(Lectures 13-14)
From last time:

- Equivalent mass density $= \alpha T^4 / c^2$
  $\sim 2.5 \times 10^{-5} h^{-2} \times \rho_{crit}$
  $\left(\Omega_{rad} \sim 2.5 \times 10^{-5} h^{-2}\right)$
- $<< \Omega_{baryon}$
- Black body spectrum stays Black Body as universe expands. $E_{photon} \sim 3k_B T$ .

**Photon to Baryon Ratio**
Since the interactions between CMBR and baryons are negligible (true after the "first three minutes"

$$\varepsilon_{rad}\left(t_0\right) \sim 4 \times 10^{-14}\, Jm^{-3} \left(\varepsilon = \frac{\alpha T^4}{c^2}\right).$$

$E_{tyical\ photon} \sim 3k_B T \sim 7 \times 10^{-4}\, eV / c^2$
→ converting to SI.
→ the number of photons $n_{photons} \sim 3.7 \times 10^8\, m^{-3}$ . "Lots".
But the present density from baryons is a few percent of $\rho_{crit}$ . Earlier, we said that this was equivalent to a few tenths of a baryon per metre cubed.
Therefore the number of photons / the number of baryons $\sim 2 \times 10^9$ . This is true now, and in the past. This will be important when we consider the very early universe and the matter / antimatter asymmetry.

**Origin of the CMBR**
Run the universe backwards to scalefactor $a \sim 10^{-6}$ (a million times more compact, and less stretched) than now.
Temperature $T \propto \frac{1}{a(t)}$ (see earlier). So $T \sim 3 \times 10^6 k$ (since $T_0 \sim 3k$ ).
The matter in the universe would be highly ionized. Hydrogen ionization potential is $13.6eV$ . At $3 \times 10^6 k$ , the energy available (through $k_B T$ ) is around 60 times this.
The universe has very strong interactions between its constituents $\left(p^+, e^-, \gamma\right)$.
Via Compton scattering. Incoming photon [as a particle] hits an electron, displacing the electron as well as the path of the photon.
Later, when energies fall so $h\nu_{photon} << m_e c^2$ , Thompson scattering occurs (the classical equivalent of Compton scattering when the photon can be treated as an EM wave).
EM field incoming. Accelerates electron up and down. The electron then re-radiates EM waves.
Electrons strongly interact with the protons by Coulomb forces. → free electrons "act as glue" between the photons and the protons (baryons). → universe is an almost perfect fluid in thermodynamic equilibrium (black body conditions).
These conditions are not so weird – it is very similar to a stellar interiors or supernovae.

Run the universe forward again. $a$ up, so $T$ down. Eventually, it cools so that electrons and protons can be combined to form hydrogen atoms in the ground state.

This stage is called "Recombination", although more accurately it would be Combination.

So the electrons are bound in quantum mechanical "embrace" with the protons. So they no longer interact strongly with the photons. So scattering process essentially stop. Photons then move freely for the rest of the history of the universe. This is called "Decoupling".

The universe previously was opaque (black body radiation), and now becomes transparent. So the CMBR we see now is a "fossil record" of the state of the universe at recombination and decoupling time.

**Thermal History of the Universe**
- Pick some interesting times to do calculations.

1. Decoupling / combination. We want the time, temperature, and the stretch factor. At the peak of the black body curve, typical photons have an energy of $\sim 3k_BT$.
$\left(k_B = 8.6\times10^{-5}\,eV\,k^{-1}\right)$.
→ $13.6eV \equiv 50{,}000k$ → everything certainly ionized.
However, there are $2\times10^{9}$ photons per baryon. We only need one photon with $E > 13.6eV$ to ionize each $e^- + p^+$ pair. So at much lower temperatures than 50,000k, the photons in the "tail" can "do the job".
At $T \sim 3{,}000k$, the universe just becomes transparent. NB: this is about half the temperature of the solar outer atmosphere.
Since $T \propto a^{-1}$, $a \sim 1/1000$ of the current size.

- Fluid $\left(\gamma, p^+, e^-\right)$ interacting.
- Combination.
- $H,\gamma$
- Decoupling
- H, $\gamma$ separate.

On this, T and Z go down, while t and a go up.
Combination, and decoupling, happen at roughly the same time.
Z at decoupling, $T \propto (1+Z)$ → $Z \approx 1000$.

CMBR is direct evidence of a "hot big bang".

