5. Beta Decay

The process allows the nucleus to reach its most stable proton-neutron ratio.

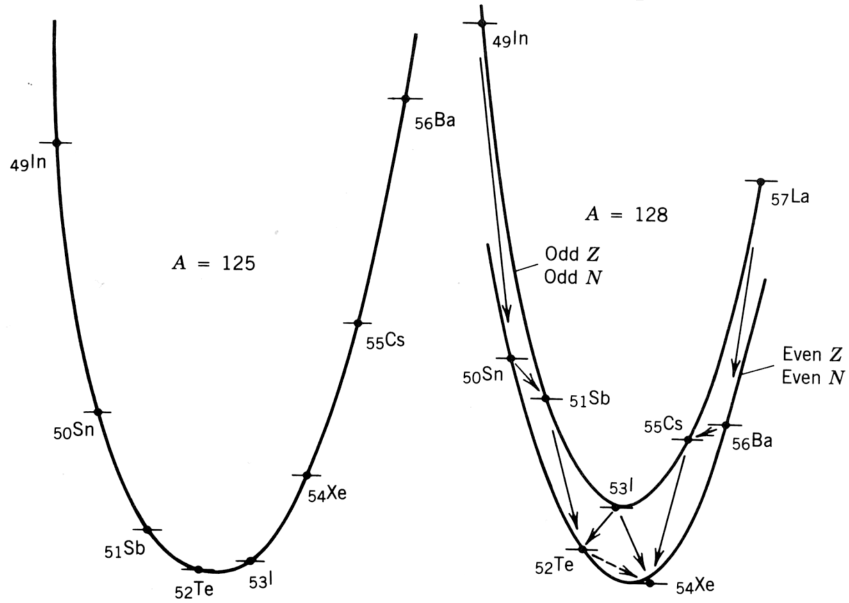


Figure 3.18 Mass chains for $A = 125$ and $A = 128$. For $A = 125$, note how the energy differences between neighboring isotopes increase as we go further from the stable member at the energy minimum. For $A = 128$, note the effect of the pairing term; in particular, ^{128}I can decay in either direction, and it is energetically possible for ^{128}Te to decay directly to ^{128}Xe by the process known as double β decay.

On the left side, β^- decay $n \to p + \beta^- + \bar{\nu}_e$. Q_{β^-}. On the right side, either β^+ or electron capture ε, with Q_ε. All particles have spin ½. The energy, momentum (and parity) is conserved in the decay.

5.1 Q Values

(Kinetic energy released)

$$Q_{\beta^-} = \left(M_{parent} - M_{daughter}\right)c^2$$

assuming that the neutrino has no mass (at the least, this is a v. good approximation)

$Q_{electron\,capture}$ is the same.

$$Q_{\beta^+} = Q_{electron\,capture} - 2m_e c^2$$

This is due to the decay products being the daughter atom + a spare e^-, plus a positron.

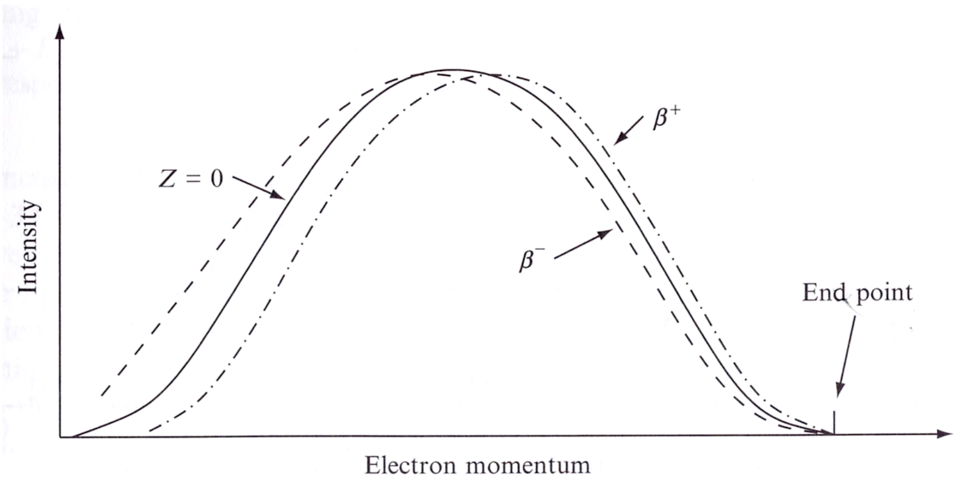
$2m_e c^2 = 1.022\,MeV$.

