q\left|B^0\right\rangle + p\left|\bar{B}^0\right\rangle$

$\left(\frac{\bar{A}}{A}\right)$ are the decay amplitudes for $\bar{B}^0$ and B^0 to decay to f_{cp}.

- If $\left|\frac{q}{p}\right| \neq 1$ → CP violation in mixing.

- If $\left|\frac{\bar{A}}{A}\right| \neq 1$ → CP violation in decay
- If $\mathrm{Im}(\lambda) \neq 0$ → CP violation in interference between decays with and without mixing.

At BABAR, measure time dependent asymmetry.

$$A_{f_{cp}}(\Delta t) = \frac{\Gamma\left(B^0(\Delta t) \to f_{cp}\right) - \Gamma\left(\bar{B}^0(\Delta t) \to f_{cp}\right)}{\Gamma\left(B^0(\Delta t) \to f_{cp}\right) + \Gamma\left(\bar{B}^0(\Delta t) \to f_{cp}\right)}$$

In terms of λ, this is:

$$A_{f_{cp}}(\Delta t) = \frac{\left(1 - |\lambda|^2\right)\cos(\Delta m_B \Delta t) - 2\,\mathrm{Im}(\lambda)\sin(\Delta m_B \Delta t)}{1 + |\lambda|^2}$$

For decays where $|\lambda| = 1$ this reduces to:

$$A_{f_{cp}}(\Delta t) = -\mathrm{Im}(\lambda)\sin(\Delta m_B \Delta t)$$