7. Supernova Remnants & The Hot ISM
DW 7.3 – 7.4

7.1 Supernovae
Two distinct types of explosions.

1) White dwarf in a binary system accretes matter from its' companion.
When the white dwarf mass exceeds the Chandrasekhar limit $\left(\sim 1.4 M_{\odot}\right)$, star implodes. → type Ia supernova.
It corresponds to the end of medium mass stars, with lifetimes of billions of years.
→ occur randomly throughout all types of galaxies.

2) Massive (O or B star) exhausts nuclear fuel and can no longer support itself against collapse → star implodes.
→ Type II or Ib, Ic, Id according to exact type of star.
End of the short (few million years) life of OB stars.
→ occur only in regions of active star formation.
All types of supernova (SNe) inject $E_* = 10^{43} \rightarrow 10^{44} J$ into the ISM.
Expanding shock region of ISM is called a Supernova Remnant (SNR).

7.2 Expansion Phases
7.2.1 Phase I – Free Expansion
Initial energy is in the form of KE of the ejecta.

$$E_* = \frac{1}{2} M v_0^{\,2}$$

Take $E \approx 10^{44} J$, $M \sim 1 M_{\odot} = 2 \times 10^{30} kg$.
→ $v_0 \approx 10^7 ms^{-1}$ $\left(10{,}000 km\, s^{-1}\right)$

- Really an average velocity.
- Associated shock is not cooling – it's adiabatic (too fast).

Initially ejecta are much denser than ISM, so expansion is unimpeded. Effect on the ISM is like a supersonic piston.
(7.2.1-1)
CD is the ejecta / ISM interface "contact discontinuity".
The reverse shock, i.e. that from the ejecta bouncing off the CD, is negligible in phase I.
Since the ISM is compressed by a fixed factor of 4, in theory:
Shock radius / contact discontinuity radius $= \dfrac{R_{shock}}{R_{CD}} = const \approx 1.1$
In reality:

- Explosions may be quite asymmetric.
- CD is unstable → mixing ejecta with the shocked ISM.

Eventually, you find that the mass of gas that has been swept up $= \dfrac{4\pi}{3} R_{shock}^{\;3} \rho_0$ will become as big as the mass that has been ejected in the first place. When the swept-up mass exceeds the original ejecta mass, then the expansion begins to slow down seriously.

7.2.2 Phase II – Taylor-Sedov Expansion
When the swept-up mass >> the mass of the ejecta, we can ignore the fact that some of the shocked gas was originally in the star. The shock is still very strong at this point, so P_0 is negligible compared to the ram pressure. The only parameters in the problem are the initial energy of the star, E_*, and the density of the gas that is being shocked, ρ_0. We essentially have instantaneous energy release at one point in a medium of finite density and zero pressure.

The solution to this problem is technically known as a blast wave, and was worked out by the English physicist G.I. Taylor in the 1950's. Sedov did the work at around the same time, but was declassified in 1959. This work was originally done in the context of the nuclear bomb.

From these two parameters alone, you can't come up with a characteristic length. All you know is how far it has expanded in a certain period of time. However, the flow must be self-similar i.e. the pattern is the same at all times apart from a scale factor. So work it out for one specific time / length, then just scale. "If you've seen one explosion, you've seen them all."

By dimensional analysis, we can work out the characteristic radius at time t:

$$R \propto E_*^{\,2} \rho_0^{\,2} t^2$$

$$[L] = \left[ML^2T^{-2}\right]^a \left[ML^{-3}\right]^b [T]^c$$

Mass: $0 = a + b$, so $b - a$

Time: $0 = c - 2a$, so $c = 2a$

Length: $1 = 2a - 3b$, so $1 = a(2-3)$ → $a = \frac{1}{5}$.

$$R = (const)\left(\frac{E_*}{\rho_0}\right)^{1/5} t^{2/3}$$

$$V_s = \frac{dR}{dt} \propto t^{-3/5}$$, so decelerating as predicted.