Q is shared between decay products consistent with momentum conservation.

Conservation of momentum means $\underline{P_m} + \underline{P_\beta} + \underline{P_\nu} = \underline{0}$.

$$Q_\beta = \underbrace{\frac{P_m^{\ 2}}{2M}}_{\substack{negligible \\ M >> m_e}} + \underbrace{E_K(\beta) + P_\nu c}_{relativistic}$$

The spectrum of the β-particle (kinetic energy of β) is continuous up to a maximum of Q_β.



End point is at Q_β.

The shape is determined by the density of the final states plus Coulomb effects.
The end point detail allows a determination of the neutrino mass. If the neutrino has mass, then the end point will be just before Q_β. The difference between the end point and Q_β will be $m_\nu c^2$.

Experiments on ${}^3H \rightarrow {}^3He + \beta^- + \nu_e$

$Q = 18,600$

Limits $m_\nu c^2 < 60eV$.

5.2 Fermi Theory of β Decay

The Fermi theory states that the decay rate λ is given by "Fermi's Golden Rule".

$$\lambda = \frac{2\pi}{\hbar}\left|\int \Psi_f * V_\beta \Psi_i dV\right|^2 \rho(E_F)$$

where $\rho(E_F)$ is the density of final states, and the integral is called the "overlap integral", and involves the β-decay operator connecting initial and final states. This is particularly large when the decaying nucleon keeps the same orbital / wavefunction – these are called "superallowed" transitions.

The shape of the electron kinetic energy spectrum (T_e) (before Coulomb correction) is found by considering the density of states $N(T_e)$ in an interval $T_e \rightarrow T_e + dT_e$. Krane shows:

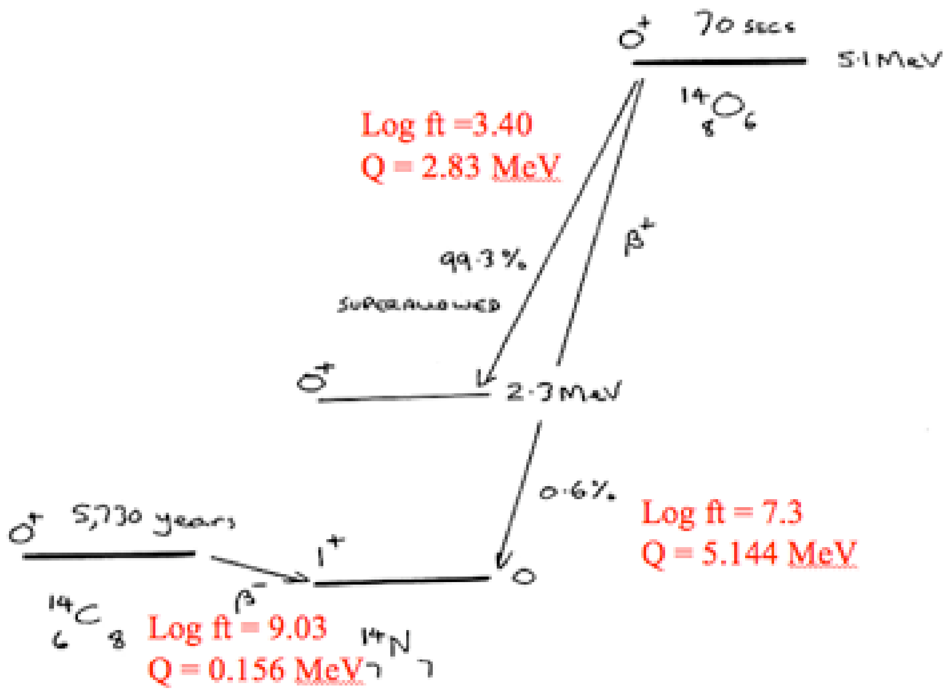
$$N(T_e) = const. \times \sqrt{T_e^2 + 2T_e m_e c^2}\left(Q_\beta - T_e\right)^2\left(T_e + m_e c^2\right)$$