**Surface of Last Scattering**
When we look out into space, we see CMBR photons coming from directions from where they were last scattered off ionizing material.
There we ions and electrons all around us – look back to large enough distances → far back in time (c is finite).
Can define a last scattering surface, i.e. a distant shell in space closer than which the universe was transparent – further away the universe was opaque.

Radiation just arriving now at Earth has been traveling uninterrupted since decoupling.

Picture the universe as a flat sheet. Expanded >>> speed of light. Photons take time (13,000 million years) to reach us after the universe has expanded by $\approx 1000$ times since decoupling.
(Picture photons as crawling ants on an expanding flat surface).

*(Picture 40)*

Phonons are stretched (red shifted) as the universe expands. $T \sim 3000k$, now $\sim 3k$. $a$ was $(1000)^{-1}$. Now, $a = 1$.

In the future, the universe will have stretched further, so the surface of last scattering will be further away. The surface of last stretching is just a locus of points in space where photons arriving now started out.

Since we believe the universe to be infinite, it could be hypothesized that there are lots of other areas outside our surface of last scattering, which each have their own limit to the distance that they can see. So each can see a bit of the overall universe, but would not know of each other's existence – light has not had enough time to travel between them (the universe can have expand much faster than c).

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Keep going back into the earlier universe. The relativistic particles in the universe are photons and neutrinos. They behave in the same way. Calculations show that the number of neutrinos in the universe should be approximately the same as the number of photons. So $\Omega_{relativistic} \sim 2\Omega_{radiation} \sim 4 \times 10^{-5} h^{-2}$. This is $<< \Omega_{matter}$, including dark matter, which is around 0.3. This implies that now,

$$\frac{\Omega_{matter}}{\Omega_{relativistic}} \approx 7500h^{-2}$$

($h^{-2} \approx 2$ since $H \sim 70 km\, s^{-1}\, Mpc^{-1} \rightarrow h \sim 0.7 \rightarrow h^2 = 0.5$)
Recall that $\rho_{rel} \propto a^{-4}$, and $\rho_{matter} \propto a^{-3}$. So

$$\frac{\rho_{rel}(t)}{\rho_{matter}(t)} \propto \frac{1}{a(t)}.$$

So as a function of time,

$$\frac{\Omega_{relativistic}}{\Omega_{matter}} \approx \frac{1}{7500h^{-2}} \frac{1}{a}$$

Now set up to calculate the full thermal history as a function of time.

$$T \propto \frac{1}{a(t)}$$

$a_{matter} \propto t^{2/3} \rightarrow T \propto t^{-2/3}$ for the matter-dominated era.

$a_{relativistic} \propto t^{1/2} \rightarrow T \propto t^{-1/2}$ for the radiation-dominated era.

Go back to combination / decoupling. $T = 3000k$, $a = 1/1000$. So at decoupling,

$$\frac{\Omega_{relativistic}}{\Omega_{matter}} \sim 0.13h^{-2}$$

This means that the universe was dominated by matter at decoupling (not hugely, however).

So since $T \propto t^{-2/3}$ in this era, $\frac{T}{2.725k} = \left(\frac{12 \times 10^9 \, years}{t}\right)^{2/3}$. The age given is now quite the currently accepted age – we haven't yet taken the $\Lambda$ term into account. Therefore for $T = 3000k$, $t = 350{,}000 \, years$, or around $10^{13}$ seconds.

Go back a bit further. Transition to radiation dominated era.
i.e. putting $\frac{\Omega_{relativistic}}{\Omega_{matter}} = 1$. So $a = \frac{1}{7500h^2} = 0.13 \times 10^{-3} h^{-2}$ (cf. $10^{-3}$ at decoupling Therefore factor of 7 smaller).
Apply the same arguments as above, which are OK back to the time of decoupling.
$T_{transition} = \frac{2.725}{0.13 \times 10^{-3} h^{-2}} \approx 20{,}000 h^2 k \approx 10{,}000k$.
$t_{transition} \approx 2 \times 10^{12} \sec \approx 60{,}000 \, years$.

Conditions at 1 second: now in the radiation-dominated era. $\frac{T}{T_{transition}} = \left(\frac{t_{transition}}{t}\right)^{1/2}$.
Substituting $T_{trans} \sim 10^4 k$ from above, and $t_{transition} \sim 2 \times 10^{12} s$, we get

$$\frac{T}{10^4} = \left(\frac{2 \times 10^{12}}{1}\right)^{1/2}$$

$$T_{1\sec} \sim 1.5 \times 10^{10} k$$

Note that you can get the same answer by the use of the Freidman equation. Problems 2, number 2.5 – also calculate for $t = 100s$.

*(Picture 41)*