Implicitly we have assumed that no, or negligible, energy is lost through radiative processes. This is known as the "energy conserving" phase. This is reasonable as long as the shock velocity is fast enough that no significant cooling happens behind it. This corresponds to a shock velocity of $V_s \geq 300\,km\,s^{-1}$.

To estimate the constant, use the shock jump conditions. (this was done nicer by Sedov, but we haven't got the time to go through that. Instead, "copying" from DW.)

Behind the strong shock;

$$E_k \text{ per unit mass } = e_k = \frac{1}{2}u_1^{\,2} = \frac{1}{2}\left(\frac{3}{4}V_s\right)^2 = \frac{9}{32}V_s^{\,2}$$

$$\text{The thermal energy per unit mass } = e_T = \frac{3}{2}nkT = \frac{3}{2}\frac{P_1}{\rho_1} = \frac{3}{2}\frac{\left(\frac{3}{4}\rho_0 V_s^{\,2}\right)}{4\rho_0} = \frac{9}{32}V_s^{\,2}$$

i.e. $e_k = e_T$.

The total mass of the shocked gas is just $M = \frac{4\pi R_{shock}{}^2}{3}\rho_0$.

We assume that the whole region has a post-shock energy (this is a fudge – it's good enough to get a rough answer, but is not at all accurate – the shock is weakening, and there are pressure forces inside the region which are changing the velocity, and hence energy. It is not correct to assume that e_k is constant. $e_k = 0$ at $r = 0$. But then, most gas is at the outside of the shock – not in the center.)

$$E_* = E_{tot} = M\left(e_k + e_T\right) = \frac{3\pi}{4}\rho_0 R_{shock}{}^3\left(\frac{dR_{shock}}{dt}\right)^2$$

This is another first order differential equation. So:

$$\int_0^{R_{shock}} R^{3/2} dR = \sqrt{\frac{4}{3\pi}\frac{E_*}{\rho_0}}\int_0^t dt$$

$$\frac{2}{5}R^{5/2} = \sqrt{\frac{4}{3\pi}\frac{E_*}{\rho_0}}t$$

So:

$$R_{sh}{}^5 = \frac{25}{4\pi}\frac{E_*}{\rho_0}t^2$$

Taking the fifth root, and using

$$R = \left(const\right)\left(\frac{E_*}{\rho_0}\right)^{1/5} t^{2/3},$$

we find that

$$const = \left(\frac{25}{3\pi}\right)^{1/5} = 1.2 .$$

In reality, the ISM density is non-uniform. So the shock advances at different speeds in different directions depending on the densities.

7.2.3 Phase III – Snowplough

When the shock velocity falls below around $V_s \le 400 kms^{-1}$, cooling becomes significant. So the energy of the system is not conserved. In a cooling shock, swept-up gas piles up in a thin shell, hence "snowplough".

Well into this phase, almost all of the shocked gas is in the shell. i.e. the gas that was heated to $> 10^6 k$ in phase II (which can't cool) is now a negligible fraction.

Then: expansion is dominated by the momentum of the shell, i.e.:

$$MV_s = const = \frac{4\pi}{3}R_s{}^3\rho_0\frac{dR}{dt}$$

i.e. at very late times, $t \propto R^4$ i.e. $R \propto t^{1/4}$. This gives a very rapid deceleration compared to phase II.

Cooling is mainly via optical lines which occur in a very thin region just behind the shock. The shock front isn't completely smooth in practice; it has ripples and dimples on it. This gives a very characteristic, filamentary nebula when seen from nearly edge on.

7.3 The Hot Ionized Medium

Phase II SNR shock-heat large volumes of gas to more than $10^6 k$. Cooling time is ~ $10^9 \to 10^{10} yrs$, so stays hot for a very long time. This gas seems to fill a substantial fraction of the ISM. $\sim 30 \to 60\%$ by volume. It is called the HIM, or coronal gas. It is detected via:

1) X-ray emission
2) Absorption lines in stellar spectra from highly ionized atoms e.g. O*VI*.

The density is very low, so is a negligible fraction by mass. We know that most of the gas is in molecular and cold atomic phases, but even in the ionized component (which is dominated by the warm phase $10^4 k$) it is negligible.