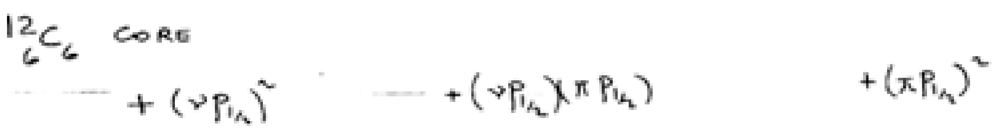
where $T_e + T_\nu = Q_\beta$.

This vanishes at $T_e = 0$, and at $T_e = Q_\beta$.

NB: if $m_e = 0$, $N(T_e) \rightarrow const \times T_e^2 T_\tau^2$.

Clearly β-decay has a strong Q_β dependence. Decay rate is roughly $\propto {Q_\beta}^5$





ft-values

If the energy dependence on the half-life is taken into account, then a comparative half life of "ft-value" can be deduced, which depends on the inverse of the matrix element $\left|\int \psi_f * V_\beta \psi_I dV\right|^2$. $\tau = \dfrac{1}{\lambda}$. Since ft values range from $10^3 \rightarrow 10^{20}\, s$, then $\log ft$ is normally quoted in literature. $\log ft = 3 \rightarrow 4$ for a superallowed transition.

5.3 Angular momentum rules

Remember alpha decay involving $\ell\hbar$: the α is emitted a distance $\dfrac{\ell\hbar}{\sqrt{2m_{\alpha E_\alpha}}}$ from the nuclear centre. Since $m_e, m_v << m_\alpha$, it is much more difficult for e^- or $\bar{v}_e$ to carry away *orbital* angular momentum. (If $E_\beta \approx 5 MeV$, then $r \approx 100\, fm$ for $\ell = 1$). We thus have these classifications:

$\ell = 0$ "allowed" decay
$\ell = 1$ "first forbidden" decay
$\ell = 2$ "second forbidden" decay.

"Forbidden" decays have much slower decay rates, hence longer half-lives.

Allowed Decays
$\ell = 0$, therefore there is no parity change between the initial and final states $\left((-1)^{\ell}\right)$.
There are two types of allowed decay depending on the total intrinsic spin carried away by the electron $\left(\underline{s} = \underline{1/2}\right)$ and the neutrino $\left(\underline{s} = \underline{1/2}\right)$.

Fermi type
$\underline{s} = \underline{1/2} + \underline{1/2} = \underline{0}$, i.e. opposite spin directions.
Angular momentum is conserved. $\underline{I_i} = \underline{I_f} + \underline{s} + \underline{\ell}$. Here, $\underline{\ell} = 0$, and $\underline{s} = 0$. So
$\Delta I = I_i - I_f = 0$.

Gamow-Teller (GT) Type
$\underline{s} = \underline{1/2} + \underline{1/2} = \underline{1}$, i.e. parallel spin directions.
Now the conservation of angular momentum leads to a vector triangle through
$\underline{I_i} = \underline{I_f} + \underline{s}$, where $\underline{s} = \underline{1}$.
$\Delta I = I_i - I_f = 0, \pm 1$.
But not $I_i = 0 \rightarrow I_f = 0$.

Classification of most fundamental process:
$n \rightarrow p + e^- + \overline{\nu}_e$
$I^{\pi}: \underline{\frac{1}{2}}^{+} \rightarrow \underline{\frac{1}{2}}^{+} + \underline{\frac{1}{2}} + \underline{\frac{1}{2}}$
So $\underline{s} = \underline{0}$ or $\underline{1}$ is possible. There is no parity change between the initial and final states.
This satisfies both Fermi and Gamow-Teller rules.
Therefore mixed transition. Experimentally, 18% Fermi, 82% G-T.

