Large scale smoothness
Universe averages out on scales $10-100MPc$ $\left(1pc = 3.26ly\right)$

- Galaxy redshift surveys *(Picture 13)*
- Radio source surveys *(Picture 14)*
  These tend to find objects further away than current galaxy surveys
  Looks like a very smooth distribution. So highly isotropic: same in all directions.
- Cosmic (Microwave) Radio Background *(Pictures 15, 16, 17)*
  15: black body spectrum implies thermodynamic equilibrium at some point (or close to).
  16 top: CMBR uniform to 1%.
  16 middle: shows the dipole at 0.1% due to Doppler effect of the Earth's motion at $100's\,kms^{-1}$ with respect to the CMBR.
  16 bottom: Dipole subtracted. Now looking at parts in $10^4 \rightarrow 10^5$. The red line is microwaves from the Milky Way.
  17: Latest satellite picture (WMAP). Deviations from uniformity $\sim 10^{-5}$ (after galaxy subtracted).
  → the universe is very isotropic.

Fundamental observer (co-moving observer)
This is an observer that moves along the flow (of the expanding universe). Therefore not moving with respect to local matter (which is carried along at the same speed).

NB: it's hard to do anything else.

Homogeneity & Isotropy
Isotropy: the universe looks statistically the same in all directions. So its properties are invariant under rotation.

Homogeneity: The universe looks statistically the same at any given time for observers at different places. So its properties are invariant under translation.

If the universe is isotropic about any point (as it is for us), and we assume cosmological principle (Copernican principle) (we are not in a special place), then it must be isotropic about all points. → homogenous.

Consider two observers, each of which can see out a distance d (i.e. a sphere of view). Let the two spheres overlap. In the overlap, both observers will see the same properties on average (e.g. average density). This argument can be extended to many spheres overlapping, and ensure that the universe has the same average properties everywhere.
(Isotropy + isotropy = homogeneity ?)

Strong assumption (now backed up by evidence): cosmological principle: the universe is the same everywhere. This greatly simplifies the subsequent analysis of the 1st order behaviour.

Cosmological Time

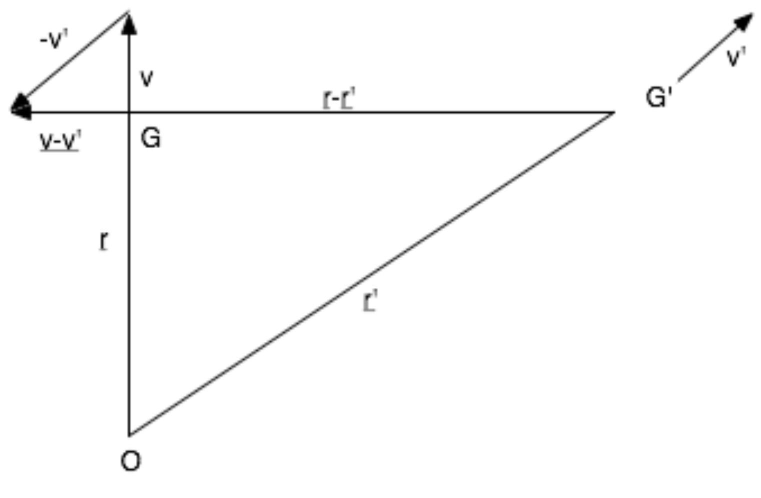
The universe appears the same to all fundamental (co-moving) observers at a given cosmological time (universe is not static – evolves). But because of the cosmological principle, these observers can "tell the time" by observing average properties, e.g. the density (averaged over $(100MPc)^3$, the current temperature of the CMB, …
i.e. all fundamental observers have the same (average) history.

The Expanding Universe
Edwin Hubble (following from Vesto Slipher 1912) obtained velocities via redshift; distances from special "standard candles" in the form of variable stars (Cepheid variables).
See pictures 4 (original), 18 (more modern data), 19 (cartoon – nearby galaxies moving slower)

Redshift is from a change in wavelength of spectral lines due to the source moving.
Redshift $Z = \frac{\lambda_{obs} - \lambda_{emitted}}{\lambda_{emitted}} = \frac{\Delta\lambda}{\lambda}$
This is a basic fact.
Locally, you can say that this is Doppler shift, and is non-relativistic.
$Z = \frac{\Delta\lambda}{\lambda} = \frac{v_{gal}}{c}$, or $v_{gal} = cZ$.
This is not true for "higher velocities" / greater distances.

