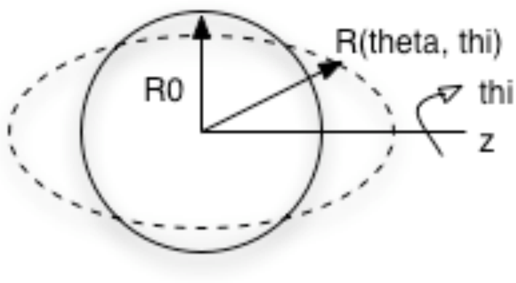
3. Collective Excitations

We return to the liquid drop model, and consider excitations involving the whole nucleus. In medium mass and heavy nuclei these excitations are often lower in energy than the single particle shell-like excitations. These states have common recognizable features occurring throughout chart of nuclides.

3.1 Nuclear Vibrations

We can describe a liquid drop vibrating about a spherical shape by radius vector $R(\theta,\phi)$ at time t.



$$R(\theta,\phi,t) = R_0\left(1+\beta_\ell \cos(\omega t) Y_{\ell,m}(\theta,\phi)\right)$$

Modes for $\ell = 1,2,3$ are shown in figure 5.18 (Krane).

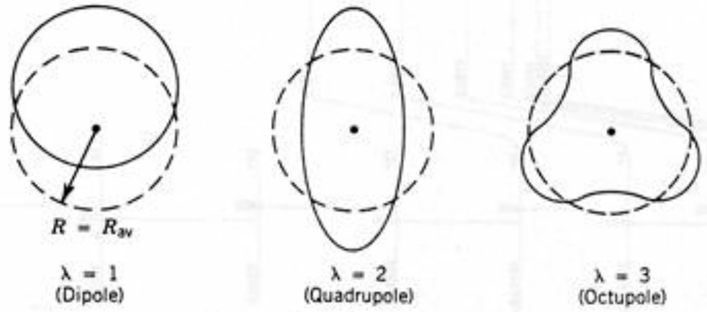


Figure 5.18 The lowest three vibrational modes of a nucleus. The drawings represent a slice through the midplane. The dashed lines show the spherical equilibrium shape and the solid lines show an instantaneous view of the vibrating surface.

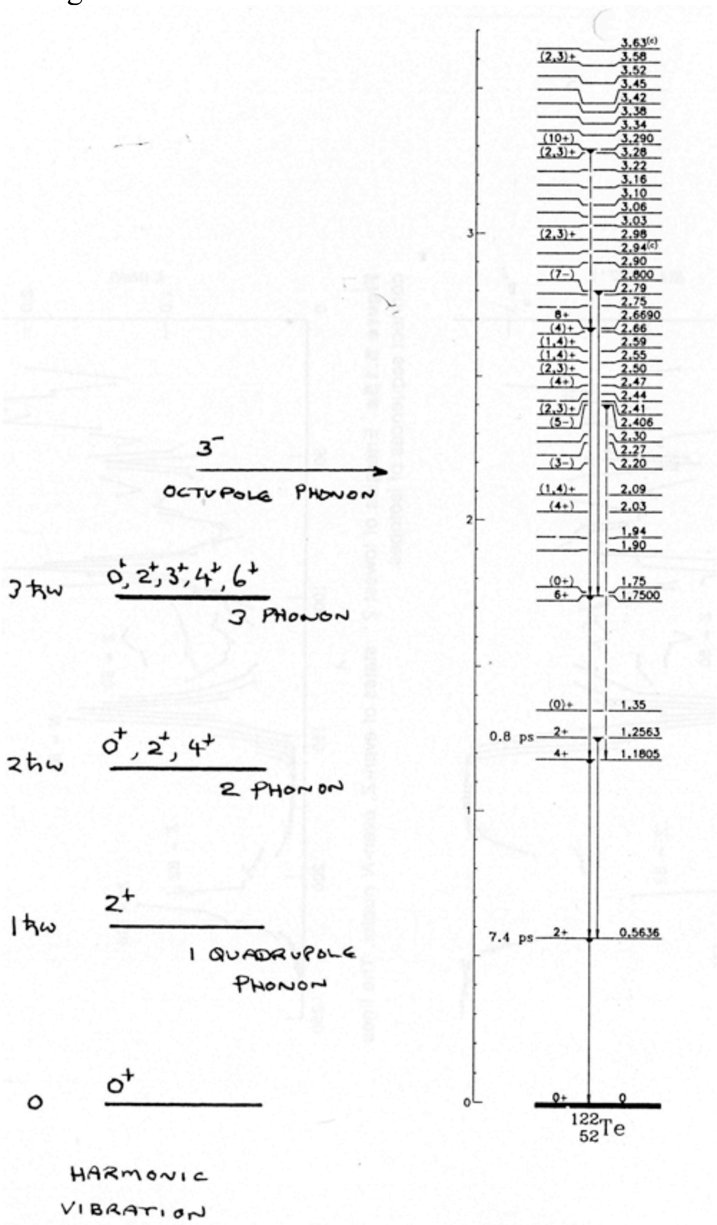
A quantum of vibration is called a Phonon. Adding a term $Y_{\ell,m}$ to the nuclear wavefunction introduces $\ell\hbar$ units of angular momentum, with a z-component m and parity $(-1)^\ell$. The vibrations carry angular momentum $\ell\hbar$, and they are bosons.