First Forbidden
$\ell = 1$ → parity change between the initial and final nuclear states. We now have:
$\underline{I_i} = \underline{I_f} + \underbrace{\underline{s}}_{=0,1} + \underbrace{\underline{\ell}}_{=1}$
Fermi 1st forbidden transition:
$s = 0$. $\Delta I = 0, \pm 1$ $(\ell = 1)$
G-T 1st forbitten transition:
$s = 1$. $\Delta I = 0, \pm 1, \pm 2$
plus parity change in both cases.

Examples of classifications
$^{14}O\left(0^+\right) \rightarrow {}^{14}N\left(0^+\right)$
Pure Fermi allowed (actually “superallowed”).
$^{14}O\left(0^+\right) \rightarrow {}^{14}N\left(1^+\right)$
Pure G-T allowed.

$^{115}In\left(\frac{9}{2}^{+}\right) \rightarrow {}^{115}Sn\left(\frac{1}{2}^{+}\right)$

$\Delta I = 4$, no parity change: $\ell = \text{even}$.
Therefore $\ell = 4$. "4th forbidden", hence why it is regarded as stable.
$T_{1/2}\left({}^{115}In\right) = 4 \times 10^{14}$ years.

If it were $9/2^{-} \rightarrow 1/2^{+}$, then it would be $\ell = 3$, $s = 1$. But it is not.

5.4 Parity Violation in β-decay

Most laws of physics are invariant under the parity operation ($\underline{r} \rightarrow -\underline{r}$ "a reflection through the origin", i.e. reversing x, y and z).
If the Hamiltonian $\left(E_k + E_p\right)$ is invariant under the parity operation, then the eigenstates have "good parity".
$\Psi(-\underline{r}) = \pm\psi(\underline{r})$, where $\pm$ is through the parity quantum number.

Parity operation on vectors
Position vector $\underline{r} \rightarrow -\underline{r}$.
Hence:
Velocity $\underline{v} \rightarrow -\underline{v}$ $(\dot{\underline{r}})$
Force $\underline{F} \rightarrow -\underline{F}$ $(m\ddot{\underline{r}})$
Electric field $\underline{E} \rightarrow -\underline{E}$ $\left(-\frac{dV}{dx}\right)$.
These are called "true" or polar vectors.
Angular momentum $\underline{I} \rightarrow +\underline{I}$ $\left(\underline{r} \times \underline{p}\right)$
Torque $\underline{\tau} \rightarrow +\underline{\tau}$ $\left(\underline{r} \times \underline{F}\right)$
Magnetic field $\underline{B} \rightarrow +\underline{B}$ $\left(I\,d\underline{\ell} \times \hat{\underline{r}}\right)$
These are called "pseudo" or 'axial" vectors.

^{60}Cs 5^{+} → (beta decay) 4^{+}, which then decays down to ^{60}Ni through gamma decay.
Nuclei are then polarized in B-field. It was found that there was a forwards-backwards asymmetry (more were emitted in the opposite direction to the magnetic field than in the same direction).
After a parity operation, the B-field and the polarity are in the same direction. However, the forwards-backwards asymmetry was reversed, i.e. more were emitted in the same direction than the opposite. This is not observed in nature, which means that nature has a handedness.
Hence it is possible to define "left" to an extraterrestrial civilization.
NB: parity of initial and final states is conserved here; the parity violation is in the physicality's of the resulting system.

The nuclear force itself must violate parity.

$V_{nuclear} = V_{strong} + V_{e/m} + V_{weak}$
The weak part of this is a very small part of the force, but it violates parity.
→ nuclear wave functions have a tiny contribution of the "wrong" parity $\left(10^{-7}\right)$ but in practice is hardly ever seen. This is generally extremely difficult to detect, except in some special circumstances.