From picture 19 – does this mean that we must be at the centre of the expansion? No. If the universe is expanding "self-similarly", then all observers see a Hubble "law" (really just a observable effect).

Consider 3 galaxies: O, G and G'.



As seen from O (us):
Hubble's law implies that:
$\underline{v}' \propto \underline{r}'$.
$\underline{v} \propto \underline{r}$
i.e. (where H_0 is the Hubble constant).
$\underline{v} = H_0 \underline{r}$
$\underline{v}' = H_0 \underline{r}'$

Then, as seen from G': (i.e. different location)
(with $\underline{v} = H\underline{r}$, $\underline{v}' = H\underline{r}'$)
$\underline{v} - \underline{v}' = H\left(\underline{r} - \underline{r}'\right)$
So we get the same constant, and the same effect, as seen from O.

This does not work (triangle of velocities matching nicely as above) if there is a non-linear relation between $\underline{v}$ and $\underline{r}$, e.g. $\underline{v} \propto \underline{r}^2$ or $\underline{v} \propto \sqrt{\underline{r}}$
(Take a 3, 4, 5 triangle in $\underline{r}$, and try to construct a right-angled triangle in v as we did above: can't be done).

So **Hubble's law ↔ Cosmological Principle**
All observers see the same thing.
(see picture 20: dots on a balloon)

What does this expansion mean?

1. Space itself is expanding and carrying matter / energy (galaxies, radiation, …) along with it.
   This is a General Relativity concept: space itself is malleable, and can be warped and stretched.
2. Objects on small scales (atoms – galaxies) are held together by strong internal forces and resist the expansion. → only concerns matter on large scales $\geq 10\,Mpc$.
3. Space can expand at $>> c$ (GR). Special Relativity refers to gravitation in free space ('flat") not in the context of stretching space. → SR is a "local" theory concerning passage of information between local observers. (see Liddle p21, sections 3.2-3)

Dynamics of the Universe
We now have enough information to start worrying about the dynamics of the universe. We will neglect GR (which is pretty much just a more complicated view of gravity), and just use Newtonian gravity to develop basic equations. A full GR view will be dealt with in the 4th year "Gravity" course.

Start with Newton's results (true for GR: Birchoff's theorem). Take a spherical distribution of matter, and consider a point within, which lies on an internal sphere. Only the matter within the internal sphere will matter; all the external matter will cancel out in terms of forces. The interior matter acts as if it were all concentrated at the centre. (arises from the inverse square law – similar result in electrostatics).

→ apply to universe: can assume an infinite spherical distribution, or just an infinite distribution.

Take a sphere on the scales of $> 10\,Mpc$ (to average properties out), and mass $M = \frac{4}{3}\pi r^3 \rho$ (1). Have a test mass m a distance r from the centre of the sphere. The only effect comes from M . The rest of the universe cancels out (statistically – assuming constant ρ …)

The Friedmann Equation

First derived by a Russian mathematician, Alexander Friedmann, using GR. Originally using a static universe, later the cosmological constant was neglected.

Test mass m .

$$F = \frac{GMm}{r^2} = \frac{4\pi}{3} G\rho r m \, .$$

We really want the gravitational potential energy…

$$E_p = -\frac{GMm}{r} = -\frac{4\pi}{3} G\rho r^2 m$$

In general, m will be moving (the universe is expanding).

$$E_k = \frac{1}{2} m\dot{r}^2$$

Conserving total energy:

$$E = E_p + E_k = const.$$

$$= \frac{1}{2} m\dot{r}^2 - \frac{4}{3}\pi G\rho r^2 m \qquad (2)$$

So E will tell us about “the fate of the universe”. If E_k dominates, it expands forever. If E_p dominates, it collapses back.