We have these possibilities:

- $\ell = 1$ dipole phonon $I^\pi = 1^-$ (spin ^ parity).
  This is not observed in the nucleus because it requires a net displacement of the centre of mass of an isolated system.
  (With high excitation, it is possible to get separation of the protons and neutrons of the nucleus, and balance them against each other. This won't be talked about in this course.)
- $\ell = 2$ quadrupole phonon. $I^\pi = 2^+$.
  Usually this is the first excited state of a spherical nucleus.

- $\ell = 3$ Octupole phonon. $I^\pi = 3^-$
  This is commonly seen at higher excitation.

Two-phonon states:
These should occur at about twice the energy of the one-phonon state.
For $\ell = 2$ there are $5 \times 5 = 25$ combinations of $(\ell, m)(\ell_2, m_2)$ ($m = -2, -1, 0, 1, 2$).
However, only 15 have the necessary symmetry (for bosons).
The 15 combinations are:
$\underline{2^+} + \underline{2^+} = \underline{4^+}, \underline{2^+}, \underline{0^+}$ (but no $\underline{1^+}, \underline{3^+}$).
$m + m = M$
For the $\underline{4^+}$ state; $M = +4, +3, ..., -4$ which gives 9 states.
For the $\underline{2^+}$ state, $M = +2, +1, 0, -1, +2$ which gives 5 states.
For the $\underline{0^+}$ state, $M = 0$, which gives 1 state.
This gives a total of 15 states.



By similar considerations, 3 phonon states can be constructed with allowed spins $\underline{6^+},\underline{4^+},\underline{3^+},\underline{2^+},\underline{0^+}$.

The best examples of vibrational nuclei are in the region between $n = 50$ and 82, with $A \approx 120$ (Z is near 50).

A good example is ${}^{122}_{52}Te$.

$E(2_1^+) \approx 500 - 1200 keV$.

Predictions:

$0^+, 2_2^+, 4^+$ at the same energy and $\approx 2 \times E(2_1^+)$ (roughly OK).

$0^+, 2_3^+, 3^+, 4_2^+, 6^+$ are at $3 \times E(2_1^+)$ (rarely OK).

$3\hbar\omega$ states start to overlap the region of shell model excited states → the number of possible states increases very rapidly with energy.

