(Lectures 11-12)
Strong limits on the number of baryons in the universe from the theory of nucleosynthesis in "the first 3 minutes".

(NB: another v. god little book, "The First 3 Minutes", by Steven Weinberg).

All light elements, $^1H, ^1D, ^3He, ^4He, ^6Li$ formed at the Big Bang, of which ~75% was ^1H , 25% $^3He, ^4He$, and only little bits of the rest.

Detailed amounts of light elements can be predicted from "Big Bang Nucleosynthesis" (BBN) calculations. It depends on the baryon density in 1^{st} 3 minutes after the big bang event.

So we can predict from known physics that the current light element abundances is $\Omega_{baryon} \sim 0.05$, i.e. around 5% of ρ_{crit}. So there are not enough Baryons in the theory, so most of dark matter must be "non-baryonic".

Summary of the mass budget:

- Luminous Baryonic (from stars via their light, which implies their mass) $\leq 1\%$ of ρ_{crit}
- Dark baryonic from MACHOS searches and BBN calculation $\leq 5\%$
- Dark matter from galaxy rotation curves, cluster virial masses, cluster gravitational Lensing. $\sim 25\%$

This gives a total of 30% of ρ_{crit}. So $\Omega_{matter} \sim 0.3$ ("normally gravitating stuff"). This means that the universe should be open.

So what is the non-Baryonic dark matter?

i. Neutrinos (we know these exist).
   3 types: e, μ, τ.
   In the first 3 minutes (see later), the same number of neutrinos as photons should have been created. If the neutrino had average masses of a few 10's of eV / c^2, we would get ρ_{crit}.
   BUT:
   a. The latest results support much smaller average masses, $\leq 1eV / c^2$.
   b. Neutrinos being light are relativistic. They are called "hot dark matter".
      → they are not bound into gravitational potential wells / don't naturally form into clumps. They spread out into a uniform background. So it is unlikely that they are the explanation for the observed clumped dark matter.

ii. WIMPS
   These are Weakly Interacting Massive ParticleS, and are the most likely suspect. They are suggested by extensions of the standard model (supersymmetry). They are weakly interacting. Around 10^6 could be passing through a fingernail per second. They are heavy, $> 100GeV / c^2$ →

slow, non-relativistic, and hence can form clumps (dark matter halo's) early on in the universe. In these gravitational potential wells, the "ordinary" baryons gather to form galaxies, stars, etc.
(Picture 37)
No-one has found them yet, but sophisticated searches are underway.

iii. It could be something completely unexpected.

Recapitulate the Friedman Equation, and its' consequences.

$$E_K + E_P = const = \frac{2U}{mx^2} \equiv -\frac{kc^2}{a^2}$$

$E_k : \left(\frac{\dot{a}}{a}\right)^2 = H^2$, Hubble Parameter.

$E_p : -\frac{8\pi G\rho}{3}$, where ρ is from all constituents.

$\frac{2U}{mx^2} \equiv -\frac{kc^2}{a^2}$ is the total energy, which is equivalent to curvature in GR. It looks like it could well be 0, i.e. the universe is just "critical".
Only 2 out of the 3 terms are independent.
The E_k is from Hubble expansion → H^2.
The E_P is from the inventory of matter constituents (light and dark) using a range of techniques → Ω (with respect to ρ_{crit}).
E_P / E_K from the deceleration parameter, the curvature of the Hubble plot at large distances *(Picture 26)*.
E_P / E_K from age estimates ($\sim H_0^{-1}$ for no gravity; numerical factor of 2/3 for $\rho = \rho_{crit}$, and different for other ρ).
$E_K + E_P$ from the curvature k via observations of the distant universe compared with local e.g. counts of galaxies, variation in large scale structure, or intrinsic properties of the CMBR (see later).

(End of Dynamics for the moment)

Aside: Olber's Paradox.

- Brief but profound aside.
- Why is the sky dark at night?
- This places limits on the properties of the universe.

The point is that the brightness (number of photons per second, i.e. the Planck black body) of stellar disk does not change with distance. It is just the area of the disk as seen proportional to r^{-2}, where r is the distance (another way of saying the inverse square law).
In an infinite universe uniformly populated with stars, all lines of sight should eventually end up on the surface of a star.
→ we should be "cooked in a 5,000k oven".

There are two parts to the resolution of the paradox:

1. The universe is expanding: it is not static, as was implicitly assumed.

2. The universe is not infinitely old; it was "born" a finite time ago.

It is not due to dust. Dust would just heat up until it was the same temperature as a stellar surface, or would disassociate into its' constituent gases / compounds / elements, which would glow at T_{star}.

Effect of expansion:

$$\frac{\lambda_{obs}}{\lambda_{emit}} = (1+Z) = \frac{a(t_0)}{a(t_{emitted})}$$

$E = \frac{hc}{\lambda}$, so photons have energy reduced by $(1+Z)$ (the energy is being stretched out; it is not loosing energy), and Doppler timescales to receive photons is increasing $(1+Z)$ as space stretches.

→ rate of energy reception $\propto (1+Z)^{-2}$ (this is over and above the inverse square law, which has already been taken into account via the area being proportional to r^{-2}).

This could be a big effect, but detailed calculations show that it reduces sky brightness by only factors of a few because effect 2 cuts in more quickly.

Effect of a finite age

Generally in physics, we know that the mean free path is $\lambda = (n\sigma)^{-1}$. Here, n is the space density of stars, and σ is the cross-sectional area of interaction, $\pi R_{star}{}^2$. So how long does a photon have to travel before it hits a star?

