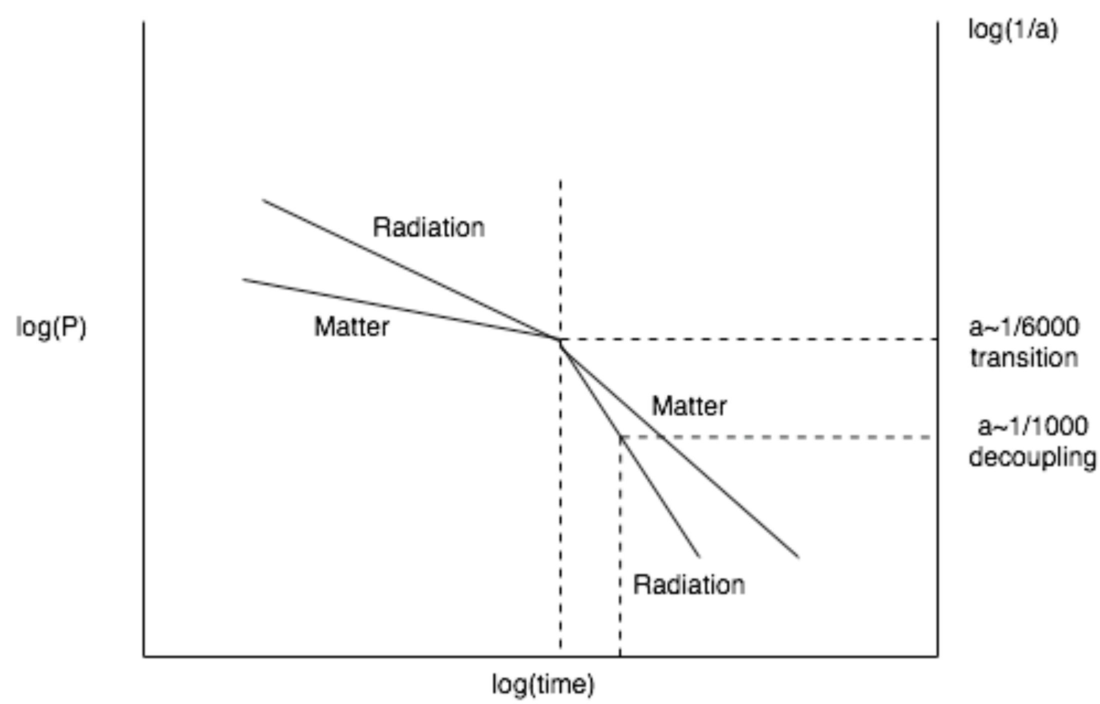
Summarizing Thermal History back to 1 second.

$t(\text{sec})$	$T_{RAD}(k)$ $\propto t^{-1/2}$ radiation era $\propto t^{-2/3}$ matter era
1	1.5×10^{10}
10^2 Density = water See Ex 2.5	10^9 Nucleosynthesis era. "first 3 minutes": conditions similar to in stars.
10^4	10^8
10^6	10^7
10^8	10^6
10^{10}	10^5
10^{12}	10^4
$\sim 2\times10^{12}$ (~60,000 years)	$\sim 10^4$ Transition to matter domination.
$\sim 10^{13}$ (~350,000 years)	3×10^3 "decoupling" and "recombination".

Matter and radiation now move independently.

Lecture pictures 41, 42, 43
Also Liddle Table 11.1.
Sketch full solutions to Friedman equation including both matter and radiation in ρ:

http://www.netlibrary.com/



The surface of last scattering is increasing into an area which, as far as we know, is infinite. Photons move off from the edge of the surface of last scattering, in all directions (at redshift at A on handout). Some photons start towards us, but they take a time E to reach us against the "tide of expansion".

Galaxies begin to form at $Z = 10$ (position C on handout). Completely formed at redshift 3 (position D). At E, $Z = 0$ (now), and the photons that set off at A $(Z = 1000)$ now reach us. Photons are also arriving from other galaxies at around $Z = 3$.