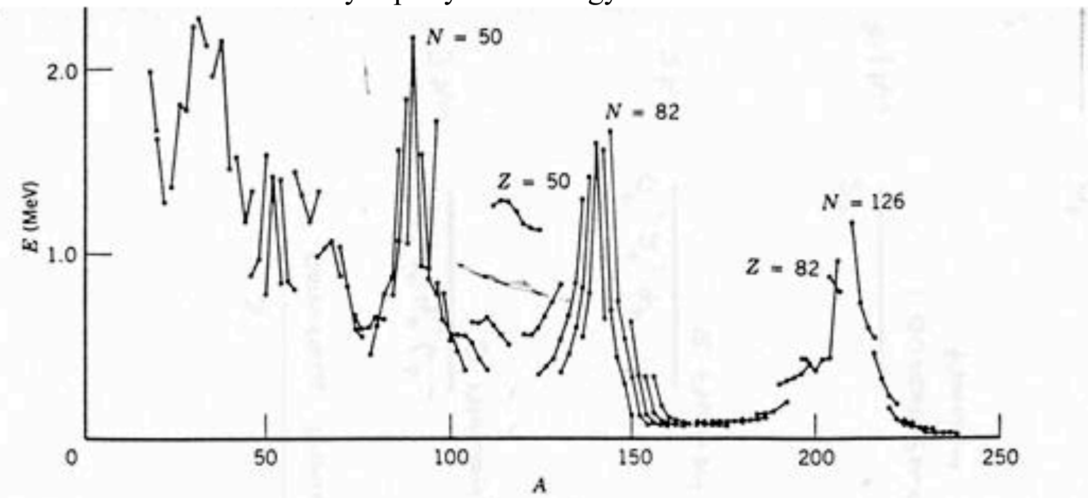


Figure 5.15a Energies of lowest 2^+ states of even-Z, even-N nuclei. The lines connect sequences of isotopes.

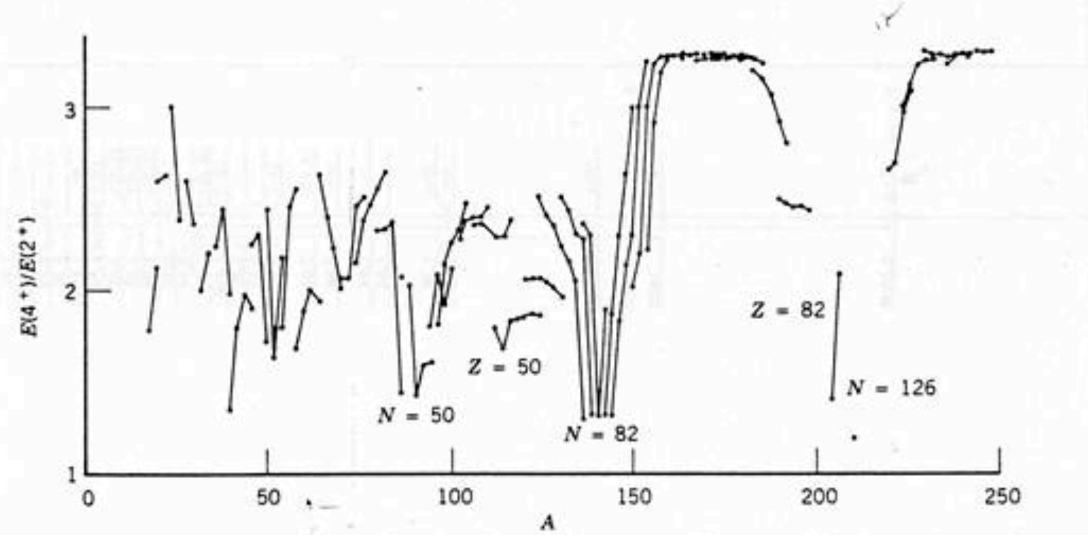


Figure 5.15b The ratio $E(4^+)\,/\,E(2^+)$ for the lowest 2^+ and 4^+ states of even-Z, even-N nuclei. The lines connect sequences of isotopes.

3.2 Nuclear Rotations

In some regions of the nuclear chart, when both Z and N lie away from shell closures, nuclei develop substantial stable distortions from spherical shape. For example, when $150 < A < 190$, and $220 < A < 300$.

The most common shape is rugby-ball like (axially symmetric, prolate).

$R(\theta,\phi) = R_0\left(1+\beta_2 Y_{2,0}(\theta,\phi)\right)$

$m=0$ means independent of ϕ. So $Y_{2,0} \propto P_2(\cos\theta)$.

With $\beta_2 \approx +0.3$. If β_2 is negative, this is the oblate Earth-shape.

For even nuclei (ground state = 0^+) this shape cannot be directly observed – the symmetry axis of the nucleus cannot be orientated in the lab for measurement – but laser spectroscopy does measure the increase in the $\langle r^2 \rangle$ of charge.

For a sphere of radius R_0, $\langle r^2 \rangle_{spherical} = \frac{3}{5}R_0^{\,2}$. For the same volume, but deformed,

$$\langle r^2 \rangle_{deformed} = \langle r^2 \rangle_{spherical}\left(1+\frac{5}{4\pi}\beta_2^{\,2}\right)$$

A deformed nucleus has a rotational degree of freedom, which can be treated classically. $KE = \frac{1}{2}\Im\omega^2$, where $\Im$ is the moment of inertia. The angular momentum $I\hbar$ is related to $\Im$ by $I\hbar = \Im\omega$.