Typical distance to stars in our galaxy is 1-2 parsecs. →

$$n \le \frac{1}{(3\times10^{16})^3} \sim 3\times10^{-50}\, m^{-3}.$$

What is $\sigma = \pi R_{star}{}^2$?

$R_{sun} = 7\times10^8\, m$.

→ $\lambda \ge 10^{15}\, pc$ ($3\times10^{15}\, ly$).

This is a "wild underestimate", as the distance between galaxies greatly increases this..

A better estimate is $n \sim 10^{-58}\, m^{-3}$, which gives $\lambda \ge 10^{23}\, pc$ $(3\times10^{23}\, ly)$.

We noted that $H_0{}^{-1}$, the Hubble Time, was ~10^{10} years.

So there are not enough stars / galaxies to fill the sky. Or our universe is not old enough to have filled up with radiation $\equiv 5000k$ uniform density of photons.

Cosmic Microwave Background Radiation (CMBR)

This is the main reason why we call it the "hot big bang".

(Pictures 15, 16, 17)

Features are almost a perfect black body spectrum. $T = 2.725 \pm 0.01k$. It is uniform to parts in 10^5 when the effect of the earth's motion and the radiation from the galaxy (milky way) has been subtracted.

→ best diagnostic of the Big Bang, and of the future development of the universe.

Energy density in the CMBR

Black body radiation the energy density $\varepsilon \propto T^4$

(Stefan-Boltzmann Law: total power radiated from the surface of a black body $= \sigma T^4$ per unit area).

It turns out that $\varepsilon = aT^4$, per unit volume, where $a = \frac{4\sigma}{c} = 7.57 \times 10^{-16} J\,m^{-3}\,k^{-4}$.

What is ε is the CMBR now?

$\varepsilon(t_0) \approx 4.2 \times 10^{-14} Jm^{-3}$ for $T = 2.725k$.

What is the equivalent mass density?

$E = mc^2$

$\Omega_{CMBR} \approx 2.5 \times 10^{-5} h^2$

where h is the fudge factor in the Hubble parameter. $H = 100h\,km\,s^{-1}\,Mpc^{-1}$.

$h \sim 0.6.7$.

This is nowhere close to ρ_{crit}, but CMBR photons dominate the photon density in the universe. It is very much greater than the sum of the radiation from stars and quasars, etc. Also, the CMBR is still very much less than the visible baryon mass density (i.e. that in stars).

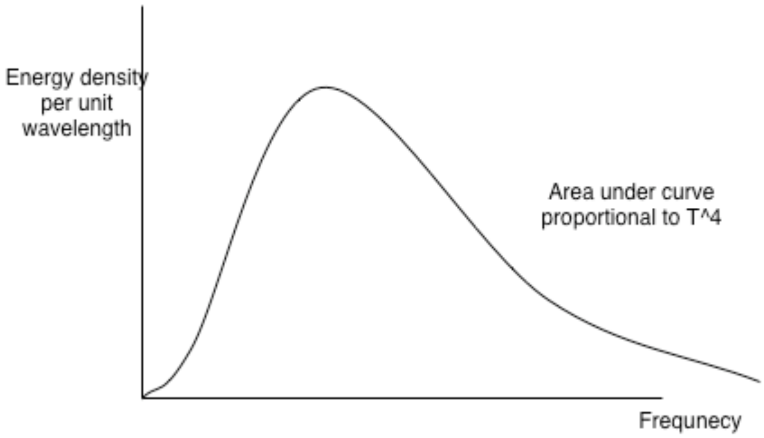
Recall from earlier that $\rho_{radiation} \propto a(t)^{-4}$. Therefore, since $\varepsilon \propto T^4$,

$$\boxed{T \propto a(t)^{-1}}$$

(or could argue $E = h\nu = \frac{hc}{\lambda}$, $\lambda \propto a(t)^{-1}$, $E \sim kT$).

In the past, the universe was much hotter.

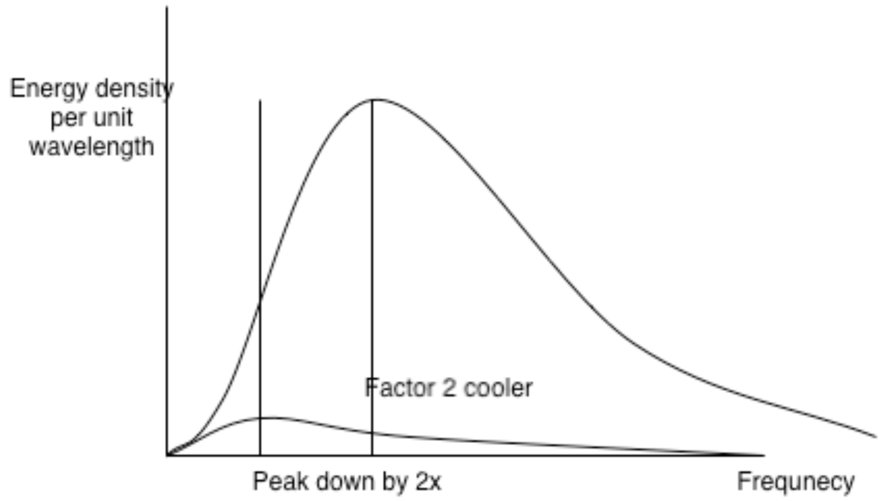
Points to remember / recall about black body radiation



$$\lambda_{peak} = \frac{2.9 \times 10^{-3}}{T} m$$

$$\nu_{peak} = \frac{2.8 k_B T}{h}$$

→ typical energy of a black body photon is $\sim 3k_B T \quad \left(E \approx h\nu_{peak}\right)$



ε down by $(2)^4 = 16$ items.

But the black body retains black body form.