NB: as the universe is infinite, there is theoretically an infinite amount of the black body photons – so we won't be running out of them any time soon (until the surface of last scattering reaches infinity).

Horizon in Cosmology
At decoupling universe, ~350,000 years old.
→ last time matter + radiation in thermal equilibrium – influenced by material only within a light travel time.
→ the horizon was 350,000 years across.

Universe stretched by ~ 1000 since then → horizon now 350×10^6 light years across.

Problem: this is much smaller than the scale of the observable universe
$\left(\sim 15 \times 10^9 \, light\ years\right)$.
→ how can CMBR be so isotropic? Regions on opposite sides of the sky have not had enough time to interact and reach mutual equilibrium!
(solution in later lectures)

Appearance of the sky at different temperature levels
$2.725k$: completely smooth at $< 1\%$ level.

$2.725 / 10^3$: "dipole" due to Earth's velocity with respect to the CMBR photons: non-relativistic Doppler effect:
$$\frac{v}{c} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta\nu}{\nu} = \frac{\Delta T}{T}$$
→ $v_{earth} \sim 600 km\ \mathrm{s}^{-1}$

$2.725 / 10^6$: Galaxy radiation – radio version of the Milky way – has to be subtracted.

After all corrections to observed maps applied…
- See tiny fluctuations in temperature
- CMBR shows Anisotropsies
- → Cosmological principle not perfect!

CMBR Isotropies

- Temperature fluctuations (ΔT)
- Very small, like a pea on top of Everest!
  $\frac{\Delta T}{T} = 10^{-5} \to 10^{-6}$
- Satellite and ground based observations can now map in great detail.
  (NB: oval shape of maps is just a flat projection of the celestial sphere)

Wayne Hu: background.chicago.edu

Anisotropy power spectrum

- To analyse the maps a statistical approach is used.
- Compare temperatures at different distances apart in the sky
- Analyse in "spherical harmonics"
- Plot as "how often a given temperature difference is found, as a function of separation angle"
- Power vs. "multipole" (spatial frequency)

CMB analysis Formalism

If T_0 is the mean CMB temperature,

$$\frac{\Delta T(\theta,\phi)}{T} = \frac{T(\theta,\phi) - T_0}{T_0}$$

This can be expressed as a sum over spherical harmonics.

$$\frac{\Delta T(\theta,\phi)}{T} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_\ell^{\,m} Y_\ell^{\,m}(\theta,\phi)$$

The $a_\ell^{\,m}$ are "multipole moments".

$\ell = 0$ is the mean temperature.

$\ell = 1$ "dipole" anisotropy

$\ell = 2$ "quadrupole" moment

etc.

ℓ is inversely related to the angular scale by $\theta \sim \frac{60}{\ell}$ degrees, or ℓ^{-1} radians.

(NB: math is the same as for solutions of the Schrödinger equation in polar coordinates for hydrogen atom)

C_ℓ's

The angular power spectrum of the temperature fluctuations is:

$$C_\ell \equiv \left\langle \left| a_\lambda^{\,m} \right|^2 \right\rangle$$

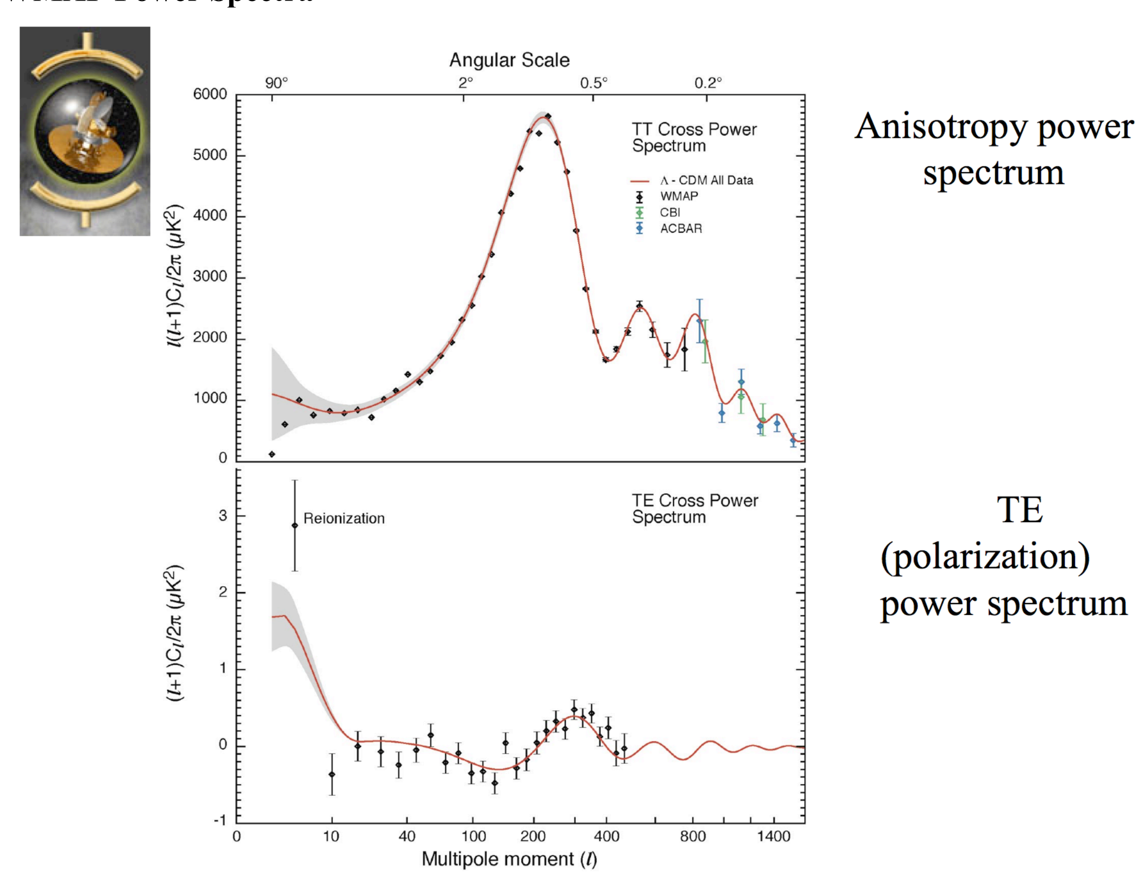
$$C_\ell^{-1} \propto \ell(\ell+1)$$

And $\ell(\ell+1)C_\ell$ should be constant at low ℓ.

This is often what is plotted as a function of ℓ.

If the fluctuations obey Gaussian statistics (which, from WMAP results, looks to be the case), each a_ℓ^m is independent and the power spectrum provides a complete statistical description of the temperature anisotropies.

WMAP Power Spectra



Harmonics – an analogy

- The CMB power spectrum is plotted as a function of spatial harmonic number (spatial wavenumber)
- Analogy to musical note, which is also a sum of harmonics (in time, rather than in space).

Why hot and cold spots in maps?

- Some CMB photons must "climb" out of areas of high density.
- General Relativity → gravitational redshift
- Loose energy → lower frequency $(\Delta E = h\Delta\nu)$
- That direction then appears colder than average.
- Known as the Sachs-Wolfe effect.

This is the reason that causes the large scale structure; there are other reasons also.

Why multiple peaks in the statistical power spectrum?

- Fluctuations from very early universe (see later lectures). (quantum fluctuations from when the universe was very small).
- In high density regions, increased gravity compresses photon-baryon fluid.
- Photon pressure resists the compression
- "Acoustic oscillations" set up.
  Over densities → hot peaks
  Under densities → cold peaks

"Hills and Springs" – cartoon
- Hills and valleys caused by gravity
- Spring represent fluid pressure.

(From Wayne Hu)
Oscillations can occur on many different scales. The question is, which is the most important scale?