So $E_k = \frac{(\Im\omega)^2}{2\Im} = \frac{\hbar^2 I(I+1)}{2\Im}$, a quantum mechanical rotor.

Even nuclei: ground state is always 0^+. Then (on symmetry grounds) the allowed rotational states are $2^+, 4^+, 6^+, 8^+, \ldots$ Prediction: if $\Im_{rigid} = \frac{2}{5}MR_0^{\,2}\left(1+0.31\beta_2\right)$, then $\frac{\hbar^2}{2\Im} \approx 6keV$ for a nucleus with $A=170$. So we expect $E\left(2_1^{\,+}\right) = 36keV$. (more like 90 – 100keV experimentally).

Second prediction: $E(4^+)/E(2^+) = 4(4+1)/2(2+1) = 20/6 = 3.\dot{3}$. This prediction is extremely good.

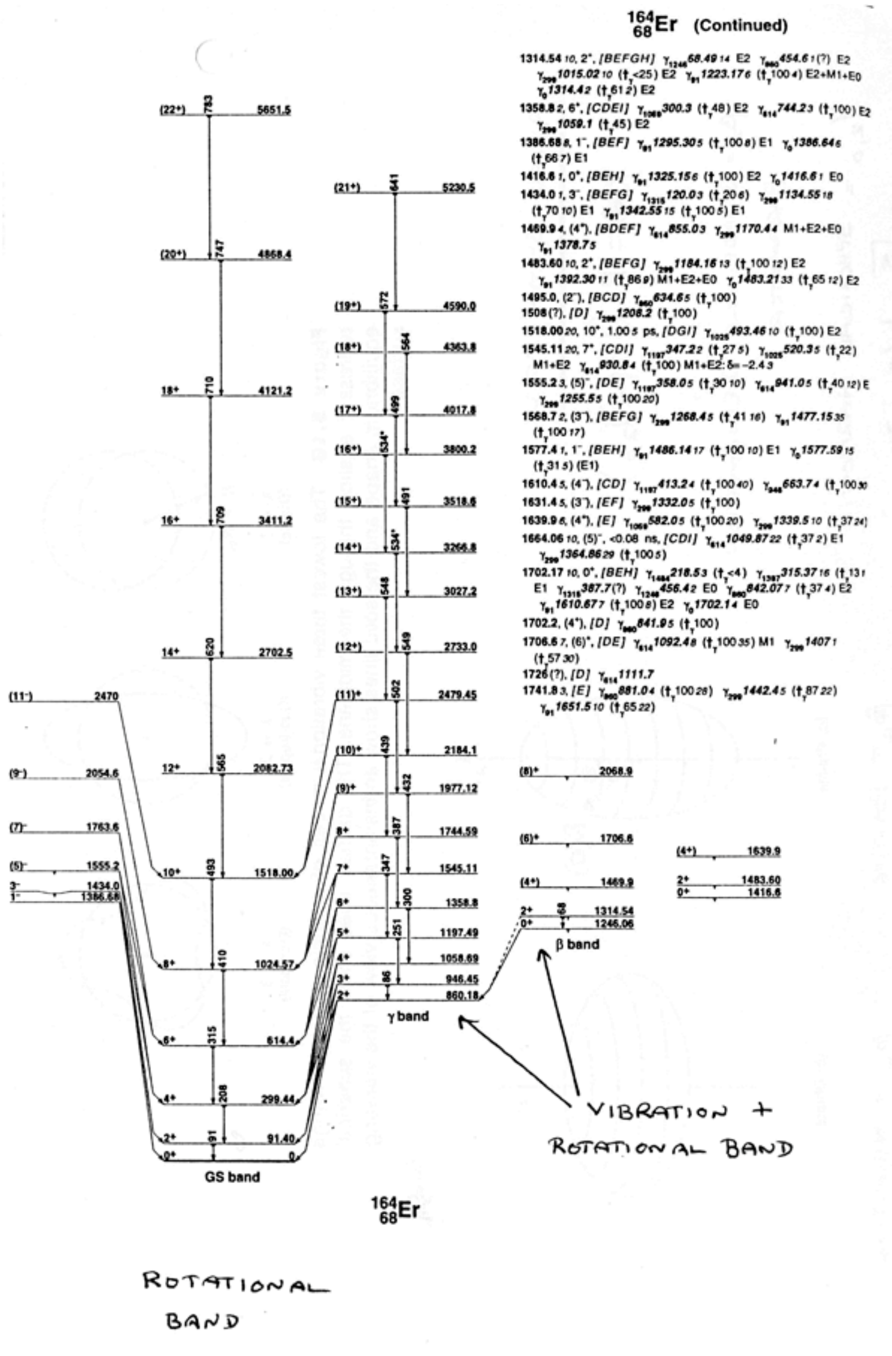
We have overestimated the moment of inertia. In fact, the completely filled lower shells of the nucleus cannot contribute to the angular momentum (all the m_j states are filled to give a spin and parity of 0^+ for the "core" of nucleus) and we should regard the rotational motion as due to the valence nucleons only.

