4. Opacity

The equation of radiative energy transport is written as:

$$\ell = \frac{16\pi acG}{3}\frac{T^4(r)}{P(r)\kappa}m(r)\nabla$$

(from equation 68)
where κ is the collisional cross section per unit mass. The mean free path s is:

$$s = \frac{1}{\rho\kappa}$$

The optical depth τ is:

$$d\tau = -\kappa\rho dr$$

4.1 Sources of Opacity

- *Bound-Bound Transitions*
  Line absorption from bound electrons
- *Bound-Free Transitions*
  Continuum absorption from ionizing transitions
- *Free-Free Transitions*
  Equivalent to Bremsstrahlung
- *Electron Scattering*
  Thomson Scattering which depends on:
    - Composition X, Y and Z – mass fractions in hydrogen, helium and 'metals'.
    - Level populations of different species (bf, bb)
    - Ionization state (bf, ff)
    - Number density of electrons

4.2 Relations

For full ionization, ignore bb and bf.
Approximate relations for continuum opacity:

$$\kappa_{ff} = 4\times10^{21}(X+Y)(1+X)\rho T^{-7/2}$$

$$\kappa_{es} = 0.02(1+X) \quad (119)$$

in units of m^2kg^{-1}
The first is known as Kramers law opacity.

4.3 Frequency Dependence

κ is an average value for the frequency dependent opacity κ_v.

$$\left(\kappa = \frac{\int_v \kappa(v)F(v)}{\int F(v)dv}\right)$$

$$\frac{1}{\kappa} = \frac{\pi}{4\sigma T^3}\int_0^\infty \frac{1}{\kappa_v}\frac{\partial B_v(T)}{\partial T}dv \quad (120)$$

κ is called the Rosseland mean opacity.