"Frozen" oscillations
- At decoupling, the state of the oscillations is "frozen in" as the photons begin to free-fly.
  → spatial inhomogeneity becomes angular anisotropy in CMBR
- Greatest contrast for those regions caught at extreme of oscillations
- To get the sizes of the hot and cold regions, we need to consider the "sound-horizon".

"Sound horizon in CMBR"
- Distance that a wave can travel before decoupling
- Speed of sound depends on the density of the photon-baryon fluid, but is $\sim \frac{c}{\sqrt{3}}$ for a highly relativistic fluid.
- Angular size of the sound horizon, i.e. the position of the first peak, indicates the density of the universe (see later).

Surface of last scattering: $\frac{350 \times 10^6}{\sqrt{3}}$ (opposite on triangle). Distance 13×10^9 light years (hypotenuse and adjacent on triangle. → angle gives around 1 degree.

Extrema → peaks
- First peak: the mode that compresses just once (→ hot)
- Second peak: the mode that compresses then rarefies (→ cold)
- Third peak: compression-rarefaction-compression (hot)
- Pure harmonic series in wavenumber (i.e. periods per radian)

  $k_1 = \frac{\pi}{(sound - horizon)}$

  $k_2 = 2k_1$

  Etc. → many modes.

The Power of the Spectrum

- Existence of multiple peaks confirms the acoustic nature of fluctuations – first predicted by Sakharov in 1960's
- Detailed spectrum sensitive to:
    - Density of the universe (and hence its geometry)
    - Proportions of baryonic matter and Dark Matter
    - … plus many other parameters – outside present scope. (About 16 parameters in most cosmologies, not all independence. Degeneracies lifted by comparison with other measurements)

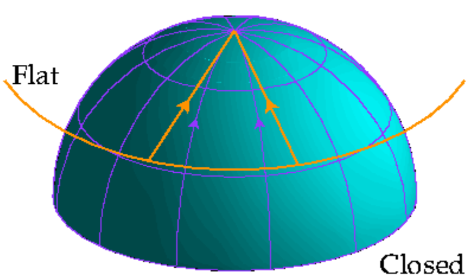
First Peak

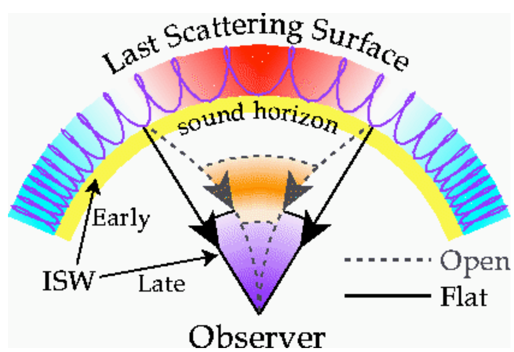
- Position now well determined. $\ell \sim 220$ (around 1 degree).
- Gives geometry of universe
- Geometry appears to be "flat" (Euclidian)
    - If it had been at higher ℓ (smaller angular scales) → negative curvature (Open geometry; we would see smaller angular scales that is actually the case)
    - If it had been at lower ℓ (larger angular scales) → positive curvature
    - (Closed geometry: "things look bigger than they really are")

Curvature

Objects in a closed universe are further away than they appear and the opposite in an open universe.

Flat universe indicates critical density (and implies missing energy – see later).





Second peak

- Baryon / photon ratio
    - Barons drag fluid into potential wells
    - More barons increase the height of compressional (odd-numbered) peaks, decrease the rarefactions (even numbered).
- Measuring ratio of first two peak heights allows baryon density (0.024 ± 0.001) to be determined.

(Effect of varying the density in baryons on the anisotropy power spectrum – Hu)

Nature and density of dark matter

- Putting more of the density into the dark matter reduces the baryon fraction, and hence reduces the peaks.
- Nature of dark matter affects power spectrum.
- E.g. Hot dark matter decreases peaks by smoothing out density condensations. (it is relativistic, so doesn't see the potential wells).