Think of it having a core which does not move, and the valence nucleons form a "sea" on top of the core, which can rotate.

This leads to the other extreme, where we might regard the nucleus as a fluid inside an ellipsoidal vessel, where just the surface wave flows.

$\frac{\hbar^2}{2\Im} \approx 90keV$, which is much too high compared to experiments.

To conclude: $\Im_{fluid} < \Im_{nucleus} < \Im_{rigid}$.



Several rotational bands built on different states are seen.

- Ground state rotational band
- Two bands built on vibrations.

  β-vibration, which elongates the nucleus in the z-direction, so what would be a circle in the x-y plane becomes an ellipse. $I^{\pi} = 0^+, 2^+, 4^+, 6^+, \ldots$

  γ-vibration, where β_2 stays roughly constant (i.e. the length in the z-direction), but the x-y plane will become elliptical in either the x or y direction. $I^{\pi} = 2^+, 3^+, 4^+, \ldots$

Information about Rotations from Gamma Decay
Rotational bands in even-even nuclei de-excite by a cascade of γ -ray transitions between adjacent states.

We know that

$$E(I) = \frac{\hbar^2}{2\Im} I(I+1)$$

so the energies of the photons given off from each level will be

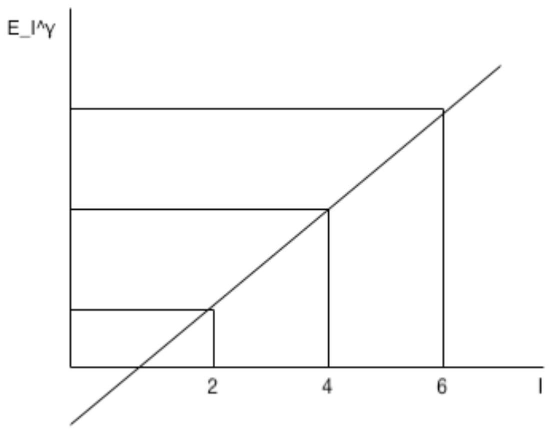
$$E_I^\gamma = E(I) - E(I-2)$$

$$= \frac{\hbar^2}{2\Im}\left[I(I+1) - (I-2)(I-1)\right]$$

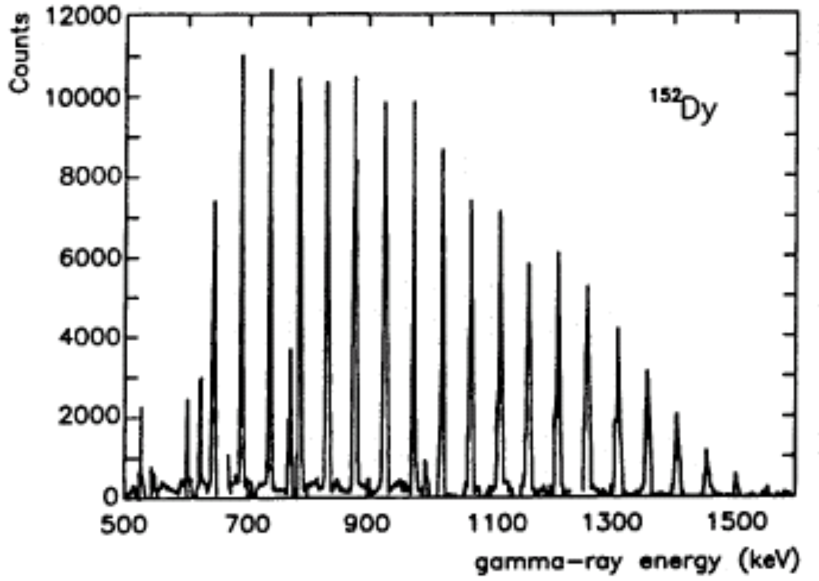
$$= \frac{\hbar^2}{2\Im}(4I-2)$$

Gamma ray energies should increase linearly with I as long as $\Im$ stays constant.

Example SD band in ^{152}Dy constant $\Im$.

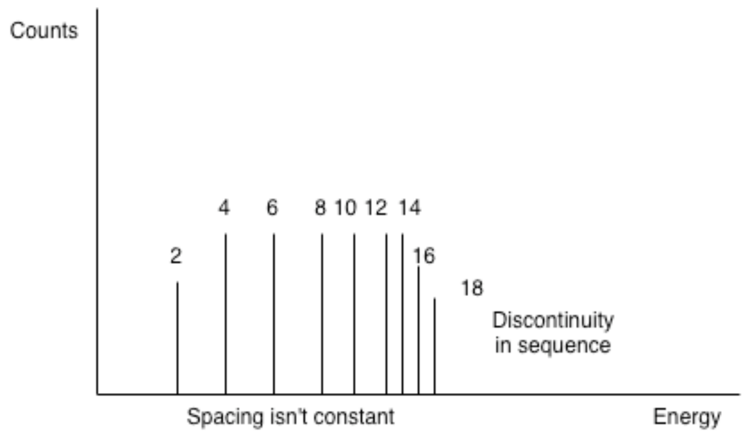


Constant spacing up the y-axis. Hence we get:

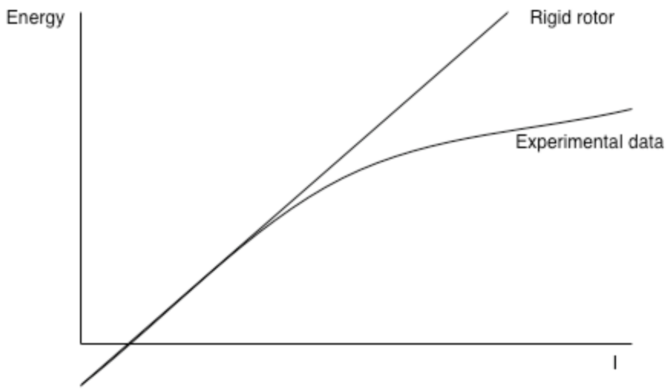


This is a rotational band for a "superdeformed" nuclear state, exhibiting a constant moment of inertia close to the "rigid" value.

Example: ground state band in ^{164}Er (which is typical)
γ -ray spectrum:



Above the discontinuity, the gaps become constant once more. The spacings above the discontinuity are less than below the discontinuity.



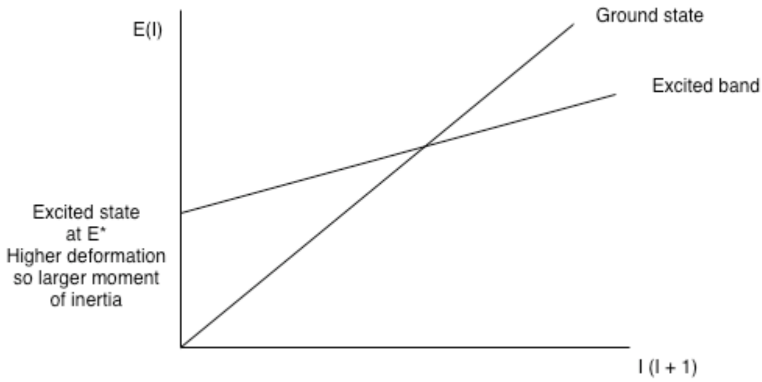
This appears consistent with an increasing $\Im$. It was originally interpreted as centrifugal stretching, but other explanations possible.

For the ground state: $E_I = \frac{\hbar^2}{2\Im} I(I+1)$

For the excited state: $E_I = E^* + \frac{\hbar^2}{2\Im_{excited}} I(I+1)$

$\Im_{excited} > \Im_{gs}$

γ -ray decays usually follow the sequence with the lowest energy for a particular spin. The technical term for this is the yrast line.



So we would follow the excited band down to the band crossing, where it meets the ground state band; from there on we would follow the ground state. The discontinuity in the gamma ray spectrum is due to the band crossing.